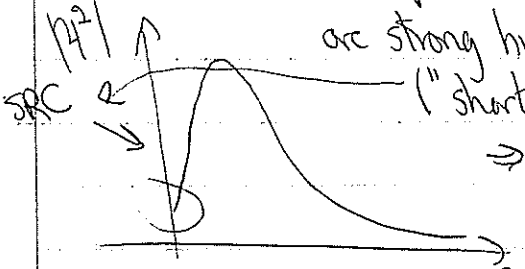
11/10/14

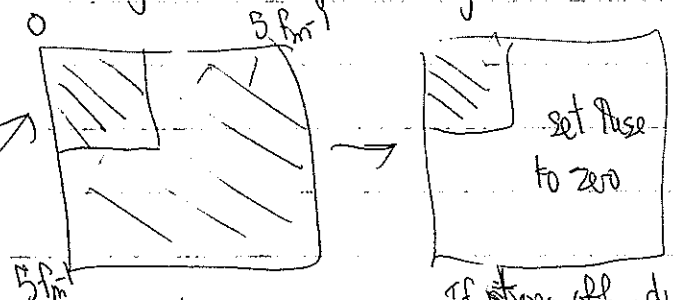
- Recap of important points from the HUGS slides (look at these first or simultaneously)

- Nuclei would be at low resolution based on Fermi momentum in large nuclei.
  - recall from exercises $\rho = \frac{2}{3\pi^2} k_f^3$ (for protons and neutrons)
    $\Rightarrow k_f = (3\pi^2 \rho/2)^{1/3}$ (see next page for reminder)
    and density of heavy nuclei about constant $\Rightarrow k_f \approx 1.1\text{-}1.3\ \text{fm}^{-1}$
  - So typical relative momentum of $\approx 1\ \text{fm}^{-1}$ (≈ 200 MeV) in a large nucleus. Even less in light nuclei
- But if the potential has a repulsive core, then there are strong high momentum components ("short-range correlations")



$\Rightarrow$
- slows down convergence of many-body nuclei. (eg. HF doesn't work)
- eg. matrices get too big (for direct diagonalization)

- low pass filter fails even for low energy.



Why? Because of quantum mechanics. $T = V + V\frac{1}{E-H_0}V + \dots$

If strong off-diagonal couplings $\Rightarrow$ can't truncate high momentum matrix elements

$$\Rightarrow \langle k|T|k\rangle = \langle k|V|k\rangle + \frac{2}{\pi}\int k'^2 dk' \frac{\langle k|V|k'\rangle\langle k'|V|k\rangle}{(k^2-k'^2)/m} + \dots$$

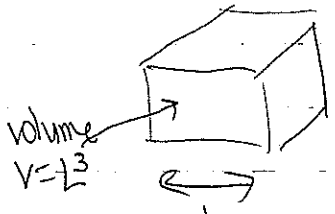
($\langle k|T|k\rangle$: $\uparrow$ low momentum)

Solution? Unitary transformation to decouple! Use RG to do it.

11/10/14:

[Aside: deriving $\rho = \frac{2}{3\pi^2} k_f^3$ ...

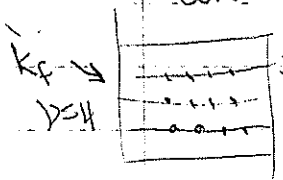
- For a non-interacting Fermi gas, imagine putting N particles in a box of side $L \Rightarrow \rho = \frac{N}{L^3}$



- when in doubt for uniform system, put in box and take $V \to \infty$ limit at end

- Let ν be the spin-isospin degeneracy:
  - $\nu = 2$ for neutrons only (spin up, spin down)
  - $\nu = 4$ for symmetry matter ($\uparrow, \downarrow, n, p$)

- Apply periodic boundary conditions (pbc) $\Rightarrow$ discrete momentum levels:



Then $N = \sum_n^{n(k_f)} \nu \cdot 1$ ← ν in each state

But pbc: $e^{i k_n (x+L)} = e^{i k_n L}$ in each dimension

$\Rightarrow k_n L = 2\pi n$ $n = 0, 1, 2, 3, \ldots$ are allowed

$\Rightarrow n = \frac{kL}{2\pi}$ or $\Delta n = 1 = \frac{L}{2\pi} \Delta k$ in each dimension

in large V limit $\Rightarrow \sum_n \to \left(\frac{L}{2\pi}\right)^3 \int d^3k = \frac{V}{(2\pi)^3} \int d^3k$

