

Some formulas for your reference:

$$\frac{d}{dx} \left(f(x) \frac{dy}{dx} \right) - g(x)y(x) + \lambda w(x)y(x) = 0$$

Fourier sine/cosine series for $0 \leq x \leq L$: $f(x) = \sum_{n=1}^{\infty} a_n \sin \frac{n\pi x}{L}$ and $f(x) = b_0 + \sum_{n=1}^{\infty} b_n \cos \frac{n\pi x}{L}$

where $a_n = \frac{2}{L} \int_0^L f(x) \sin \left(\frac{n\pi x}{L} \right) dx$, $b_0 = \frac{1}{L} \int_0^L f(x) dx$, $b_n = \frac{2}{L} \int_0^L f(x) \cos \left(\frac{n\pi x}{L} \right) dx$

$$\int_0^1 \sin(n\pi x) dx = \frac{1 + (-1)^{n+1}}{n\pi} \quad \int_0^1 x \sin(n\pi x) dx = \frac{(-1)^{n+1}}{n\pi}$$

$$\int_0^1 \cos(n\pi x) dx = 0 \quad \int_0^1 x \cos(n\pi x) dx = -\frac{1 + (-1)^{n+1}}{n^2\pi^2}$$

$$\tilde{F}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx \quad f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{F}(k) e^{ikx} dk \quad \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} dt = \delta(\omega)$$

$$\int_0^{\infty} x^n e^{-x} dx = n! \quad \delta(ax) = \frac{1}{|a|} \delta(x) \quad \delta[g(x)] = \sum_{i=1}^N \frac{\delta(x - x_{0i})}{|g'(x_{0i})|}$$

$$\nabla^2 \psi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2}$$

$$\Phi(\mathbf{x}) = \sum_{l=0}^{\infty} \sum_{m=-l}^l \left(A_{lm} r^l + \frac{B_{lm}}{r^{l+1}} \right) Y_{lm}(\theta, \phi) \quad \int_0^{\pi} \sin \theta d\theta \int_0^{2\pi} d\phi Y_{lm}^*(\theta, \phi) Y_{l'm'}(\theta, \phi) = \delta_{ll'} \delta_{mm'}$$

$$\text{normalized } Y_{lm}(\theta, \phi)'s : \quad Y_{00} = \frac{1}{\sqrt{4\pi}}, \quad Y_{10} = \sqrt{\frac{3}{4\pi}} \cos \theta, \quad Y_{1\pm 1} = -\sqrt{\frac{3}{8\pi}} \sin \theta e^{\pm i\phi}$$