12/1/14

Monday 8805 Class

Left over from 11/24/14 $\Rightarrow$ do 1, 2, 3

- In next 3 pages 2, 3, 4, we recap the 3-body discussion of eliminating degrees of freedom and connect to RG transformations

- Then continue with HUGS 4-37 to 4-44 with overview of infinite matter calculations

- Then finish with a discussion of the tensor interaction on 5-7 and look at the pictures in the handout.

Next: Chiral EFT in brief (see handouts)

Finale: Density functional theory (DFT)

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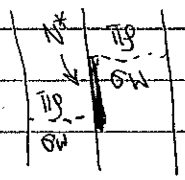
· Revisiting 3-body force discussion...

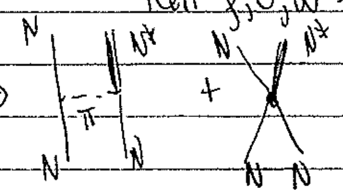
· We discussed the examples of the Earth-Moon-Sun system and the interaction of neutral atoms as places where the low-energy theory has three-body forces.
  · The general feature is that three-body forces arise from the elimination of degrees of freedom
  · if we included positions & individual masses or interactions between all electrons, then two-body only.
  · eliminating these variables (degrees of freedom) in favor of collective coordinates (center-of-mass position) required three-body forces.

· So what about the nuclear case?

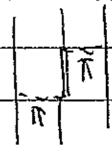


· In this diagram two nucleons exchange a boson, maybe a pion, maybe a heavier meson, exciting an N* (excited state of nucleon) for a brief period.
· N* could mean a Δ, could mean something else.

· Suppose our theory had both N's and N*'s explicitly and pions but no ρ, σ, ω (treated as heavy)
  · then $\rho, \sigma, \omega \to$ expanded as contacts plus derivatives of contacts (derivative expansion)
  $\left[\text{cf. } \frac{1}{(\vec{k}-\vec{k}')^2+\mu^2} \to C_0 + C_2(\vec{k}^2+\vec{k}'^2) + C_2' \vec{k}\cdot\vec{k}' + \ldots \right]$

$\Rightarrow$ 

would be diagrams included in the low-energy theory, but no 3-body, because |_|\ , etc. is two successive 2-body interactions.

· But now if we eliminate N* $\Rightarrow$ 

$\rightarrow$ 

$\Rightarrow$ This is a 3-body force

· 3-body if can't be broken into successive two-body interactions

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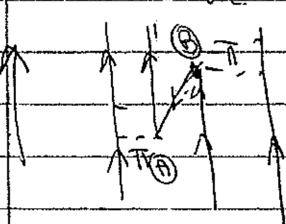
- When is it good to replace the N^* excitation? When we don't resolve that it was excited.
  - By the uncertainty principle, if we excite by ΔE a virtual state, it can last for $\Delta t = \hbar/\Delta E$, which is short if ΔE is large ⇒ endpoint are close enough so they are not <u>resolved</u> ⇒ replace by contact • and derivatives
  - So this is a danger if $M_\Delta - M_N \approx 300$ MeV, then it will break down much sooner than for energy differences & 500–1000 MeV (such as heavier meson exchanges),
    - We will keep coming back to this!
    - Expansion parameter $Q/(M_\Delta - M_N)$ may be smaller than we want!

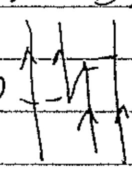
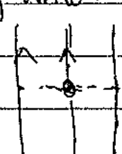
- How about a process like:

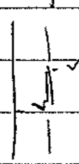
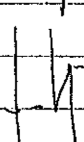
time 

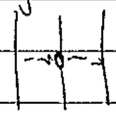
- So the idea is that a nucleon emits a pion that becomes a nucleon-<u>anti-nucleon</u> pair at Ⓐ. The anti-nucleon annhilates with a 2^{nd} nucleon at Ⓑ emitting a pion absorbed by a 3^{rd} nucleon.

Class: • In the previous case, we had $\delta E \gtrsim M_\Delta - M_N$. What is it here?