$\Rightarrow N = \frac{V}{(2\pi)^3} \int_0^{k_f} d^3k \, \nu$

↑ volume of sphere in large V limit

or $\boxed{\frac{N}{V} = \rho = \frac{1}{8\pi^3} \cdot \frac{4}{3}\pi k_f^3 \cdot \nu = \frac{\nu k_f^3}{6\pi^2}}$

]

W10/14

RG is a good thing to

i) make problems more perturbative (how do you measure this?)
- convergent vs. non convergent but also the rate
- exercise: Weinberg eigenvalue as diagnostic of convergence

$T = V + VG_0V + VG_0VG_0V + \dots$

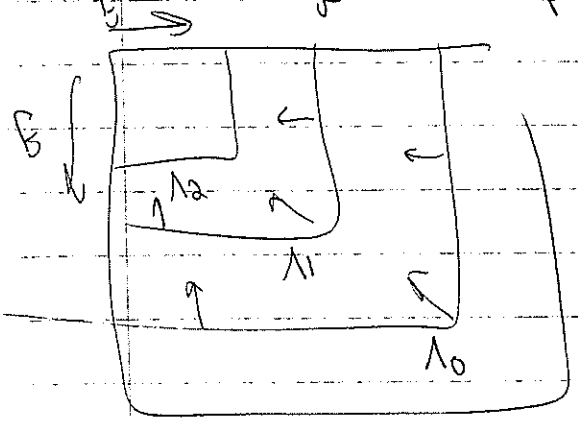
↖ look at eigenvectors of this operator (in k, k')

ii) Revealing universal characteristics by filtering out model dependent short-distance details

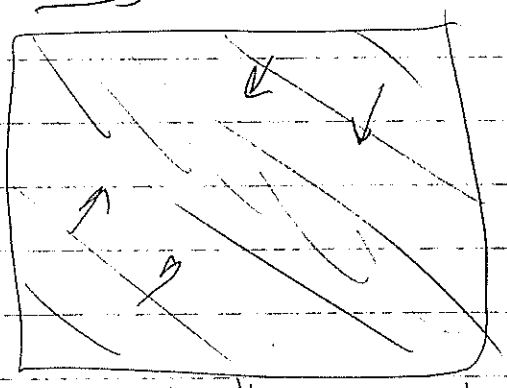
iii) simplify nuclear structure/reactions calculations
- quantum chemistry methods work with nuclear interactions – coupled cluster, configuration interaction, ...
- note: chiral EFT potentials are already "soft"

↑ no coupling to very high momentum

Two ways to decouple



lower cutoff step by step in Λ and demand something invariant, eg. $\frac{d}{d\Lambda} T(k,k';k^2)$



drive the Hamiltonian toward the diagonal with "flow equation" (Wegner; Glazek/Wilson 1990s)

01/10/14

- Historical perspective on flow equations and SRG
  - In the early 1970's, Ken Wilson and Franz Wegner both made important contributions to understanding critical phenomena and the renormalization group (RG)

  - Then 20 years later in the early 1990's they independently innovate again
    - unitary RG flow to make many-particle Hamiltonians increasingly energy diagonal

  - Glazek and Wilson "Renormalization of Hamiltonians" in 1993 ⇒ SRG for QCD on the light front.

  - Wegner "Flow equations for Hamiltonians" (1994)
    ⇒ application to condensed matter problems
    - S. Kehrein, "Flow equation approach to many-particle systems"
    - Dissipative quantum systems to correlated electron physics to non-equilibrium problems to ...

  - Particularly well suited for low energy nuclear physics
    - only applied since 2007
    - technically simpler and more versatile than other methods

11/10/14

SRG Flow Equation

We wish to transform an initial Hamiltonian

$$H = T + V$$

(T: kinetic energy; V: potential energy)

which you should imagine are stored as a matrix in some basis. We transform with a unitary transformation

$$U_s^\dagger U_s = U_s U_s^\dagger = 1$$

$$\Rightarrow H_s = U_s H U_s^\dagger \equiv T + V_s \quad \leftarrow \text{define } V_s \equiv H_s - T$$

so that the kinetic energy is always the same.

