5. Scattering Theory - 2

- Here we consider what nucleon-nucleon (NN) phase shifts can tell us about the nuclear (two-body!) force.

- Consider first the quantum numbers that specify NN scattering
  - orbital angular momentum $\vec{L}$; $L = 0, 1, 2, \ldots$
    - not conserved
  - total spin $\vec{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$ $S = 0, 1$
    - conserved
  - total angular momentum $\vec{J} = \vec{L} + \vec{S}$ $J = |L-S|, \ldots L+S$
    - conserved by rotational symmetry
  - total isospin $\vec{T} = \frac{1}{2}(\vec{\tau}_1 + \vec{\tau}_2)$ $T = 0, 1$
    - conserved

Adding angular momenta: $J = \begin{cases} L, & S=0 \\ |L-1|, L, L+1 & S=1 \end{cases}$

- What are the allowed partial waves if we account for the Pauli principle for fermions $\Rightarrow$ totally antisymmetric wave function

even $L = 0, 2, 4, \ldots$ $\rightarrow$ $S=0$ $\Rightarrow$ $T=1$
spatial wf is symmetric × antisymmetric × symmetric = antisymmetric ✓
$\rightarrow$ $S=1$ $\Rightarrow$ $T=0$
× symmetric × antisymmetric "

odd $L = 1, 3, 5, \ldots$ $\rightarrow$ $S=0$ $\Rightarrow$ $T=0$
spatial wf is antisymmetric × antisym. × antisym = antisym. ✓
$\rightarrow$ $S=1$ $\Rightarrow$ $T=1$
× symmetric × symmetric "

So if given L, S, J, then T is specified by Pauli principle

· We use a spectroscopic notation to specify NN scattering <u>channels</u>:

$$^{2S+1}L_J \quad \text{with} \quad \begin{array}{ccccc} L=0, & 1, & 2, & 3, & 4, \ldots \\ S & P & D & F & G \end{array}$$

At low energies, low L will dominate (why?). Lowest are:

$$^1S_0 \; ^3S_1 \; ^1P_1 \; ^3P_0 \; ^3P_1 \; ^3P_2 \; ^1D_2 \; ^3D_1 \; ^3D_2 \; ^3D_3 \; ^1F_3 \; ^3F_2 \; ^3F_3 \; ^3F_4$$

· S, J conserved, but L can change from interactions
⇒ called <u>coupled channel</u>. Only S=1, given J couples |l−1|, l+1

· J=L don't mix because of parity.
· Coupled channels mean an additional 2×2 matrix structure to the partial wave potential, S or T matrix.

$$\begin{array}{ccc} & J_< = |l-1| & J_> = l+1 \\ J_< & \left(\begin{array}{c|c} & \\ \hline & \end{array} \right. & \left. \right) \\ J_> & & \end{array} \qquad \text{e.g.} \qquad \begin{array}{ccc} & ^3S_1 & ^3D_1 \\ ^3S_1 & \left(\begin{array}{c|c} & \\ \hline & \end{array} \right. & \left. \right) \\ ^3D_1 & & \end{array}$$

tensor force couples these

· How do we parametrize the S-matrix in this case? Convention "bar" phase shifts $\delta_{J_<}$, $\delta_{J_>}$, and mixing angle ε_J

$$S\text{-matrix} = \begin{pmatrix} e^{2i\delta_{J_<}} \cos 2\varepsilon_J & i e^{i(\delta_{J_<}+\delta_{J_>})} \sin 2\varepsilon_J \\ i e^{i(\delta_{J_<}+\delta_{J_>})} \sin 2\varepsilon_J & e^{2i\delta_{J_>}} \cos 2\varepsilon_J \end{pmatrix}$$

· note in $\varepsilon_J \to 0$ limit, usual decoupled result.

Here we will consider strong interactions, but note that a quantitative comparison with experimental cross section requires (long-range) electromagnetic interactions.

- Phys. Rev. C 51, 38 (1995) on the Argonne V_{18} potential is a good source.

$V_{em}(pp)$ = one- and two-photon exchange Coulomb terms
Darwin-Foldy correction
vacuum polarization $e^+ e^-$
magnetic moment interactions

$V_{em}(pn)$ = Coulomb term due to neutron charge distribution
mag. moment interaction

n p π^- n

$V_{em}(nn)$ = mag. moment interaction

OK, let's look at NN phase shifts using the Nijmegen partial-wave analysis available on nn-online.org/NN.

- Start with S-waves. $^1S_0$ and $^3S_1$: are these attractive or repulsive? Are there bound states?

