· Before a section, look through the two-minute and discussion questions so you will recognize important points.

2-1

2. QCD-1

- Low-energy nuclear physics is the physics of QCD and electroweak interactions (which includes electron and neutrino scattering and beta decay). But this region is very difficult to solve.

- We will use low-energy effective theories, but to do so we need to build on some knowledge of QCD. (However, we will introduce but not resolve subtleties)
  - We will have much to say later about effective theories and their field theory implementations (EFTs).
  - For now, we need to identify the degrees of freedom and what symmetries (or other properties) are inherited at low energies, and how QCD might simplify in some limits, e.g. $m_u = m_d \doteq 0$, $m_c, m_b, m_t \to \infty$

- QCD Lagrangian
  - dofs are quarks (spin 1/2 fermions) and gluons (spin 1 gauge bosons)
  - quark field ψ has Lorentz, color, and flavor indices, function of space-time x^μ
    - Lorentz: 1,2,3,4; color: r,g,b; flavor: u,d,s,c,b,t

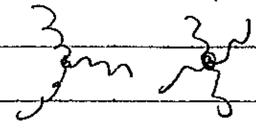
$$\mathcal{L}_{QCD} = \sum_{q = u,d,s,\ldots} \left(\bar\psi_{q,i}(x)\left[(i\gamma_\mu D^\mu)_{ij} - m_q\delta_{ij}\right]\psi_{q,j}\right) - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}$$

$q = u,d,s,\ldots$ flavors; $i,j = 1,2,3$ color (red, blue, green)

$$D^\mu = \partial_\mu - igA^a_\mu \frac{\lambda^a}{2} \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, +\vec\nabla\right)$ note + sign

- Lorentz indices are implicit
- λ^a are 8 ($a = 1,\ldots,8$) Gell-Mann matrices (cf. Pauli matrices) $\Rightarrow (\lambda^a)_{ij}$ — 3x3 traceless, Hermitian
- gluons are "flavor blind" (don't distinguish between flavors)
- gluon has quadratic but also cubic and quartic terms $\Rightarrow$ self-interactions (but only with ∂_μ's $\Rightarrow$ no mass term)
- quark emitting gluon changes color (in general)



- Often $A_\mu \equiv A^a_\mu \frac{\lambda^a}{2}$ as a matrix field (SU(3) space)
- Rules from Lagrangian: quadratic parts give propagators, others give interaction vertices. Coefficients give coupling strength. (so mass term from $\bar\psi\psi$ and no A^2 means massless gluon)

inputs to QCD: g and "current" quark masses m_q ← specify values

	charge	mass	
u (up)	$2/3$	2-3 MeV	2 clearly light quarks
d (down)	$-1/3$	4-6 MeV	⇒ relevant for low-energy nuclear physics
c (charm)	$2/3$	≈ 1.3 GeV	3 heavy c, t, b on nuclear scales
s (strange)	$-1/3$	≈ 100 MeV	← what to do about strange?
t (top)	$2/3$	≈ 170 GeV	Is it part of low-energy nuclear physics?
b (bottom)	$-1/3$	≈ 4.5 GeV	Where have you heard of it?

3 generations

- defining mass is subtle given confinement (see below)
- quark mass is like a coupling constant ⇒ depends on momentum scale and renormalization scheme.
- u, d, s defined with $\overline{MS}$ at $\mu = 2$ GeV (see PDG for more up-to-date values)

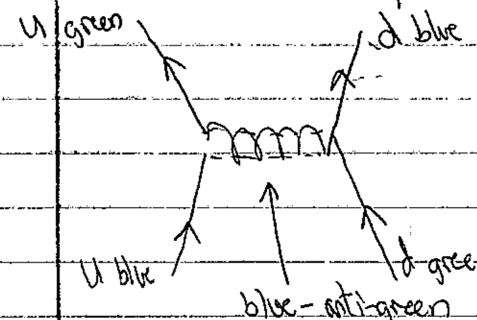
What does QED Lagrangian look like if we suppress indices?

