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7701 Lecture 16

- Gameplan: Catch up on delta function / PS#5 material after brief examples of catching errors on exam. Remember: The goal is for everyone to get better and succeed.
- Strategies for finding errors:

① Make a personal checklist for different types of problems

eg. for vector/δ/ϵ problems:

$\vec{\nabla}\times(\vec{\nabla}\times\vec{v})$
$= \epsilon_{ijk}\epsilon_{kln}\partial_j\partial_l V_m$

a. Are free indices the same on both sides of the equation?

b. Do any repeated indices appear more than twice?

$\delta_{ad}B_d \rightarrow B_d$

c. Are free indices in summed terms the same? (cf. $A_i + B_j$)

- Think of how you would write a program (eg. Mathematica) to check your problem ⇒ concrete rules

- This is a good exercise: a paradigm for thinking scientifically
  - It's not that you will be a bad physicist if you don't master a particular set of math manipulations. We will only do a subset
  - But you need to be able to transfer procedures to other formal and experimental situations.
  - That is why memorizing is insufficient (or sub-optimal). We do need to know some lists of facts, but we want more.
  - Earlier I said that if you commit all mistakes you will be able to recognize them: the key is realizing that the error in a vector problem (for example) is the analog of an error elsewhere.

two ways to succeed:
a) memorize specifics of particular problems
b) recognize general principles and similarities / analogs
⇒ physics way (build intuition)

② Testing your answer.

- If your answer has particular characteristics (real or imaginary, positive or negative, specific dependence on parameters), look for ways to verify these characteristics.

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Problem 4: $I = \int_{-\infty}^{\infty} d\omega \, \frac{\omega e^{-i\omega t}}{\omega^2 + \omega_0^2} = \begin{cases} -\pi i \, e^{-\omega_0 t} & t > 0 \\ +\pi i \, e^{+\omega_0 t} & t < 0 \end{cases}$

a) Answer is purely imaginary — can this be correct?

$e^{-i\omega t} = \cos \omega t - i \sin \omega t$

$\frac{\omega \cos \omega t}{\omega^2 + \omega_0^2}$ is odd under $\omega \leftrightarrow -\omega \Rightarrow$ vanishes

$\Rightarrow I = -i \underbrace{\int_{-\infty}^{\infty} d\omega \, \frac{\omega \sin \omega t}{\omega^2 + \omega_0^2}}_{\text{real}} = -i * (\text{real number})$ ✓

b) What about the sign change? If $t \to -t$, $\sin \omega t$ changes sign. ✓

c) What about $\int_{-\infty}^{\infty} d\omega \frac{\omega \sin \omega t}{\omega^2 + \omega_0^2} > 0$ for $t > 0$?

More difficult because $\sin \omega t$ goes negative. But $\frac{\omega}{\omega^2 + \omega_0^2}$ puts more weight at smaller ω, so for at least some choices of ω_0 and t we can argue the positive part of sin dominates.

d) Does it make sense that the answer gets smaller as $|t| \to \infty$? Yes: more wiggles in $\sin \omega t$ so it averages out.

e) Does it make sense that the answer gets smaller as $\omega_0 \to \infty$? Yes: the integrand is smaller.

f) Can you think of more?

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(3) Use units!

- For problem 4, $w_0 t$ must be dimensionless and I must be dimensionless, So you know e^{w_0} or $\frac{1}{t}e^{-w_0 t}$ would be incorrect.

- In problem 6: $\frac{d}{dt}x(t) + \beta x(t) = g(t) \quad 0 < t < T_0$

$\Rightarrow$ β has units of $\frac{1}{[T]}$ (and g is in units where $v = \frac{x}{t}$ is 1)

So if your answer has $\frac{1}{\left(\frac{T_0}{2\pi n}\right)^2 + \beta^2}$ in it, the dimensions don't match in the denominators and you would suspect something like $\frac{1}{\left(\frac{2\pi n}{T_0}\right)^2 + \beta^2}$.

$\Rightarrow$ many, many more opportunities for units.

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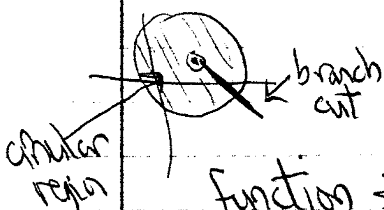
Other midterm comments

- A branch point doesn't have a Laurent expansion about it because that require the function to have an annular region (e.g. $a < |z - z_0| < b$ where z_0 is the expansion point) in which it is analytic. If we have a branch point, we have to introduce a branch cut to make the function single valued. This cut must go through any angular region and the function is not analytic on the cut.



- Leaving an answer in the form $(i)^{1/3}$ or $(-1)^{1/3}$ is insufficient because we need to know what branch of $z^{1/3}$ we are on. That is, with $z = re^{i\theta} e^{in2\pi}$ $(i)^{1/3}$ could be $(e^{i\pi/2})^{1/3} = e^{\pi i/6}$ or $(e^{i(\pi/2+2\pi)})^{1/3} = e^{5\pi i/6}$ or $e^{i(\pi/2+4\pi)} = e^{9\pi i/6}$.
Similarly with $-1 = e^{\pi i + 2\pi n i}$.

- The canonical example of an essential singularity is $e^{1/z}$. In the midterm, we had $\dfrac{1}{1-e^z}$ which is not $e^{1/z}$

Even if the 1 was missing, it would be $e^{-z} \neq e^{1/z}$.
Check by asking what happens as z becomes small:
$e^z \to 1 + z + \ldots$ while $e^{1/z}$ diverges very badly.

- If you rationalize a denominator and get (as in problem 6)

$$\frac{1}{\left(\frac{2\pi n}{T_0}\right)^2 - \beta^2} \Rightarrow \text{must be wrong.}$$

minus sign, so could be negative if β is large.

$$\frac{1}{x+iy} \cdot \frac{x-iy}{x-iy} = \frac{x-iy}{x^2+y^2}$$

positive definite