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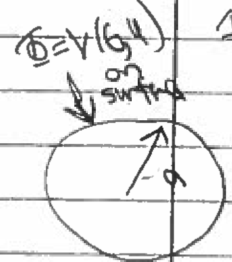
7701 Lecture 35

∗ • Please complete online SEI course evaluation

• 7701 has been evolving significantly and may continue to do so ⇒ your comments are very valuable!

• PS #11 comments ⇒ really good problems!



1. Best preparation for exam is to do hollow sphere problem using our two main methods for solving $\nabla^2 \Phi(\vec{x}) = 0$

a) Green's function and master formula

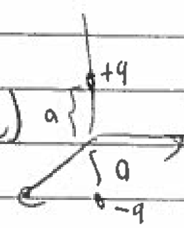
$$\Phi(\vec{x}) = -\frac{1}{4\pi}\oint_S V(\theta',\varphi')\frac{\partial G_D(\vec{x},\vec{x}'_s)}{\partial r}\Big|_{r=a} d^2x'_s \quad \leftarrow a^2 d\Omega'_s$$

• G_D for sphere: image charges!

$$G_D \to \frac{1}{|\vec{x}-\vec{x}'|} - \frac{a}{r'}\frac{1}{|\vec{x}-\frac{a^2}{r'^2}\vec{x}'|}$$

b) Expansion in Y_{lm}'s (or P_l's, or ...)

2. Use expansion $\frac{1}{|\vec{x}-\vec{x}'|} = 4\pi \sum_{lm} \frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}} Y^*_{lm}(\Omega')Y_{lm}(\Omega)$



Simplifies to $\Phi(\vec{x}) = \frac{q}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\left[1-(-1)^l\right]\frac{r_<^l}{r_>^{l+1}}P_l(\cos\theta)$

(so only odd l's)

• For inside grounded shell: guess solution or use method of images on charges and take limit of two images.

3. Use $Q^{(l)}_{ijk} = \int d^3x\, \rho(\vec{x})\, x^i y^j z^k$. If all $q_{lm} = \int d^3x'\, Y^*_{lm}(\theta',\varphi')\, r'^l \rho(\vec{x}')$ vanish for $l < l'$, then all $Q^{(l)}_{ijk}$ do as well (linear combinations!)

• What happens if we shift origin to (x_0, y_0, z_0)?

I.e. $x^i \to (x-x_0)^i$, etc. What is difference $\Delta Q^{(l)}_{ijk}$?

4. a) $\rho(\vec{x}) = \frac{1}{64\pi} r^2 e^{-r} \sin^2\theta \Rightarrow Y_{20} = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right),\ Y_{00} = \frac{1}{\sqrt{4\pi}}$

$\Rightarrow \sin^2\theta = 1-\cos^2\theta = \frac{2}{3}\sqrt{4\pi}\left(Y_{00} - \frac{1}{\sqrt{5}}Y_{20}\right) \Rightarrow q_{lm}$ is trivial given $\int Y^*_{lm} Y_{l'm'}\, d\Omega = \delta_{ll'}\delta_{mm'}$

b) Use $\Phi \propto \int \frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|}$ ← expand $\frac{1}{|\vec{x}-\vec{x}'|}$ in $\sum Y^*_{lm} Y_{lm}$'s

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Comments on final exam:

- Formula sheet was an (unintentional) "Spot-the-Error" exercise. Thanks for corrections!
- Structure:
  - series of short-answer "Spot-the-Error" problems.
    - e.g. what's wrong with this expansion or this application of the master formula
    - can cover more topics in short time
  - 2 longer problems
    - 1 on expansion example, one on Green function
    - 2nd one requires knowing Green function methods and concepts (constructing, applying)

- Topics:
  - 9, 10, 11 topics. No long problem on multipoles
  - Expansions in different coordinate systems for Φ
    - On formula sheet, have P_ℓ and $Y_{\ell m}$ expansions, plus cylindrical (Cartesian you are expected to remember! We've mostly done Fourier series.)

    using specific $Y_{\ell m}$'s
    - Separation of variables construction
    - Use of BC's at origin, ∞, on surfaces to determine coefficients
    - Projecting coefficients using orthogonalization formulas.
    - Expansion for $\frac{1}{|\vec{x}-\vec{x}'|}$ in spherical coordinates
  - Green functions (Dirichlet)
    - $G = \frac{1}{|\vec{x}-\vec{x}'|} + \Lambda(\vec{x},\vec{x}')$, $\nabla^2 \frac{1}{|\vec{x}-\vec{x}'|} = -4\pi\delta^3(\vec{x}-\vec{x}')$, $\nabla^2\Lambda = 0$
    - physical interpretation
    - how to construct them
      - method of images, division of region, eigenfunction expansion
      - use of symmetry $G_D(\vec{x},\vec{x}') = G_D(\vec{x}',\vec{x})$
    - Master formula application
  - Multipole expansion — what is a multipole moment, PS#11 examples.

- Checks of your answers ~ reminders on formula sheet.