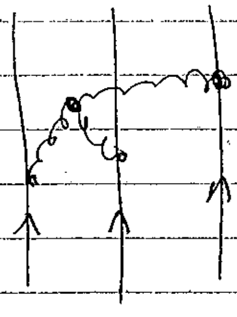
$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu \underbrace{D_\mu}_{\partial_\mu - ieA_\mu} - m)\psi - \frac{1}{4}\underbrace{F_{\mu\nu}}_{\partial_\mu A_\nu - \partial_\nu A_\mu} F^{\mu\nu}$$

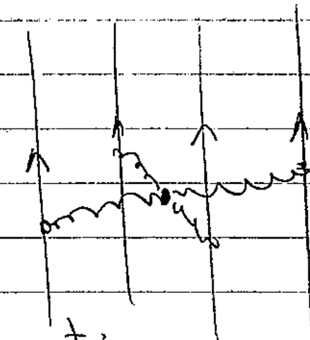
only 1 photon field (cf. $a = 1, \ldots 8$ gluons)

QED		QCD
e^+, e^- electric charge q	→	$q, \bar{q}$ quarks color charge g
1 photon	→	8 gluons (massless)
gauge group: $U(1)$	→	$SU(3)$

Forces between quarks mediated by massless gluons (cf. photons)







3 and 4 gluon vertices

color

$$L_{int} = g \sum_q \bar{\psi}^i_q(x) \left(\frac{\lambda^a}{2}\right)_{ij} \gamma^\mu \psi^j_q(x) A^a_\mu(x)$$

flavor

$\Rightarrow$ 3 and 4 quark interactions (many body forces!...)

- a summed $1, \ldots, 8$
- q summed $u, d, s, \ldots$
- i, j summed $1, 2, 3$
- implicit Lorentz indices on ψ's and γ^μ summed $1, 2, 3, 4$

Note: only 8 gluons $= 3^2 -$ color blind singlet $\frac{1}{\sqrt{3}}(|r\bar{r}\rangle + |b\bar{b}\rangle + |g\bar{g}\rangle)$

r = red, b = blue, g = green

- Coupling $\frac{g^2}{4\pi} \equiv \alpha_s$ "strong coupling"

Compare to $\alpha_{QED} = \frac{e^2}{4\pi} \doteq \frac{1}{137}$ (at $q^2 = 0 \Rightarrow$ it runs like α_s as well for $q^2 > 0$, So this means the value for long distances, e.g. Coulomb force in atom)

<u>Running couplings</u>

- From electromagnitism, we know that the effective Coulomb force in a polarizable medium is altered by a dielectric constant ε

$$F = \frac{q_1 q_2}{4\pi r^2} \rightarrow \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2} \text{ with } \varepsilon > 1 \text{ from charge screening (or use } \varepsilon_0 \text{ and } \varepsilon)$$

We could say the charge becomes $\tilde{q}_i = \frac{q_i}{\sqrt{\varepsilon}} < q_i$

- The QED vacuum is like a polarizable medium because there are fluctuating e^+-e^- pairs

e^- e^+ e^- e^+ e^- e^+ e^-

Key observation: screening is different $\Rightarrow$ coupling strength varies with distance (or equivalently momentum by Fourier transform)

This variation is called "running" and will be a key concept for us considering forces between nucleons at low energies.

- We have made a physical picture for QED, but later we will consider how the running coupling emerges more formally.

· The change in the coupling as we change a momentum scale μ is given by a renormalization group equation

$$\mu \frac{d}{d\mu}\alpha(\mu) = \beta(\alpha)$$ ← right side is called the "beta function"

· For QED, $\beta_{em} \doteq \frac{2\alpha_{QED}^2}{3\pi} > 0 \Rightarrow \frac{1}{\alpha_{QED}(\mu)} = \frac{1}{\alpha_{QED}(\mu_0)} - \frac{1}{3\pi}\ln\frac{\mu^2}{\mu_0^2}$

after integrating $\mu\frac{d}{d\mu}\alpha_{QED} = \frac{d\alpha_{QED}}{d\ln\mu} = \frac{2\alpha_{QED}^2}{3\pi}$ from μ_0 to μ.