- s is called the "flow parameter" — it is just a label for where we are in the evolution of H_s.
- For the RG, we imagine changing s a little, so we can differentiate H_s:

$$\frac{dH_s}{ds} = \frac{dU_s}{ds} H U_s^\dagger + U_s H \frac{dU_s^\dagger}{ds} \qquad (H_{s=\infty} = H_{s=0} \text{ doesn't depend on } s)$$

($U_s^\dagger U_s$ and $U_s U_s^\dagger$ are inserted between the factors)

$$= \underbrace{\frac{dU_s}{ds} U_s^\dagger}_{\equiv \eta_s} \underbrace{U_s H U_s^\dagger}_{H_s} + \underbrace{U_s H U_s^\dagger}_{H_s} \underbrace{U_s \frac{dU_s^\dagger}{ds}}_{\eta_s^\dagger}$$

claim: $\eta_s^\dagger = -\eta_s$ [exercise]

$$= [\eta_s, H_s]$$

η_s is anti-Hermitian $\Rightarrow$ can choose $\eta_s = [G_s, H_s]$ with G_s Hermitian.

$$\Rightarrow \boxed{\frac{dH_s}{ds} = \frac{dV_s}{ds} = [[G_s, H_s], H_s]}$$

This is the SRG flow equation.

11/10/14

- The pattern of the flow is determined by $\hat{G}_s$.

- Most of the nuclear applications have used $\hat{G}_s = \hat{T}_{rel}$, the relativ. kinetic energy. $\Rightarrow \frac{d\hat{V}_s}{ds} = \left[\left[\hat{T}_{rel}, \hat{H}_s\right], \hat{H}_s\right]$

- Evaluate in a partial wave momentum basis
  $\Rightarrow |k l m\rangle \equiv |k\rangle$ with l implicit
  - Completeness:
  $$1 = \frac{2}{\pi}\int_0^\infty |q\rangle q^2 dq \langle q|$$
  with $\hbar^2/m = 1$ units.
  - $\hat{T}_{rel}|k\rangle = \frac{\hbar^2}{m}k^2|k\rangle \Rightarrow k^2|k\rangle$
  - $\langle k|\frac{d\hat{V}_s}{ds}|k'\rangle \equiv \frac{d}{ds}V_s(k,k')$
  - $\hat{H}_s = \hat{T}_{rel} + \hat{V}_s$

- This is all you need to derive [exercise]

$$\frac{dV_s}{ds}(k,k') = -(k^2-k'^2)^2 V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2 dq\, (k^2+k'^2-2q^2) V_s(k,q) V_s(q,k')$$

[Hint: Write out the double commutator and take $\langle k|\ |k'\rangle$ matrix elements. $\left[\left[T_{rel}, T_{rel}+V_s\right], T_{rel}+V_s\right]$ and insert complete sets of states as needed.
Eg. $\left[\left[T_{rel}, V_s\right], T_{rel}\right] = T_{rel}V_s T_{rel} - T_{rel}T_{rel}V_s - V_s T_{rel} T_{rel} + T_{rel}V_s T_{rel}$
and $\langle k|T_{rel}V_s T_{rel}|k'\rangle = k^2\langle k|V_s|k'\rangle k'^2$
(act left, act right)]

- This is an equation that is easy to discretize and solve as a set of coupled, first-order differential equations

11/10/14

If we consider the two terms on the right side of the partial wave equation, then we find that the first one dominates for off-diagonal momenta.

$$\Rightarrow \frac{dV_s}{ds}(k,k') \doteq -(k^2-k'^2)V_s(k,k') \quad \text{for } k \gg k' \text{ or } k' \gg k$$

But now the equations are all decoupled and the solution is [in the exercises]

$$V_s(k,k') = V_{s=0}(k,k')\, e^{-s(k^2-k'^2)^2}$$

so off-diagonal matrix elements are driven to zero as s increases, and they go to zero faster the further off diagonal they are.

