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Wednesday 8805 Class

- We'll go over portions of the HUGS lectures (indicated by lecture number and slide number; eg. HUGS2.4–5 means slides 4 and 5 from the 2nd lecture) as well as the RG notes from 11/10/14.

- Start with a recap of the need for decoupling
  HUGS2.4–5 "Why did our low-pass filter fail?" Even at low energy!
  - Also RG-1 ⇒ go over those points: if strong coupling, wave functions get high momentum components
  - bottom line: achieve decoupling by modifying the Hamiltonian (in effect the potential because we choose to keep the kinetic energy fixed: the decomposition $H = T + V$ is not unique; we only want $H \underset{r\to\infty}{\to} T$).

short-range correlations ⇒ hard to solve!

- Modifications will be by a series of infinitesimal (in principle, small in practice) unitary transformations.
  - RG: small steps leading to decoupling
  - Why can we get away with this? Slide HUGS 2.6 says that we can change Hamiltonian and other operators and the wave function so that measurable quantities don't change.
  - What is measurable about a wavefunction?

do this first

    - See HUGS6.47–49
    - distinguish other quantities that depend on the scale (how far we have evolved → resolution scale) and the scheme (eg. what generator G_s and initial $H_{s=0}$).
  - Note the changes in the deuteron wf on HUGS6.48
    - Asymptotic normalization (tail of wf) and energy unchanged
    - relative normalization of S and D parts changes (D-state prob.)

- What RG is good for: we'll look at "perturbativeness" and "universality".
  - Suggested by evolution of potential for low-momentum nucleons
    ⇒ HUGS2.35 shows same gentle potential from two initial potentials (

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- Side note: For low-momentum nucleons, evolved potentials are <u>attractive</u> ⇒ no sign problem for lattice EFT.
  - Used by U. Washington group (Bulgac et al.) to do calculations with AFQMC ⇒ see Phys. Rev. Lett. 113, 182503 (2014).
- The tensor part (S-D mixing) leads to the deuteron having both ~~a~~ S-wave and D-wave parts to its wave function (total J=1 from S=1 ⊕ L=0 or S=1 ⊕ L=2)

- Recall from HUGS 2.36-39 how the SRG with <u>one</u> choice of G_s (namely $G_s = T$, the kinetic energy) decouples by driving the potential matrix toward energy diagonal — that is, matrix elements connecting different energies are driven to zero, and faster if the energy difference is greater.
  - All automagically! Just turn the differential equation crank.
  - What does the 2nd term here "do"? (What must be maintained?)
  - Where does "physics" go? We are shifting strength in the Hamiltonian matrix, can we understand this in simple terms? (see below!)

- Alternative decoupling: block (rather than band) diagonal
  - HUGS 2.41 reduce Λ vs. λ. Unitary? Why not?
  - What if we want to keep it unitary?
    ⇒ HUGS 2.42
  - General rule: choose G_s to match desired final pattern (see Husky potential and custom potential HUGS 2.54-64 ⇒ for the latter, what was G_s?)

- Side question: Why is it useful to have multiple potentials at different values of λ or Λ? ⇒ see RG-7
  - test for observables or determine scale dependence
  - test for errors in solution method
  - test approximations — theoretical error bar.
- cf. finding the "best" potential

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Computational aspects

- make everything look like linear algebra (eg. matrix equations)
- see HUGS 2, 67–68

- approximate integrals by quadrature, eg. Gaussian

$$\int_a^b dk\, f(k) \approx \sum_{n=1}^{N} w_n f(k_n) \quad \text{with } \{k_n, w_n\}\ (n=1,N)$$

determined by N, a, b (not f)

- Then $I(p,q) \equiv \int_a^b dk\, k^2\, V(p,k) V(k,q) \rightarrow I_{ij}$ matrix $(i,j=1,N)$

$$I_{ij} = \sum_{n=1}^{N} k_n^2 w_n V_{in} V_{nj} = \sum_{n=1}^{N} \tilde{V}_{in} \tilde{V}_{nj} \quad \text{with } \tilde{V}_{ij} = \sqrt{w_i}\, k_i V_{ij} k_j \sqrt{w_j}$$

matrix multiplication symmetric

- Computationally very efficient because matrix operations are highly optimized (instead of loops)

- If we need intermediate values other than k_n, interpolate! (because functions are smooth)

* - Look at RG10 – RG13 for demonstrations that Wegner and block diagonal generators will always decouple.
  - Using $G_s = T_{rel}$ is an approximation to $G_s = H_D$ (Wegner choice), so it works in most cases.

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Good things from RG: improved perturbativeness

- Intuitive from

it off-diagonal is reduced, this is smaller than before!

$$\langle k|T(E)|k'\rangle = \langle k|V|k'\rangle + \frac{2}{\pi}\int q^2 dq \frac{\langle k|V|q\rangle\langle q|V|k'\rangle}{(E - q^2/m)} + \ldots$$

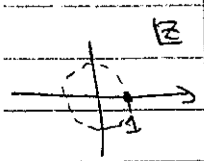
- What is a quantitative measure of "perturbativeness", which is relevant even if the series is not convergent.