Note: $\alpha_{QED}(\mu=0) = \frac{1}{137.035}$ and $\alpha_{QED}(\mu>0) > \alpha_{QED}(0) \Rightarrow$ gets stronger at higher energies or shorter distances

- But for QCD it is different

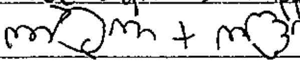
$$\beta_{QCD}(\alpha_s) = -\frac{\beta_0}{2\pi}\alpha_s^2 + \ldots \quad \text{with } \beta_0 = 11 - \frac{2}{3}n_f$$ ← # of "active" quark flavors (if $m_q^2 \gg Q^2$ then not active)

(This is at "one loop") gluon "anti-screening" $q\bar{q}$ screening like e^+e^- in QED

$$\Rightarrow \frac{1}{\alpha_s(\mu)} = \frac{33-2n_f}{6\pi}\ln\left(\frac{\mu}{\Lambda_{QCD}}\right)$$ where Λ_{QCD} replaces the integration constant $\alpha_s(\mu_0)$

(or $\alpha_s(\mu)^{-1}$ in terms of $\alpha_s(\mu_0)^{-1}$ → relate two different measurements as in figure)

For $\mu = Q \gg \Lambda_{QCD}$ (high momentum), for $n_f \leq 16$ $\alpha_s(Q) \to 0$ "asymptotic freedom"

- Gross, Politzer, Wilczek won Nobel Prize for this (noting the minus sign)
- Key is the gluon contribution flips the sign ⇒ happens because of gluon self-interaction  in QCD.

· See the figure with $\alpha_s(Q)$ vs. Q and $\alpha_s(m_Z)$ from different experiments and theory (LQCD is best!)

Λ_{QCD} is the scale of QCD ("standard kilogram") $\Rightarrow$ replace g as input
- depends on renormalization scheme and number of active quark flavors
- For $\overline{MS}$ and 3 quark flavors, $\Lambda_{QCD} \approx 250$ MeV

- From $\frac{1}{\alpha_s(Q)} = \left(\frac{33-2n_f}{6\pi}\right) \ln \frac{Q}{\Lambda_{QCD}}$

- we see for $Q \gg \Lambda_{QCD}$, $\alpha_s(Q) \to 0$ (but slowly)
- As $Q \to \Lambda_{QCD}^+$, if we could believe the formula still (we can't!), then we have $\alpha_s(Q) \to \infty$.
  $\Rightarrow$ at least perturbation theory doesn't work anymore.

- At high energies, perturbation theory works, as verified by many calculations (see slide on quark jets, which are result of initial high energy $q\bar{q}$ pair or $q\bar{q}$ + gluon, which become a shower of particles in the initial direction).

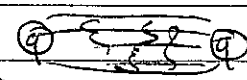
- So Λ_{QCD} is a dividing scale.
  - The nonperturbative physics below this scale leads us to use effective field theory for nuclear forces and nuclei.
  - There are two major nonperturbative consequences
    1) color confinement
    2) chiral symmetry breaking

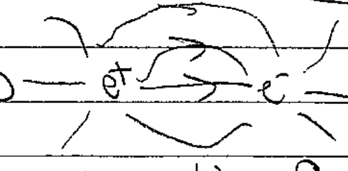
1) means quarks cannot be isolated; confined to color singlet (colorless) hadrons.
- Confinement is indicated experimentally and by lattice calculations, but not formally proven (Millenium problem!)