- Both channels have large scattering length compared to range ~1-2 fm

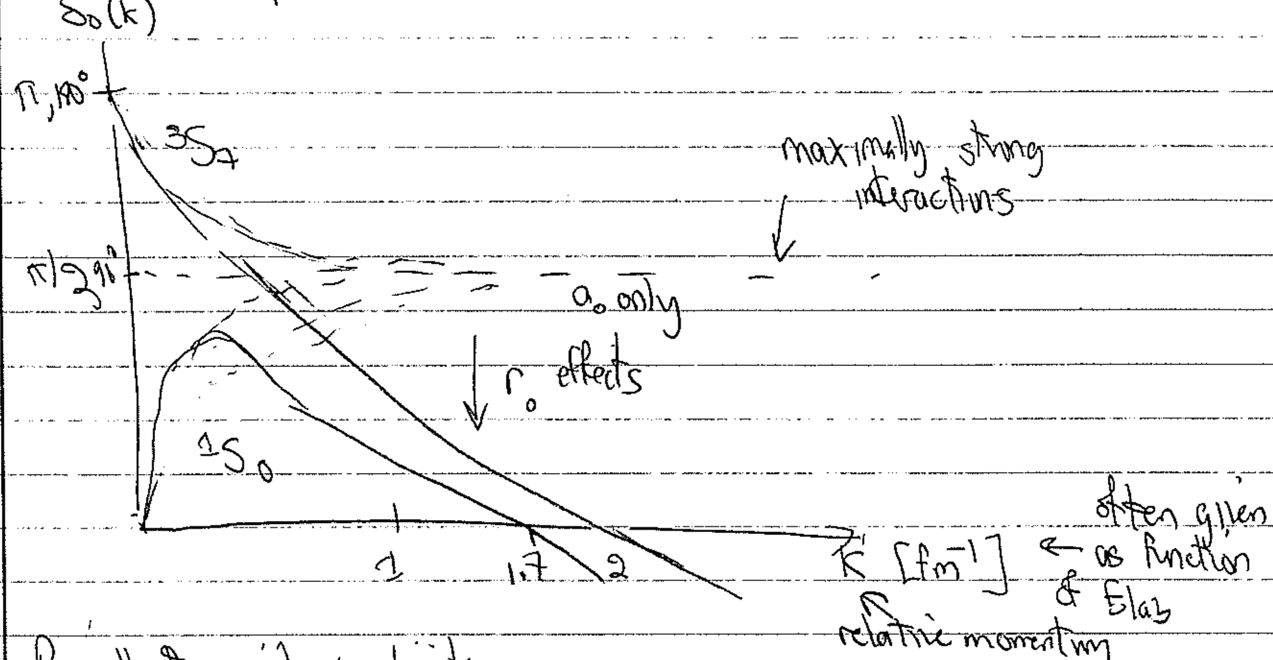
$$^1S_0 \quad a_{nn} \doteq (a_{pp} - \text{em effects}) \approx -18\,\text{fm}$$

$$a_{np} \doteq -23.7\,\text{fm} \rightarrow \text{almost bound (resonance)}$$

$$^3S_1 \quad a_{np} = +5.4\,\text{fm} \rightarrow \text{bound deuteron}$$

mixing angle $\varepsilon < 5^\circ$ small for $E_{lab} < 300$ MeV

Sketch of phase shifts



Recall the unitary limit

$$\frac{d\sigma}{d\Omega} = \frac{1}{(k\cot\delta)^2 + k^2} \le \frac{1}{k^2} \Rightarrow \delta = \frac{\pi}{2} \text{ for all } k$$

is strongest ($\cot\delta = \cos\delta/\sin\delta = 0$)

- This would imply $\frac{1}{a} = 0$ and all effective range parameters $= 0$

- So we see that the NN interaction in S-waves is strong at low energies from large a_0 but weakens at higher energies due to effective range effects

- Recall $\delta(k) > 0 \Rightarrow$ attractive potential; $\delta(k) < 0 \Rightarrow$ repulsive potential
  - S waves are attractive at low energies, repulsive at high energies

not unique!
$\Rightarrow$ if local potential, needs a "hard core" repulsion (Try this with VPA mathematica notebook)
$\Rightarrow$ if non-local potential, then repulsive momentum dependence

The large scattering length in both spin channels leads to an approximate low-energy symmetry
$\Rightarrow$ approximate $SU(2)_{isospin} \otimes SU(2)_{spin} = SU(4)$ symmetry $\equiv$ Wigner symmetry
- spin-orbit symmetry breaks spin $\uparrow$ and $\downarrow$ symmetry $\Rightarrow$ broken in nuclei