- Note that s has dimensions of $[L]^4$ [so fm^4 here]. We define $\lambda = s^{-1/4}$, which has dimensions of fm^{-1}, a momentum.
- λ^2 gives roughly the width in k^2 of V_s [exercise]

* Go back to slides and see k^2 width and universality.
  - also teaser for 3-body forces and block diagonalization.
  - If we decouple high momentum states, we expect to get 3-body (and higher body) forces
    $\Rightarrow$ eliminating degrees of freedom

11/10/14

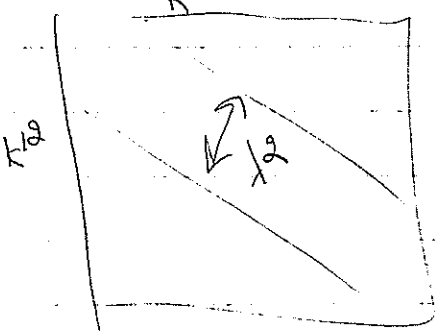
* Plazza, exercises preview

* Why would we want to repeat nuclear structure calculations for different values of the SRG λ (or s).
  - Observables are supposed to be unchanged
    ⇒ test if a quantity is an observable (example, clear demonstration that D-state probability in the deuteron is not)
    ⇒ determine the scale dependence of a quantity
  - Test for errors
  - Test approximations
    - We will see this particularly in considering many-body potentials and other operators.

* General equation is $\frac{dH_s}{ds} = [\eta(s), H_s] = [[G_s, H_s], H_s]$
  - T_{rel} (or T) doesn't change by construction
  - What if we used $T = T_{com} + T_{rel}$ for G_s instead of T_{rel}?
    (answer: $[T_{com}, H_s] = 0$, so no difference!)
  - other choices for G_s?

* $\langle k| \frac{dH_s}{ds} |k'\rangle \equiv \frac{dV_s(k,k')}{ds} = -\left(\frac{k^2}{k} - k'^2\right)^2 V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2 dq \left(k^2 + k'^2 - 2q^2\right) V_s(k,q) V_s(q,k')$

- If $\left(\frac{k^2}{k} - k'^2\right)^2 V_s(k,k')$ dominates then $V_s(k,k') \doteq V_{s=0}(k,k') e^{-\left(k^2 - k'^2\right)^2 / \left(\lambda^2\right)^2}$ $k \neq k'$ (must be sufficiently off diagonal)



- Look at slides.

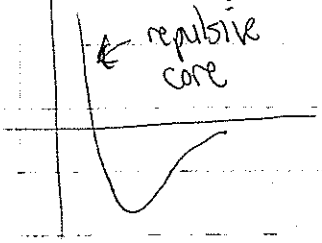
11/10/14

Visualizing the softening of NN interactions

- In momentum space we associate softening of a potential with the decreased coupling between high and low momentum:

$$\langle k_{high}|V|k_{low}\rangle \to 0$$

- But what does this do to our picture of potentials having strong short-range repulsion?



- Visualizing is not so easy, because the potential becomes non-local, so it is a functional of r and r'

- note that the $-(\vec{k}^2-\vec{k}'^2)^2 V_s(\vec{k},\vec{k}')$ term in the SRG equation (not partial wave projected) can be written using

$$\vec{k}^2-\vec{k}'^2 = (\vec{k}+\vec{k}')\cdot(\vec{k}-\vec{k}') \to \vec{p}\cdot\vec{q}$$

as an explicit function of total momentum $\vec{p}=(\vec{k}+\vec{k}')$ and not just momentum transfer $\vec{q}=\vec{k}-\vec{k}' \Rightarrow$ nonlocal

why so partial waves not mix? (except for already coupled) ⇒ insert expansions in k^2, k'^2, q^2 dependence from SRG

- Plan: use a local projection
  - The high-momentum tails of low-energy wavefunctions are suppressed by RG evolution which implies the wavefunction variation over short distances is small. So in the non-local Schrödinger equation:

$$-\frac{1}{2\mu}\nabla^2\psi(\vec{r}) + \int d^3r' V(\vec{r},\vec{r}')\psi(\vec{r}') = E\psi(\vec{r})$$

(treat as constant over range of V nonlocality)