Slide shows calculation of potential energy between quarks in calculation without sea quarks (so no $q\bar{q}$ pairs from vacuum),

- We see Coulombic ($-\frac{1}{r}$) behavior at short distances but linear growth at large r ⇒ like a string stretching

- The energy to separate $q\bar{q}$ is expressed as $E = \sigma r$ ← "string tension"

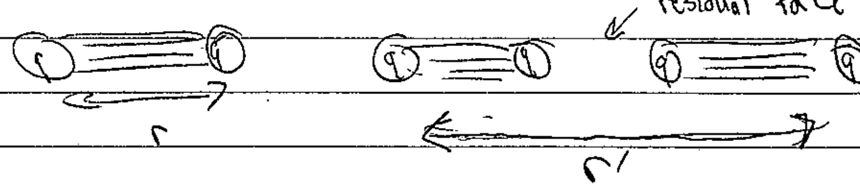
- understood as color flux tube 

(as opposed to QED 

spread out flux)

- If $q\bar{q}$ pairs allowed, string/gluon flux tube breaks as r increases when E can create a new $q\bar{q}$ pair
  ⇒ one meson becomes two mesons!

residual force between mesons



Bottom line: Confinement implies that any strong interaction system (at zero temperature and density) must have a color singlet wave function at distances larger than $1/\Lambda_{QCD}$.

⇒ degrees of freedom at low energies are hadrons

masses of hadrons ⇒ look at figure of lattice QCD calculations
- π, K, Σ input (need $m_u = m_d$, m_s, g ⇒ $N_f = 2$ light + 1 heavy quark ← degenerate)
  then all predictions!
  ⇒ strong evidence that QCD is nonperturbatively correct and calculable! (why not just do nuclei?)

bosons: mesons ($q\bar{q}$) $\pi, \rho, \ldots$ focus on hadrons with u, d valence quarks
fermions: baryons (qqq) $N, \Delta, \ldots$ Need $q\bar{q}$ or qqq for color singlet

- light vs. heavy quarks from comparison to Λ_{QCD}: $m_u, m_d \ll \Lambda_{QCD}$
  What about the strange quark?

- Note that $M_{hadron} \sim 1$ GeV except for light π (and maybe K)
  ∴ But where are the different charge states?
- Key concept: QCD symmetries of quarks are reflected in symmetries in hadron spectrum (eg. energies of ρ^0, ρ^+, ρ^- compared to each other).

$\frac{m_u - m_d}{\Lambda_{QCD}} \ll 1$ so consider limit that this is zero

$\Rightarrow$ u, d quarks form an isospin multiplet

$|u\rangle = |\text{isospin} \uparrow\rangle = |T=\frac{1}{2}, m_T = \frac{1}{2}\rangle$ where isospin operator is
$|d\rangle = |\text{isospin} \downarrow\rangle = |T=\frac{1}{2}, m_T = -\frac{1}{2}\rangle$ $\vec{T} = \hbar \frac{\vec{\tau}}{2}$
with Pauli matrices $\vec{\tau}$ (same as $\vec{\sigma}$, but different symbol to keep track that it is a different space)

- gluons don't care about flavor and $m_u, m_d \approx 0$ on Λ_{QCD} scale, so near isospin symmetry makes sense.

Isospin symmetry (approximate because $m_u \neq m_d$) is clearly seen in the hadron spectrum (see PDG tables).

baryons: nucleon $N(\frac{1}{2}^+)$ ← J^π $|n\rangle = |T=\frac{1}{2}, m_T = -\frac{1}{2}\rangle$ d(ud) ← diquark coupled to spin 0
≈ 940 MeV $|p\rangle = |T=\frac{1}{2}, m_T = +\frac{1}{2}\rangle$

Isospin doublet in nonstrange (S=0, strangeness not spin) part of baryon octet (see the figure)

Delta isobars $\Delta(\frac{3}{2}^+)$ 1232 MeV

$|\Delta^-\rangle$ $|\Delta^0\rangle$ $|\Delta^+\rangle$ $|\Delta^{++}\rangle$
ddd udd uud uuu
$|T=\frac{3}{2}, m_T = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\rangle$