Initially $3 \times M_N$ + kinetic, in the middle an extra $2M_N$ ⇒ $\Delta E \gtrsim 2M_N$, which is large ⇒ Δt is small

⇒  →  is a 3-body force that is a good approximation,

<u>Important</u>:  or  may be good or bad, or incomplete models — maybe it requires quarks and gluons to describe. As long as

 contains <u>all</u> allowed (by symmetries) vertices, then we don't care — we will be <u>model</u> <u>independent</u> with our EFT!

but there can be signatures in the couplings

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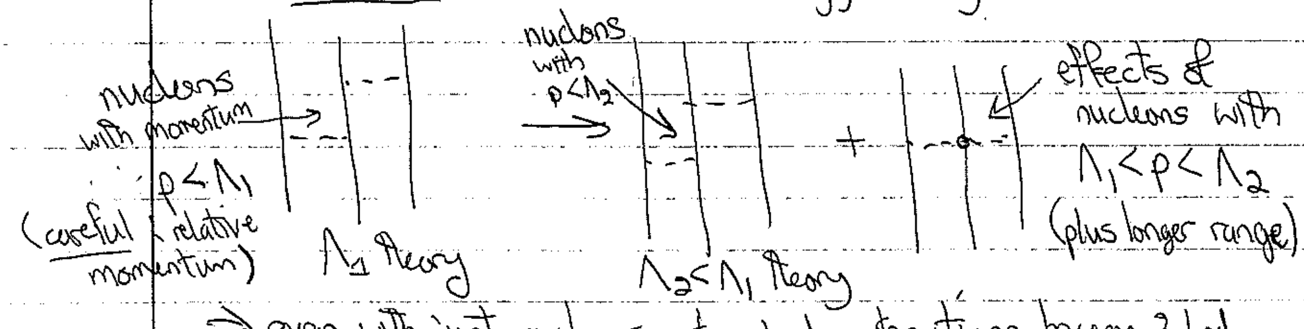
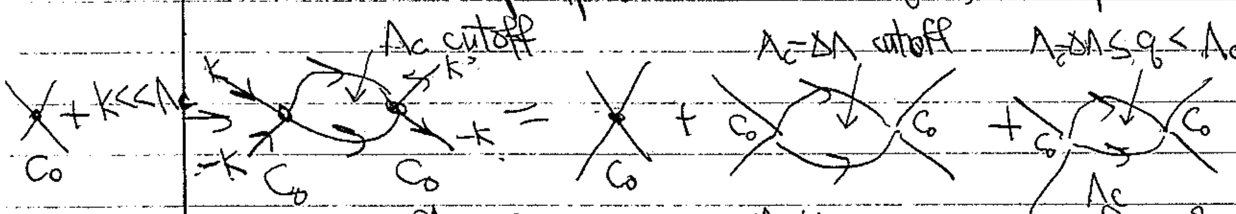
- Moral: whether we have a 3-body force or not and how large a contribution depends on our choice of degrees of freedom.
  - But this includes where our cutoff eliminates <u>nucleons</u> from our low-energy theory



$\Rightarrow$ even with just nucleons, two-body interactions become 3-body if we eliminate degrees of freedom (in this case by lowering Λ_c)

- This is the same principle as we had with $C_0(\Lambda)$ in the pionless theory:



$$\Rightarrow C_0(\Lambda)^2 \frac{m}{2\pi^2}\int_0^{\Lambda_c} dq\, \frac{q^2}{k^2-q^2+i\varepsilon} \longrightarrow \int_0^{\Lambda_c-\Delta\Lambda} + C_0(\Lambda)^2\frac{m}{2\pi^2}\int_{\Lambda_c-\Delta\Lambda}^{\Lambda_c} dq\, \frac{q^2}{k^2-q^2+i\varepsilon}$$

convince yourself: same as uncertainty principle argument

But $k^2 \ll q^2$ in 2nd integral because $k \ll \Lambda_c - \Delta\Lambda \Rightarrow \frac{1}{k^2-q^2+i\varepsilon} \to \frac{-1}{q^2}\left(1 + \frac{k^2}{q^2} + \dots\right)$ (small)

$$\Rightarrow \int_{\Lambda_c-\Delta\Lambda}^{\Lambda_c} dq\, \frac{q^2}{k^2-q^2+i\varepsilon} \to \int_{\Lambda_c-\Delta\Lambda}^{\Lambda_c} dq \frac{-q^2}{q^2} + k^2\int_{\Lambda_c-\Delta\Lambda}^{\Lambda_c} dq\, \frac{-q^2}{q^4} + \dots = -\Delta\Lambda\left(1 + O\!\left(\frac{k^2}{\Lambda_c^2}\right)\right)$$

$\Rightarrow$ the [bubble diagram] contribution for $\Lambda_c - \Lambda_c < q < \Lambda_c$ looks mostly like a constant: [contact vertex]

What if different regulator? Just factors!