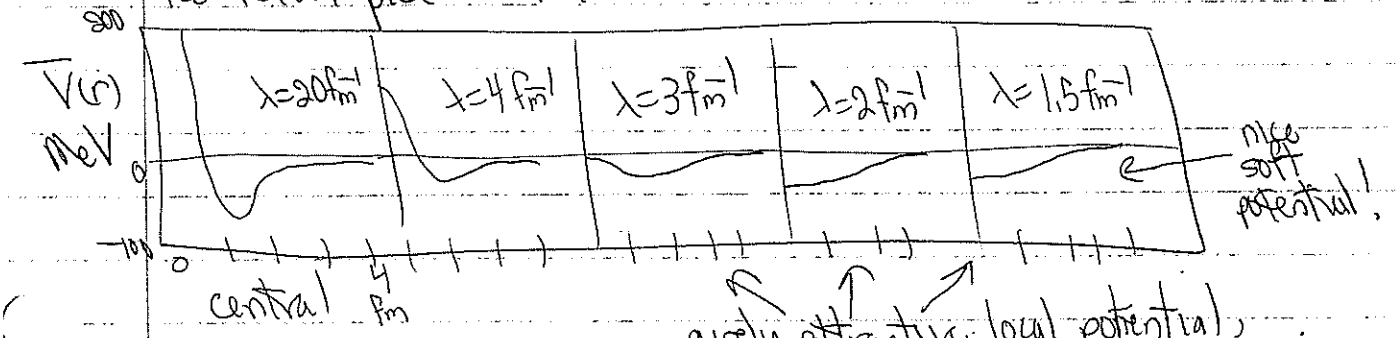
$$\to -\frac{1}{2\mu}\nabla^2\psi(\vec{r}) + \psi(\vec{r})\int d^3r' V(\vec{r},\vec{r}') \approx E\psi(\vec{r})$$

11/10/14

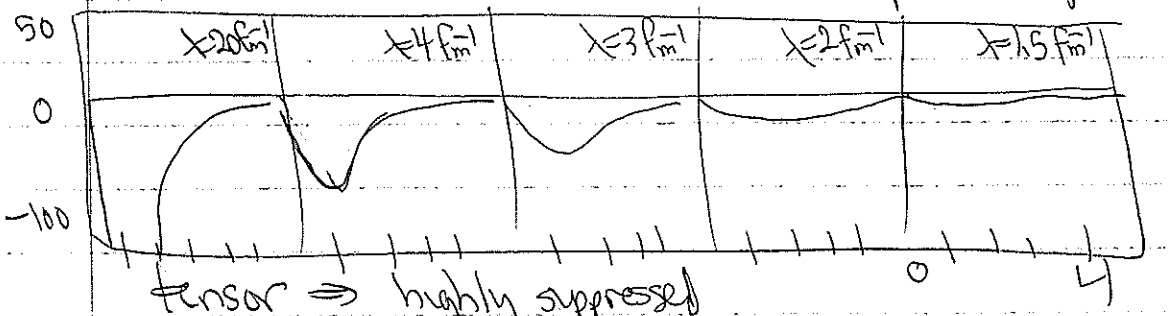
⇒ define $\overline{V_\lambda}(r) = \int d^3r' V_\lambda(\vec{r}, \vec{r}')$ as the local projection.

- Kyle Wendt has developed this idea further, to apply beyond S-waves (which is all that survives the angular integral)

- We'll sketch the result for the AV18 potential and look at the actual pictures later.



purely attractive local potential; so phase shift must fail to change sign ⇒ non-local part at higher momentum



tensor ⇒ highly suppressed

- D-state probability changes greatly
- But asymptotic D-S ratio unchanged!
- What about quadrupole moment?

- Different potentials evolve to same in both momentum rep. (at momentum below λ and in local projection),
- Where do you expect high energy contributions to go? cf. $\Lambda_0 \to \Lambda_0 - \Delta\Lambda + \Delta\Lambda$

⇒ see slides (same thing here!)

11/10/14

- Wegner chose for the flow-equation generator (to see an alternative)
  ⇒ use the diagonal of H_s in whatever basis you are in. E.g., $H_d = T_{rel} + V_s(k,k)$ ← diagonal

- Let's consider the general case with $H_{ii} \equiv e_i$ where we are labeling the basis elements $|i\rangle, |j\rangle, \ldots$
  - Note that these could be plane waves, harmonic oscillators, 2 particle or more, ...

$$\langle i|\frac{dH_s}{ds}|j\rangle = \frac{dH_{ij}}{ds} = \langle i|[[[H_d,H_s],H_s],H_s]|j\rangle$$

$$= \langle i| \overset{\leftarrow}{H_d}H_sH_s - H_sH_dH_s - H_sH_dH_s + H_sH_sH_d |j\rangle$$
(a) (b) (c) (d)

insert $\sum_k |k\rangle\langle k|$ and use $H_d|j\rangle = e_j|j\rangle$, etc. between H_s's

$$\Rightarrow \frac{dH_{ij}}{ds} = \sum_k (\overset{(a)}{e_i} - \overset{(b)}{e_k} - \overset{(c)}{e_k} + \overset{(d)}{e_j}) H_{ik}H_{kj} \quad \Leftarrow \text{simple matrix multiplication!}$$

$i=j \Rightarrow dH_{ii}/ds = 2\sum_k (e_i - e_k)|H_{ik}|^2$

We want to ask: what can we say about $\frac{d}{ds}\sum_{i\neq j}|H_{ij}|^2$?
- This is the sum of the squares of the off-diagonal ($i\neq j$) parts. Does it decrease?