Simple constituent quark mass model: $m_{constituent} = \frac{m_{\Delta^{++}}}{3} \doteq 400 \text{ MeV} \sim \Lambda_{QCD}$ ← an effective mass

$m_N = 3\, m_{constituent} - B_{diquark}$ ← diquark ud S=0 binding energy ≈ 300 MeV
≈ 900 MeV

$m_p \approx 938.3$ MeV, $m_n \doteq 939.6$ MeV $\Rightarrow$ explain why heavier: $m_u \neq m_d$, electromagnetism?

2-8

mesons: ($L=0$ states)

pseudoscalar pions: $\pi(0^-)$ (scalar; pseudo (parity odd from $P_q P_{\bar{q}} (-1)^L = (+1)(-1)(+1) = -1$), $L=0$) $|\pi^-\rangle, |\pi^0\rangle, |\pi^+\rangle$ — $\bar{u}d$; $\bar{u}u - \bar{d}d$, $\bar{d}u$

140 MeV $|T=1, m_T = -1, 0, 1\rangle$

vector meson $\rho(1^-)$ $|\rho^-\rangle, |\rho^0\rangle, |\rho^+\rangle$

~ 770 MeV

Check constituent quark mass model expectations

$m_\rho \simeq 2 m_{constituent} \simeq 800$ MeV ($S=1$ so no $B_{diquark}$)

$\Rightarrow$ consistent

but $m_\pi \simeq 140$ MeV $\ll 2m_{constituent} - B_{diquark} \doteq 500$ MeV

Why is it so light? Answer: it is an approximate Goldstone boson of spontaneously broken chiral symmetry

Consider $\mathcal{L}_{QCD}$ symmetries with massless quarks

- If $m_q = 0$, spin is either in direction of motion (right handed) or in opposite direction (left-handed)
  [Discussion question: How does this follow formally?]

- The corresponding quark fields can be projected:

$$\psi = \psi_L + \psi_R = \tfrac{1}{2}(1-\gamma_5)\psi + \tfrac{1}{2}(1+\gamma_5)\psi$$

- If no mass term in $\mathcal{L}_{quark}$, then splits into two non-communicating $\mathcal{L}$'s: (QCD interaction does not couple massless left and right-handed quarks)

$$\mathcal{L}_q = \bar{u}\, i\not{D}\, u + \bar{d}\, i\not{D}\, d = \bar{u}_L i\not{D} u_L + \bar{u}_R i\not{D} u_R + \bar{d}_L i\not{D} d_L + \bar{d}_R i\not{D} d_R$$

- So $\mathcal{L}_q$ is symmetric under independent rotations in u,d space of L- and R-handed quarks
  (i.e. mix u_L and d_L separately from mixing u_R and d_R)

The symmetry group is

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_L \otimes U(1)_R$$

$$= \underset{\substack{\shortparallel \\ \text{vector} \\ \shortparallel \\ \text{isospin}}}{SU(2)_{L+R}} \otimes \underset{\substack{\shortparallel \\ \text{axial} \\ \shortparallel \\ \text{chiral}}}{SU(2)_{L-R}} \otimes \underset{\substack{\uparrow \\ \text{baryon number symmetry}}}{U(1)_V} \otimes \underset{\substack{\uparrow \\ \text{broken by quantum effects (anomaly)}}}{U(1)_A}$$

rotate L and R together

rotate opposite

$SU(2)_{isospin}$ is reflected in hadron spectrum (examples?)