$\Rightarrow$ see HUGS 2.72–86

Highlights:
- consider the Born series in operator form with $E \to z$ (complex)

$$\hat{T}(z) = \hat{V} + \hat{V}\frac{1}{z-\hat{H}_0}\hat{V} + \hat{V}\frac{1}{z-\hat{H}_0}\hat{V}\frac{1}{z-\hat{H}_0}\hat{V} + \ldots$$

where we must remember that $\frac{1}{z-\hat{H}_0}$ is the inverse of $z-H_0$.



- If this were a power series in x, we would know when it converges and how close it is to convergent
  e.g. $1 + x + x^2 + \ldots$ is convergent for $|x| < 1$, and very non-convergent for $|x| \gg 1$. Pole at $x = 1$. (sum is $\frac{1}{1-x}$)

- We also know (or should know!) that matrix elements of $\hat{T}(z)$ have poles at the bound-state energies.
  - How do we see this? Let $(H_0 + V)|b\rangle = E_b|b\rangle$ ("b" for bound)

$$\Rightarrow \hat{V}|b\rangle = (E_b - \hat{H}_0)|b\rangle \quad \text{or} \quad \frac{1}{E_b - H_0}V|b\rangle = |b\rangle$$

So $T(z=E_b)|b\rangle = V|b\rangle + V\frac{1}{E_b-H_0}V|b\rangle + V\frac{1}{E_b-H_0}V\frac{1}{E_b-H_0}V|b\rangle + \ldots$

$$= V(1 + 1 + 1 + \ldots)|b\rangle = \infty$$

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We convert to numbers by knowing that $|b\rangle$ is an eigenvector of $\left(\frac{1}{E_b - H_0}\right)V$.

$\Rightarrow$ Generalize at <u>any</u> E by finding eigenvalues $\eta_\nu(E)$ and eigenvectors $|\Gamma_\nu(E)\rangle$ of $\frac{1}{E-\hat{H}_0}V$

$$\Rightarrow \frac{1}{E-\hat{H}_0} V |\Gamma_\nu(E)\rangle = \eta_\nu(E) |\Gamma_\nu(E)\rangle$$

(just give the matrix $\langle k| \frac{1}{E-\hat{H}_0} V |k'\rangle$, for example, to a diagonalization program.)

$$\Rightarrow \hat{T}(E)|\Gamma_\nu\rangle = V|\Gamma_\nu\rangle (1 + \eta_\nu + \eta_\nu^2 + \ldots)$$

$\Rightarrow \hat{T}(E)$ diverges at E if $|\eta_\nu(E)| > 1$

- Equivalent to saying $\frac{1}{E-\hat{H}_0}\left(\frac{V}{\eta_\nu(E)}\right)$ has a bound-state at E for potential $V/\eta_\nu(E)$.

- Look at HUGS 2.78-82 for $E = -2.22\,\text{MeV}$ in $^3S_1$ channel.
  - There is a deuteron bound state at this energy, so there is a <u>positive</u> eigenvalue $\eta_\nu(-2.22\,\text{MeV}) = 1$ at any SRG λ or Λ.
  - Never changes because eigenvalues of $\hat{H} = \hat{H}_0 + \hat{V}$ are preserved
  - But η_ν can also be negative $\Rightarrow V/\eta_\nu$ flips the potential. When is there a bound state at $E = -2.22\,\text{MeV}$?

hard core $\Rightarrow$ for large Λ, $|\eta_\nu| \gg 1$, but decreases below 1 as Λ decreases.

  - At finite density, <u>both</u> decrease from Pauli blocking

$$\frac{2}{\pi}\int q^2\, dq \, \frac{\langle k|V|q\rangle\langle q|V|k'\rangle}{(E - q^2/m)}$$

exclude states that are already filled!

- $\Rightarrow$ perturbative in many-body context!

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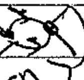
Now consider the flow toward "universality".

- First recall: "What does changing a cutoff do in an EFT?" ⇒ HUGS 3.53

⇒ as the cutoff Λ_c changes, we shift contributions from the sum over intermediate states (loop integral ) to the coupling constant [e.g. $C_0(\Lambda_c)$ or similarly in QCD]

Claim: analogous effect from RG on V!

- Go through HUGS 3.55–67. Take-away points:
  - different starting NN potentials that are "phase equivalent" (reproduce same phase shifts) and have same long-distance behavior (one pion exchange), flow to the same matrix elements
  - Major (but not entire) change is from a (regulated) constant interaction. Regulated means a constant only up to some momentum.
  - So in what sense are these different potentials to start with? How does this compare to QCD running coupling?
  - Brian Dainton showed that the potential collapse to the same form only happens when phase equivalent (HUGS 3.64–65.)
  - The decoupling is also local: matrix elements only talk to nearby neighbors.