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```
// file: harmonic_oscillator.cpp
//
// Functions to generate normalized harmonic oscillator wave functions
//
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//
// Revision history:
// 01/24/04 original version, translated from harmonic_oscillator.c
//
// Notes:
// * The potential is  $V(r) = (1/2)m \omega^2 r^2$ .
// * The oscillator parameter  $b$  is related to  $m$  and  $\omega$  by
//    $\hbar\omega = \hbar^2/(m b^2)$ .
// * We use units with  $\hbar = 1$ .
// *  $b$  sets the length scale; so  $q \equiv r/b$  is the dimensionless
//   coordinate.
// * The oscillator state is specified by the radial quantum number
//    $n$  and the angular momentum quantum number  $l$ .
// * Conventions NOT the same as Fetter and Walecka section 57.
//   * Laguerre polynomials not normalized with cube
// * Normalization is:  $\int_0^\infty dr [u_{nl}(r)]^2 = 1$ 
// * Uses gsl library; compile and link with:
//   g++ -c harmonic_oscillator.c
//   g++ -o ... harmonic_oscillator.o -lgsl -lgslcblas -lm
//
// To do:
// * should check that  $n$  and  $l$  are non-negative
//
//*****

// include files
#include <cmath>

#include <gsl/gsl_math.h>
#include <gsl/gsl_sf_gamma.h>
#include <gsl/gsl_sf_laguerre.h>
#include <gsl/gsl_errno.h>

// function prototypes
double ho_radial (int n, int l, double b, double r);
double norm (int n, int l, double b);
double ho_eigenvalue (int n, int l, double b, double m);

//*****
//
// Calculate the eigenvalue of a harmonic oscillator radial function
//
// *  $\epsilon_{nl} = \hbar\omega (2(n-1) + 1 + 3/2)$ 
// *  $\hbar=1$  here
// *  $\hbar\omega \equiv \hbar^2/(m b^2)$ 
//
//*****
double
ho_eigenvalue (int n, int l, double b, double m)
{
    return ((2. * ((double) n - 1.) + (double) l + 3. / 2.) / (m * b * b));
}

//*****
//
// Calculate a normalized harmonic oscillator radial function
//
// * evaluate at position  $r$ 
// *  $u_{nl}(q) = N_{nl} q^{l+1} e^{-q^2/2} L^{l+1/2}_{n-1}(q^2)$ 
//
//*****
double
ho_radial (int n, int l, double b, double r)
{
```

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double q = r / b;
double qsq = q * q;
double a = (double) l + 1. / 2.;

return
    norm (n, l, b) * gsl_pow_int (q, (l + 1)) * exp (-qsq / 2.)
    * gsl_sf_laguerre_n ((n - 1), a, qsq);
}

//*****
//
// Normalization factor for a harmonic oscillator radial function
//
// *  $N_{nl} = 2(n-1)!/[b \Gamma(n+1/2)]$ 
// * verified by checking against Mathematica for different  $n, l, b$ 
//
//*****
double
norm (int n, int l, double b)
{
    double arg = (double) n + (double) l + 1. / 2.;

    return sqrt (2. * gsl_sf_fact ((unsigned) (n - 1)) /
                (b * gsl_sf_gamma (arg)));
}
```