$SU(2)_{axial}$ would imply degenerate parity partners.
E.g., for the nucleon $N(1/2^+)$ and $N(1/2^-)$
but $m_N^{1/2^+} \doteq 940$ MeV $\neq$ 1535 MeV

$\Rightarrow$ chiral symmetry is spontaneously broken in the QCD ground state (i.e., the vacuum)

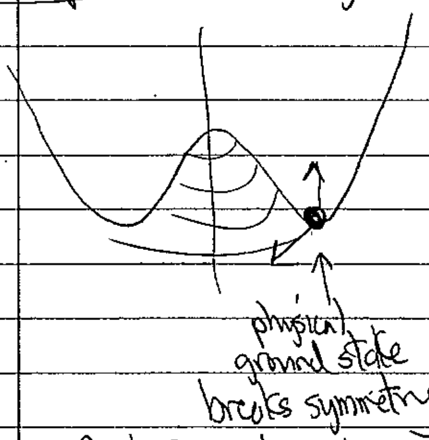
In addition, $SU(2)_A$ is explicitly broken because $m_u, m_d \neq 0$, and a mass term mixes L, R

$$\mathcal{L}_{q,mass} = -\bar{u}_R m_u u_L - \bar{u}_L m_u u_R - \bar{d}_R m_d d_L - \bar{d}_L m_d d_R$$

Note: we'll return to this multiple times!

Spontaneous Symmetry Breaking

eg. for a scalar field



Effective potential tells us about possible ground states.
- Here there is a symmetry: all choices in the bottom of the valley have the same energy.
- But one is picked out in the vacuum $\Rightarrow$ "SSB"

- But then low-lying excitations in the original symmetry direction cost very little energy $E \sim \frac{h}{\lambda}$
  (long-wavelength modes are almost degenerate)

$\Rightarrow$ SSB leads to massless Goldstone bosons

Other examples of SSB and Goldstone bosons ....

phase	broken symmetry	Goldstone boson
crystal	translations	phonon = lattice vibrations
ferromagnet $\uparrow\uparrow\uparrow\uparrow\uparrow$	rotations	magnons $\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow$ spin wave λ

Light pions are the Goldstone bosons of chiral symmetry breaking
Gell-Mann-Oakes-Renner relation $\Rightarrow m_\pi^2 \sim m_q$
- so finite pion mass because $m_{u,d} \neq 0$.
- but note that $m_\pi^2 / m_g^2 \ll 1$

Consequences for nuclear forces
- π's interact weakly at low momentum $\sim q$
- self-interaction of π's

Chiral symmetry breaking also responsible for dynamical mass generation of $m_{constituent} \approx 400\,MeV \gg m_{u,d}$

QCD Phase diagram (see slide)

- At high temperatures and densities $\Rightarrow$ high momenta
  $\Rightarrow$ asymptotic freedom

- There are transitions to deconfinement (not confined to hadrons) and chiral symmetry restorations ($m_{const.} \to m_{u,d}$)

- For chemical potential $\mu = 0$, lattice QCD says $T_c \gtrsim 170$ MeV $\sim 10^{12}$ K (depends on # of flavors)

- But naive expectation of weakly interacting plasma is not found! $\Rightarrow$ topic of RHIC physics.

- We will focus on the low T, low baryon density region of the QCD phase diagram
  $\Rightarrow$ degrees of freedom (dofs): nucleons and pions (and Δ's) and sometimes only nucleons. (When?)

Some comments on units ...

- We will work with units where $\hbar = c = 1$
- Use $\hbar c = 197.327$ MeV fm ≈ 200 MeV-fm
  to convert MeV $\leftrightarrow$ fm^{-1} or fm $\leftrightarrow$ MeV^{-1}

e.g. pion mass $m_\pi = 140 \text{ MeV} = \frac{140 \text{ MeV}}{\hbar c} = \frac{140 \text{ MeV}}{200 \text{ MeV-fm}} \doteq 0.7 \text{ fm}^{-1}$

put anywhere because it is 1!

(inverse de Broglie wavelength)

We'll also use $\frac{\hbar^2}{m_N} = 41.4 \text{ MeV-fm}^2$ a lot.

Can you derive $\frac{\hbar^2}{m_N} \approx 40$ MeV quickly?