$\Rightarrow$ Change C_0 to compensate: $\Delta C_0 = C_0(\Lambda)^2 \frac{m}{2\pi^2}(-\Delta\Lambda)$ or $\frac{d}{d\Lambda} C_0(\Lambda) = \frac{m}{2\pi^2}(C_0(\Lambda))^2$ ($\Delta C_0 \to dC_0$, $\Delta\Lambda \to d\Lambda < 0$)

which is the RG equation from an earlier lecture (sign does work!)

(sign does work: if $d\Lambda < 0$, then C_0 decreases ✓)

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- Where does the tensor interaction come from?
  - One source is the pion, and the pion tensor interaction has important effects on nuclear structure.
- Let's do a quick derivation of the one-pion exchange potential starting from the interacting Hamiltonian density

$$\mathcal{H}_{\pi N} = \frac{g_A}{2F_\pi} N^\dagger \vec{\sigma}\cdot(\vec{\nabla}\pi \cdot \underset{\sim}{\tau}) N \quad \text{(lots of hidden indices!)}$$

$\leftarrow$ 2x2 matrix in isospin

$\uparrow$ 2x2 matrix in spin

$g_A \approx 1.27$

$F_\pi \approx 92.4$ MeV

In 2nd quantized form

create nucleon with $\vec{k}', m_s', m_t'$

$$\hat{H}_{int} = -i\frac{g_A}{2F_\pi}\int \frac{d^3k\, d^3k'}{(2\pi)^6}\, b^\dagger(\vec{k}', m_s', m_t')\, b(\vec{k}, m_s, m_t)\, \vec{\sigma}_{m_s' m_s}\cdot \vec{q}\, (\underset{\sim}{\tau})^i_{m_t' m_t}$$

$\leftarrow$ T=1, isotriplet

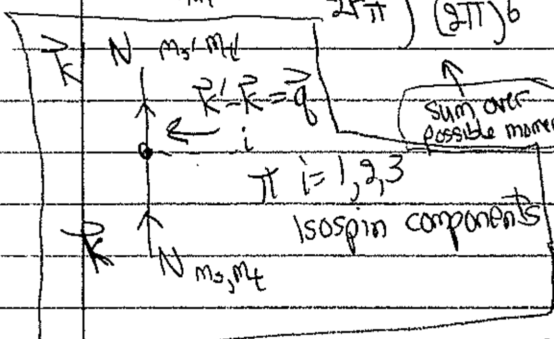
$$\times \frac{1}{\sqrt{2\omega_q}}\left(a_i^\dagger(-\vec{q}) + a_i(\vec{q})\right)\Big|_{\vec{q}=\vec{k}'-\vec{k}}$$

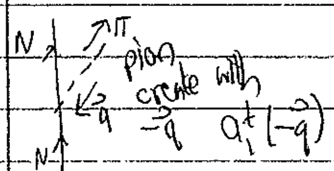
sum over possible momenta

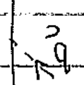
pseudoscalar, Goldstone boson (derivative)

create pion in isospin state i with $-\vec{q}$

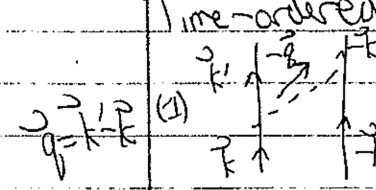
$\omega_q = \sqrt{m_\pi^2 + q^2}$ normalization factor for pion





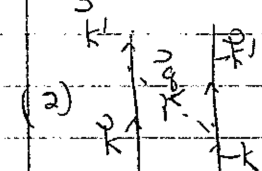


Time-ordered perturbation theory (2nd order)



$$\langle \vec{k}'|V_{OPE}^{(1)}|\vec{k}\rangle = \sum_{n=1\,\text{pion}} \frac{\langle \vec{k}'|\hat{H}_{int}|n\rangle\langle n|\hat{H}_{int}|\vec{k}\rangle}{E_i - E_n} \quad \leftarrow \; = -\sqrt{q^2+m_\pi^2} = -\omega_q$$

$$= -\frac{1}{\omega_q}\left(-i\frac{g_A}{2F_\pi}\right)^2 \vec{\sigma}_1\cdot\vec{q}\,\frac{1}{2\omega_q}\,\vec{\sigma}_2\cdot(-\vec{q})\;\vec{\tau}_1\cdot\vec{\tau}_2$$



$\leftarrow$ same with $\vec{\sigma}_1\cdot(-\vec{q})\,\vec{\sigma}_2\cdot\vec{q}$