- The full sum is $\sum_{ij}|H_{ij}|^2 = \sum_{ij} H_{ij}H_{ji} = \mathrm{Tr}\, H_s^2 =$ constant! The trace is invariant.

So $\frac{d}{ds}\sum_{i\neq j}|H_{ij}|^2 = -\frac{d}{ds}\sum_i |H_{ii}|^2 = -2\sum_i \underbrace{H_{ii}}_{e_i}\underbrace{\frac{dH_{ii}}{ds}}_{\text{from eqn's}} = -4\sum_{i\neq k} e_i(e_i - e_k)$

$$= -2\sum_{i\neq k}(e_i-e_k)^2|H_{ik}|^2 \leq 0 !$$

⇒ except for degeneracies, off-diagonal elements decrease.

14/10/14

- The use of T_{rel} instead of $H_0 = T_{rel} + H_{ij}$ is ok for nuclear physics, at least in the momentum basis, because $T_{rel} \gg (V_S)_{ij}$ so $H_0 \approx T_{rel}$

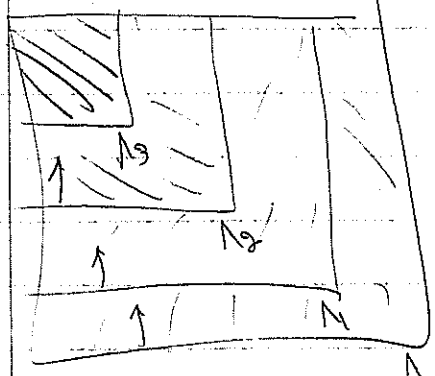
25 Schlussel

- It can fail, though: ⇒ see Wendt et al. with large Λ leading order forces,

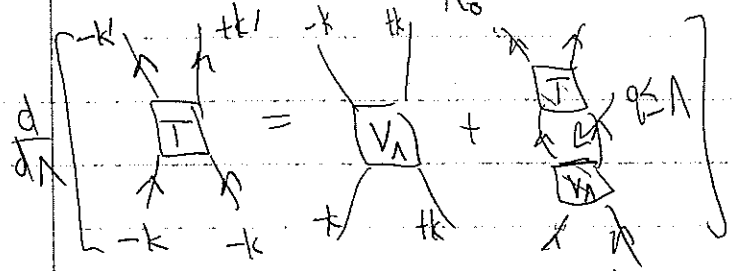
⇒ good example of decoupling.

11/10/14

• $V_{low\,k}$ RG equation – Bogner, Kuo, Schwenk (2001)



- based on requiring the half-off shell T matrix to be invariant with a change in cutoff on the sum over intermediate states.



initial $T(k',k;k^2) = V_{NN}(k',k) + \frac{2}{\pi}\mathcal{P}\int_0^{\Lambda_\infty} \frac{V_{NN}(k,p)T(p,k;k^2)}{k^2-p^2}\,p^2dp$

($\mathcal{P}$: principal value sum/integral; half-on-shell because $k;k^2$ but $p \neq k$)

for all $k,k' < \Lambda$, with cutoff $= V^{\Lambda}_{low\,k}(k',k) + \frac{2}{\pi}\mathcal{P}\int_0^{\Lambda} \frac{V^{\Lambda}_{low\,k}(k,p)T(p,k;k^2)}{k^2-p^2}$

Take $\frac{dT}{d\Lambda} = 0 \Rightarrow \frac{d}{d\Lambda}V^{\Lambda}_{low\,k}(k',k) = \frac{2}{\pi}\frac{V^{\Lambda}_{low\,k}(k,\Lambda)T(\Lambda,k;\Lambda^2)}{1-(k/\Lambda)^2}$ ← derivation is not immediate, (see Bogner et al.)

