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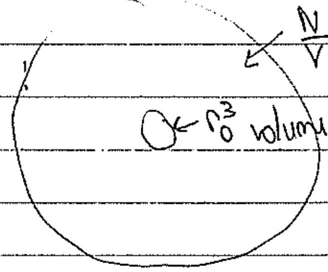
- Comments on Piotr_Magierski_path-integral-qmc-rmcn.pdf

slide 1: · always attraction, but different regimes based on value of $k_F a_0$ ⇒ our expansion parameter. ($a_0 \equiv a_s$)
· note that $1/k_F a_s$ is plotted, so the origin is very large scattering length: just bound or just unbound state at zero energy.

slide 2: Joaquin Drut is Casey's supervisor for REU/thesis project. Former OSU postdoc in NTG, now faculty at UNC

slide 4:



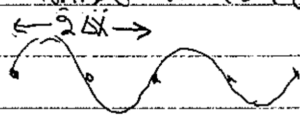
$\frac{N}{V} = \frac{1 \text{ particle}}{\rho \text{ density}}$ ⇒ $n r_0^3 \ll 1$ means the ratio of the volume over which a particle interacts is $\ll$ the volume for that particle, ⇒ dilute gas!

$\boxed{n \equiv \rho}$

· universality follows from lack of scale: $r_0 \to 0$, $a_0 \to \pm\infty$
so dimensional analysis implies $E \propto E_{\text{Fermi gas}}$ ⇒ dimensionless ratio
and for $x = T/\varepsilon_F$ scaling parameter (dimensionless)
ε_F ← Fermi energy ⇒ $E(x) = \xi(x) E_{FG}$

slide 8: quantities of interest: energy, pressure, momentum distribution $n_\sigma(k)$
· talk of $n_\sigma(k) \sim C/k^4$ ⇒ C is called the "contact"

slide 9: Same Hamiltonian we've considered $g \to -C_0$ (so $g>0$ is attractive)
· $\nu = 2$: only spin up and down.
· $\Delta \equiv \nabla^2$
· Given a spacing Δx and periodic boundary conditions, what is the minimum wavelength ⇒ maximum momentum wave number k ($\equiv k_{cut}$)?



← minimum wavelength $\lambda_{min} = 2\Delta x$
$$\Rightarrow k_{cut} = \frac{2\pi}{\lambda_{min}} = \frac{\pi}{\Delta x}$$

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slide 10: $k_{cut} \equiv \Lambda_{UV} = \frac{\pi}{\Delta x}$

What about $k_{min} \equiv \Lambda_{IR}$? $e^{ik_n x} = e^{ik_n(x+L)}$ PBC

$\Rightarrow k_n L = 2\pi n \Rightarrow k_{min} = 2\pi/L = \Lambda_{IR}$

How big and fine a lattice do we need?

Want the minimum kinetic energy and spacing to be much smaller that the Fermi energy

$$\Rightarrow \frac{\Lambda_{IR}^2}{2m} \ll \varepsilon_F$$

Want the energy cutoff to be much larger than the pairing gap (?)

$$\frac{\Lambda_{UV}^2}{2m} \gg \Delta$$

- real space $\overset{FFT}{\Longleftrightarrow}$ momentum space means use "Fast Fourier Transform" algorithm to switch back and forth. Kinetic energy is trivial in momentum scale, potential is trivial in coordinate (real) space. [eg. trivial ⇒ diagonal matrix]

slide 11: Running coupling

- See equation (29) from section 7b on "Non-perturbative matching"

$$C_0(\Lambda_c) = \frac{4\pi}{m}\frac{1}{\frac{1}{a_0} - \frac{2}{\pi}\Lambda_c} \Rightarrow \frac{1}{C_0} = \frac{m}{4\pi}\left(\frac{1}{a_0} - \frac{2}{\pi}\Lambda_c\right)$$

sharp cutoff

$$= \frac{m}{4\pi a_0} - \frac{m}{2\pi^2}\Lambda_c$$

$$= -\frac{1}{g}\left(= \frac{m}{4\pi\hbar^2 a} - \frac{m k_{cut}}{2\pi^2\hbar^2}\right)$$

The unitary limit is when $a_0 \to \infty$ so

$$\boxed{\frac{1}{g} = \frac{m k_{cut}}{2\pi^2} = \frac{m\,\pi/\Delta x}{2\pi^2} = \frac{m}{2\pi\Delta x}}$$

Trotter expansion: breaks up partition function into the product of many small imaginary time / temperature steps.

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slide 12: Splitting up the kinetic and potential parts costs only a τ^3 error (you might have thought it was worse ⇒ see Piazza).

· Eliminate fermion integration in favor of auxiliary field σ.

* · for basic idea, see 5 on 11/3/14