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Putting it together, class: so this is a local potential (any particle exchange at long distance)

$$V_{OPE}(\vec{k}', \vec{k}) = V_{OPE}(\vec{q} = \vec{k}' - \vec{k})$$

(1) + (2)

$$= -\frac{g_A^2}{(2f_\pi)^2} \vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}\, \frac{2}{2\omega_q^2} \vec{\tau}_1 \cdot \vec{\tau}_2$$

$$= -\frac{g_A^2}{4f_\pi^2} \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + m_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2$$

In one of the Piazza exercises, you carry out the Fourier transform showing

$$\int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2 + m_\pi^2} e^{i\vec{q}\cdot\vec{r}} = \frac{1}{4\pi} \frac{e^{-m_\pi r}}{r}$$

(standard, but remind yourself)

and then evaluating the derivatives in $\vec{\sigma}\cdot\vec{q}$. The bottom line is

$$V_{OPE}(\vec{r}) = \frac{m_\pi^3}{12\pi}\left(\frac{g_A}{2f_\pi}\right)^2 \vec{\tau}_1 \cdot \vec{\tau}_2 \left[3T(r) S_{12}(\hat{r}) + Y(r)\vec{\sigma}_1 \cdot \vec{\sigma}_2\right]$$

where $T(r) = \frac{e^{-m_\pi r}}{m_\pi r}\left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right)$ ← singular $\frac{1}{r^3}$!

$$Y(r) = \frac{e^{-m_\pi r}}{m_\pi r}$$

and $S_{12}(\hat{r}) = \left[(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - \frac{1}{3}\vec{\sigma}_1 \cdot \vec{\sigma}_2\right]$.

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Let's talk a bit about the impact of the tensor force on NN bound states. (also see handout)

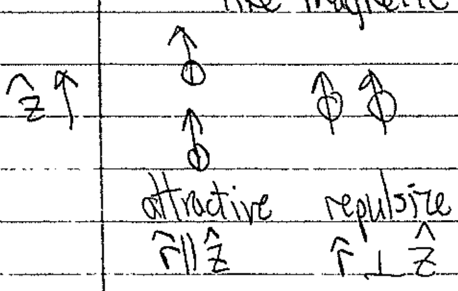
- nn and pp have no bound states
- np has one shallow bound state (large scattering length!)
  - $\Rightarrow$ deuteron $T=0$
- What's special about $T=0$?

- Deuteron properties: (measured!)
  - binding energy $-2.224575(9)$ MeV small!
    - $\Rightarrow$ rms radius $\sqrt{\langle r^2 \rangle} \doteq 1.9752(1)$ fm large! (tail)
  - $J^P = 1^+$ (angular momentum 1, parity +)
  - isospin $T=0$, $M_T=0$ (np)
  - electric quadrupole moment $Q_d \doteq 0.2859(3)\, e\, \text{fm}^2 > 0$

- Two spin $1/2$ nucleons $\Rightarrow S=0,1$ $\quad J-1 < l < J+1 \Rightarrow l = 0,1,2$
  - parity $+ \Rightarrow l=0,2 \Rightarrow$ space symmetric so $S=1$ (since $T=0$)
  - So ${}^3S_1$, ${}^3D_1$ possible
  - Expect ${}^3S_1$ energetically but $Q_d \neq 0 \Rightarrow l=2$ admixture
    - $\Rightarrow$ tensor force mixes ${}^3S_1 - {}^3D_1$ (recall T1b)
  - Attractive tensor $\Rightarrow$ extra binding (cf. nn)

- Is attractive tensor in $T=0$ consistent with $Q_d > 0$?
  - like magnetic dipole-dipole

ze
ides



$\hat{Q} = e(3z^2 - r^2) = er^2(3\cos^2\theta - 1) \Rightarrow Q_d > 0 \Rightarrow \langle z^2 \rangle > \langle x^2 \rangle, \langle y^2 \rangle$

- like spins ($M_S = \pm 1$) prefer to be oriented head to tail
  - $\Rightarrow$ prolate $\Rightarrow Q_d > 0$

[note $Q_d = \langle \psi_D\, M=J | \hat{Q} | \psi_d\, M=J \rangle$]

- unlike spins ($M_S = 0$) prefer side-by-side configuration

- Wave functions: S-wave $u(r)$, D-wave $w(r)$ parts

$u(r) \xrightarrow[r\to\infty]{} A_S e^{-\gamma r}$ $\quad \gamma = \sqrt{2E_d}$ binding momentum

$w(r) \xrightarrow[r\to\infty]{} \eta_d A_S \left(1 + \frac{3}{\gamma r} + \frac{3}{(\gamma r)^2}\right) e^{-\gamma r}$ $\quad \eta_d$ and A_S asymptotic normalizations

$\Big\}$ measurable