cf. partial wave SRG equation (with $G_S = T_{rel}$)

$$\frac{d}{d\lambda}V_\lambda(k,k') = \left(-\frac{4}{\lambda^5}\right)\left(k^2-k'^2\right)^2 V_\lambda(k,k') + \frac{2}{\pi}\int_0^\infty \left(k^2+k'^2-2q^2\right)V_\lambda(k,q)V_\lambda(q,k')\,q^2dq$$

from $\frac{d}{ds} = \frac{d\lambda}{ds}\frac{d}{d\lambda}$ and $s = 1/\lambda^4$

- Compare rhs: T matrix for $V^{\Lambda}_{low\,k}$ but just potential for SRG

⇒ SRG much easier for $A > 2$ (otherwise need T matrix in all channels).

11/10/14 =

- Can we get a $V_{low\,k}$-like potential from the SRG flow equation by an appropriate choice of G_s? Yes!
  - Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s(\Lambda) = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}_\Lambda$
  - Choose Λ and then G_s is the running Hamiltonian with the off-diagonal blocks defined by Λ set equal to zero.
    - P and Q are projection operators, $P+Q=1$.
- Proof that this does what we want [Gubankova et al.]
  - A measure of off-diagonal coupling is QH_sP (Flavor) so this is the part that does the coupling. (Yes)
  - $\Rightarrow \sum_n \langle \eta_n | (QH_sP)^\dagger (QH_sP) | \eta_n \rangle = \mathrm{Tr}[PH_sQH_sP] \geq 0$ (since $Q^2=Q$, $P^2=P$)
    $[G_s, H_s] = [PH_sP + QH_sQ, H_s]$
- Now check how this changes with s using $\frac{d}{ds}H_s = [\eta_s, H_s]$

$$\frac{d}{ds}\mathrm{Tr}[PH_sQH_sP] = \mathrm{Tr}\{P\eta_s Q(QH_sQH_sP - QH_sPH_sP)\}$$
$$+ \mathrm{Tr}[(PH_sPH_sQ - PH_sQH_sQ)Q\eta_sP]$$

you are invited to prove it $\Rightarrow$ $= -2\,\mathrm{Tr}[(Q\eta_sP)^\dagger(Q\eta_sP)] \leq 0$

$\Rightarrow$ The off-diagonal QH_sP block will decrease (or not increase) as s increases.

- Two examples: $G_s = \mathrm{Tr}\left(\;\right)$ $\rightarrow$ goes to $\left(\;\right)$

$G_s = \left(\;\right)$ $\Rightarrow$ goes to $\left(\;\right)$,

Does it always evolve to the pattern of G_s? See pictures!

11/10/14

(RG-14)

SRG Flow of Operators

- We'll have more to say about operators in a future lecture; just some basic ideas here.

- When we transform $H_s = U(s) H_{s=0} U^\dagger(s)$, the wave functions also get transformed: $|\psi_n^s\rangle = U(s)|\psi_n^{s=0}\rangle$, so that energies are unchanged $E_n = \langle\psi_n^{s=0}|H_{s=0}|\psi_n^{s=0}\rangle = \langle\psi_n^s|H_s|\psi_n^s\rangle$
- So this means that any operator O must be transformed:

$$O_s = U(s) O U(s)^\dagger$$

- We can calculate this directly by constructing $U(s)$ and $U(s)^\dagger$. We do this by first evolving H to H_s, then finding all the eigenstates $|\psi_n^s\rangle$ of H and $|\psi_n^{s=0}\rangle$ of $H_{s=0}$.

- Then we have $U(s) = \sum_n |\psi_n^s\rangle\langle\psi_n^{s=0}|$
  - in a basis like momentum space, this would give us the matrix element $\langle k|U(s)|k'\rangle = \sum_n \langle k|\psi_n^s\rangle\langle\psi_n^{s=0}|k'\rangle$
  - just an outer product.

- This works fine in practice but there are two other ways
  - i) Evolve O_s with its own flow equation
  - ii) evolve $U(s)$ " " " " " and then use $O_s = U(s) O U(s)^\dagger$

What are the equations? ⇒ you do that for exercises!

11/10/14

How do we know that SRG evolution of operators (including the Hamiltonian) must generate many-body terms?

- In exercises: think about physics. Here: formal discussion.

