7213

7201 Lecture 35

x

Please complete online SET course evaluation—7201 has been evolving significantly and may continue to do so; your comments are very valuable.

*PS #11 comments => really good problems.*

1. Best preparation for exam is to do hollow sphere problem using our two main methods for solving $\mathbf{V} \cdot \mathbf{A} = 0$
   a. Green's function and second formula:
   \[
   \Phi(x) = \frac{1}{4\pi} \int \frac{G_0(x_0, x_s) \phi(x_0) dx_0}{|x - x_s|}
   \]
   $G_0$ for sphere: image charges!
   \[
   G_0 = \frac{4}{|x - x_s|} - \frac{a^2}{|x - x_s|^3}
   \]
   b. Expansion in $\phi_m$s (or $\bar{p}_i$s, or ...)

2. Use expansion $\mathbf{\Phi}(x) = \frac{4}{4\pi} \int \frac{G_0(x_0, x_s) \phi(x_0) dx_0}{|x - x_s|}$
   Simplifies to $\mathbf{\Phi}(x) = \frac{A}{4\pi} \sum_{-\infty}^{\infty} (1 - \frac{a^2}{|x - x_s|^2}) Y_0^{j_0}(\theta, \phi) Y_{lm}(\theta, \phi) a_l^m$
   (or only add $l=2$)

   ForMake grounded shell: guess solution or use method of images on charges and take limit of two images.

3. Use $B_{jk}^m = \int_0^R \mathbf{R}(x_0) \times \mathbf{p}(x_0) \times \mathbf{R}(x)$. If all $q_{lm} = \int_0^R \mathbf{Y}_m^{\ast}(\theta, \phi) \mathbf{R}(x) \times \mathbf{R}(x_0) \times \mathbf{Y}_m(\theta, \phi) dx_0$ vanish for $l < l'$, then all $B_{jk}^m$ do as well (linear combinations).

   What happens if we shift origin to $(x_0, y_0, z_0)$?
   $x \rightarrow (x - x_0)$, etc., What is difference $\Delta B_{ij}^m$?

4. $A^2 = \frac{\int_0^R \mathbf{R}(x_0) \times \mathbf{R}(x) \cdot \mathbf{R}(x_0) \times \mathbf{R}(x) dx_0}{\int_0^R \mathbf{R}(x_0) \times \mathbf{R}(x) dx_0}$
   $\Rightarrow \mathbf{A} = \frac{1}{2} \mathbf{R}(x_0) \times \mathbf{R}(x)$

   $\Rightarrow \mathbf{B} = \frac{\int_0^R \mathbf{R}(x_0) \times \mathbf{R}(x) \cdot \mathbf{R}(x_0) \times \mathbf{R}(x) dx_0}{\int_0^R \mathbf{R}(x_0) \times \mathbf{R}(x) dx_0}

   \Rightarrow q_{lm} \text{ is trivial given } \int_0^R \mathbf{Y}_m^{\ast}(\theta, \phi) \mathbf{R}(x_0) \cdot \mathbf{R}(x_0) \times \mathbf{Y}_m(\theta, \phi) dx_0$
   b. Use $\Phi$ or $\Phi(x) = \exp(\frac{i}{\epsilon} \cdot \mathbf{p} \cdot \mathbf{x})$ in $\mathbf{F} = \mathbf{F}_0 + \epsilon \mathbf{F}_\epsilon$
Comments on Final exam:

- Formula sheet was an unintentional "Spot-the-Eror" exercise. Thanks for corrections.
- Structure:
  - series of short-answer "Spot-the-Eror" problems
    - eg. what's wrong with this expansion or this application of the master formula
  - can cover more topics in short time
- Longer problems
  - 1 expansion example, one on Green function
  - and one requires knowing Green function methods and concepts (constructing, applying)
- Topics:
  - 9, 10, 11 topics. No long problem on multipoles
  - Expansions in different coordinate systems, for \( E \)
    - On formula sheet, have \( P_\ell \) and \( V_\ell \) expansions plus cylindrical (cartesian you are expected to remember! We've mostly done Fourier series)
    - Separation of variables construction
  - Use of BC's at origin, or, on surfaces, to determine coefficients
  - Inverting coefficients using orthogonality formulas
  - Expansion for \( \psi_{\ell j} \) in spherical coordinates
  - Green functions (Dirichlet):
    - \( G(x, x') = \frac{1}{4\pi|x-x'|} \), \( \nabla^2 G = -\delta(x-x') \), \( \nabla^2 G = 0 \)
  - Physical interpretation
  - How to construct them
    - Method of images, division of region, eigenfunction expansion
    - Use of symmetry: \( G_0(x, x') = G_0(x', x) \)
    - Master formula application
    - Multipole expansion - what is a multipole moment; AS1111 examples.
- Checks of your answers - reminders on formula sheet.