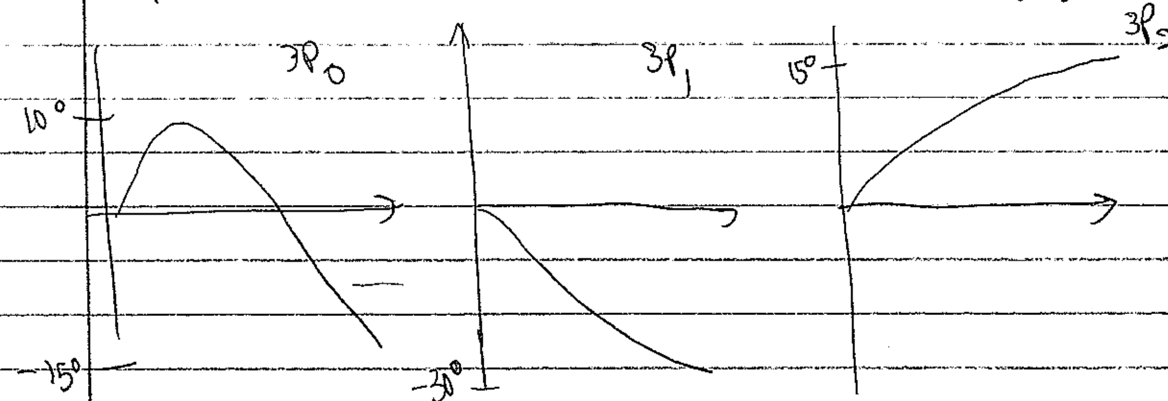
Range of the NN force

check Machleidt

- One of the exercises for Scattering 2 is to compare $^1S_0$ to $^1D_2$ phase shifts and use this to estimate the radial extent of the repulsive core of a local potential.
  - The idea is that differ by a centrifugal barrier in the $^1D_2$ channel. What should this do and how can you use this to estimate the range of the core?
  - What sets the extent of the long-range attraction?

Triplet P waves and further insights into the nuclear force

Triplet $\Rightarrow S=1$ P-wave $\Rightarrow L=1 \Rightarrow J=0,1,2$



warm-up: Do we see attraction or repulsion?

central interaction: $V_{\mathbb{1}} + V_{\sigma_1 \cdot \sigma_2}\, \vec{\sigma}_1 \cdot \vec{\sigma}_2$

- How would the contribution to $^3P_{0,1,2}$ compare? The same! (Why?)
  - We can isolate the central part of ^3P waves by averaging, weighted by $2J+1$

$$\Rightarrow \bar{\delta}_{L=1}(k) = \frac{\sum_J (2J+1)\,\delta^J_{L=1}(k)}{\sum_J (2J+1)}$$

In exercise: $\bar{\delta}_{L=1} < 5°$ for $E_{lab} < 150$ MeV
$\Rightarrow$ central ^3P interactions are small

- So what can contribute to the splitting of the ^3P waves?

$\Rightarrow$ spin-orbit force $\propto \langle \vec{L}\cdot\vec{S} \rangle = \langle \frac{1}{2}(\vec{J}^2 - \vec{L}^2 - \vec{S}^2) \rangle$

$$= \frac{1}{2}\left(J(J+1) - S(S+1) - L(L+1)\right)$$

$$= \begin{cases} -2 & ^3P_0 \\ -1 & ^3P_1 \\ 1 & ^3P_2 \end{cases}$$

$\Rightarrow$ need $V_{\vec{L}\cdot\vec{S}}\, \vec{L}\cdot\vec{S}$ with attractive spin-orbit $V_{\vec{L}\cdot\vec{S}} < 0$

$\rightarrow$ $^3P_2$ attractive ✓ , $^3P_1$ repulsive ✓

but $^3P_2$ only works for high energy and only the sign

- What else is there?

$\Rightarrow$ tensor interactions $S_{12}(\hat{r}) = \vec{\sigma}_1\cdot\vec{r}_1\, \vec{\sigma}_2\cdot\vec{r}_2 - \frac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2$

- couples spin and space $\Rightarrow$ $J_<$ and $J_>$ partial waves coupled.

Question: Why doesn't $\vec{L}\cdot\vec{S}$ couple partial waves?

Scales in nuclear forces   momentum scales Q

effective range parameters

i) $\left|\frac{1}{a_0}\right| = \left|\frac{1}{-20\,\text{fm}}\right|$ or $\frac{1}{5\,\text{fm}} \approx 10\text{–}40$ MeV   (recall $\hbar c \simeq 200$ MeV-fm)

ii) $m_\pi = 140$ MeV $\frac{1}{r_0} \simeq \frac{1}{2.7\,\text{fm}} \sim m_\pi/2$

iii) $m_\Delta - m_N \sim 2m_\pi$

iv) $m_{\text{heavy}=\rho,\omega} \sim 1$ GeV (roughly)

$\Rightarrow$ separation of scales: 0 — $\left|\frac{1}{a}\right| \ll m_\pi < m_\Delta - m_N \ll m_{\text{heavy}}$ $\rightarrow Q$