- Consider 2nd quantization. This is defined with two ingredients:
  1. a single-particle basis (e.g. plane waves [in a box] or HO wfns)
  2. a reference state that serves as the "vacuum".

examples: - could be the actual vacuum
- or a filled core (Fermi sea or a closed shell) $\{ q_i|0\rangle = 0$

- Kinetic energy: $T = \sum_{ij} \frac{\hat{p}_i^2}{2m} a_i^\dagger a_i \to \sum_{ij} a_i^\dagger \langle i|T|j\rangle a_j$ (the labels i, j, ... could refer to (discretized) momenta, or to harmonic oscillators, or ...)

- Two body potential: $\frac{1}{4}\sum V^{(2)}_{ijkl} a_i^\dagger a_j^\dagger a_l a_k$

- 3-body-potential: $\frac{1}{36}\sum V^{(3)}_{ijklmn} a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l$

or $\hat{A} = \sum_{ij} a_i^\dagger \langle i|T|j\rangle a_j + \frac{1}{4}\sum_{ijkl} a_i^\dagger a_j^\dagger \langle ij|V^{(2)}|kl\rangle a_l a_k + \frac{1}{36}\sum_{ijklmn} a_i^\dagger a_j^\dagger a_k^\dagger \langle ijk|V^{(3)}|klm\rangle$ (anti-symmetrized matrix element $\Rightarrow$ $\frac{1}{4}$, $\frac{1}{36}$)

- These operators have anti-commutation relations:

$$\{a_i, a_j^\dagger\} = a_i a_j^\dagger \pm a_j^\dagger a_i = \delta_{ij}, \quad \{a_i, a_j\} = \{a_i^\dagger, a_j^\dagger\} = 0$$

Claim: $\frac{dV_s}{ds} = \left[\left[\underbrace{\sum a^\dagger a}_{G_s = T}, \underbrace{\sum a^\dagger a^\dagger a a}_{2\text{-body}}\right], \underbrace{\sum a^\dagger a^\dagger a a}_{2\text{-body}}\right] = \ldots + \underbrace{\sum a^\dagger a^\dagger a^\dagger a a a}_{3\text{-body!}}$

- And this is just one time step!
- $\Rightarrow$ A-body operators generated
- Is this a problem?
  - depends: we need to be able to truncate $\Rightarrow$ need hierarchy
  - also need to be able to calculate with minimal (usually 3-body)

show this in the exercises

In-medium SRG $\Rightarrow$ - Alternative: Pick a different reference state $\Rightarrow$ reshuffles what is many-body!

11/10/14

SRG: technology to evolve 3-body forces.

Three methods now exist:

slides by Angelo Calci

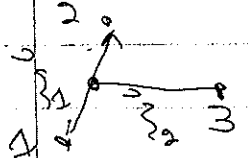
i) evolve in a discrete harmonic oscillator basis (Eric Jurgenson)
⇒ applied to No-Core Shell Model (tomorrow)
many developments in Darmstadt, P. Navratil

ii) evolve in a partial-wave momentum basis (Kai Hebeler)
no disconnected pieces
⇒ separate evolution of 2 and 3 body parts
⇒ applied to neutron matter (next week)

iii) evolve in a hyperspherical basis (Kyle Wendt)
⇒ good features, visualization

- more later on these comparisons
- Recent! 4-body evolution! (see Angelo Calci talk from Trento)

- Oscillator evolution is in a 3-body Jacobi basis
  - generalization of center-of-mass and relative



$$\vec{\xi}_0 = \sqrt{\frac{1}{3}}\left[\vec{r}_1+\vec{r}_2+\vec{r}_3\right] \leftarrow \text{potential doesn't depend}$$

$$\vec{\xi}_1 = \sqrt{\frac{1}{2}}\left[\vec{r}_1-\vec{r}_2\right] \leftarrow \text{relative between 1 and 2.}$$

$$\vec{\xi}_2 = \sqrt{\frac{2}{3}}\left[\frac{1}{2}(\vec{r}_1+\vec{r}_2)-\vec{r}_3\right] \leftarrow \text{relative btwn 3 and com of 1 and 2}$$

(com of 1,2)

hard part: must antisymmetrize HO basis: $|\alpha\rangle = |[(N_1 L_1 S_1) J_1 (N_2 L_2 S_2) J_2] J M_J, (T_1, T_2) T M_T\rangle$

- momentum space evolution $|p q \alpha\rangle_i \equiv |p_i q_i; [(LS) J (l s_i) j] J\rangle_z (T t_i) T M_T\rangle$