Physics 8805: Nuclear Few- and Many-Body Physics
Discussion questions and exercises for Scattering 2

[Last revised on September 16, 2014 at 07:37:23.]

1. Two-minute questions on the effective range expansion and phase shifts.

   (a) If you had two candidate potentials that had the same effective range expansion coefficients (out to the order you could measure them), how could you tell them apart experimentally? [Hint: trick question!]

   (b) Find the scattering length and effective range and shape parameter (i.e., the first three terms of the effective range expansion) for a hard sphere of radius R (that is, the potential is infinite for $r < R$ and zero for $r > R$). Are the parameters natural or unnatural? (E.g., can they all be reasonably estimated by naive dimensional analysis.) What is the radius of convergence for the expansion in this case? (That is, for what range of k does the expansion converge?)

   (c) Why is the deuteron binding energy different from $\frac{\hbar^2}{ma_S^2}$, where a_S is the scattering length?

   (d) Which S-waves contribute to np scattering?

   (e) The central part of the one-pion-exchange NN potential is given by $V_{\mathrm{OPE}}(r) \sim m_\pi^2 \, e^{-m_\pi r}/r$. Discuss how the pion-mass dependence can explain why the scattering length in the $^1S_0$ channel is more attractive for np, $a_S = -23.7\,\mathrm{fm}$, than for nn, $a_S = -18.5\,\mathrm{fm}$.

   (f) List all coupled channels for $J \leqslant 4$.

   (g) Why do only central interactions contribute to the average P-wave phase shift (defined in the scattering theory 2 lecture)?

2. What do we mean when we say that probes or particles at low energy do not "resolve" fine detail? What size structure would a $5\,\mathrm{MeV}$ neutron resolve? [Hint: recall diffraction.]

3. What is the first (i.e., lowest energy) source of inelasticity in NN scattering? Find an expression in terms of the pion and nucleon masses for the laboratory energy of the inelastic threshold. Why are phenomenological potentials typically fit to elastic phase shifts for $50\,\mathrm{MeV}$ above this threshold?

4. Questions about nucleon-nucleon phase shift plots (e.g., from NN-Online).

   (a) Is the partial wave analysis leading to these graphs model independent? That is, are there assumptions made? If so, what are they?

   (b) Go to the NN-OnLine website linked under Miscellany→Links→Nuclear Resources and generate graphs of neutron-proton scattering phase shifts from 0 to $350\,\mathrm{MeV}$ lab energy for some different channels.

(c) Why is the central potential contribution to each of the triplet P-waves the same?

(d) Why doesn't $\mathbf{L} \cdot \mathbf{S}$ couple partial waves the way the tensor force does?

(e) For the D-wave phase shifts, which ones have attractive interactions and which ones have repulsive interactions?

(f) Compare $^1S_0$ to $^1D_2$ and use the results to estimate the radial extent of the repulsive core for a local potential. (Hint: What does the centrifugal barrier in the D state do? You can use a classical argument.) [It would be great to test out such an argument with the numerical calculations!]

(g) Extract the scattering lengths (and effective ranges, if possible) from the phase shift data for np scattering and compare to quoted answers.

(h) Neutrons form Cooper pairs in neutron stars. At low densities/momenta, neutrons pair in the $^1S_0$ channel where the NN interaction is most attractive. As the S-wave interaction becomes repulsive with increasing density/momentum, in which channel are neutrons expected to pair?

5. Repulsive-attractive square well as a test laboratory using the VPA. By this we mean a combined repulsive square well of radius R_c and height V_c (the "core") and an attractive square well of radius R_0 and depth $-V_0$. This is already implemented for the VPA in the Mathematica notebook `square_well_scattering.nb` and in an iPython notebook.

(a) Play with the value of V_c with R_c set to a reasonable value (given that we are in units where $R_0 = 1$); what do you observe?

(b) Convince yourself that a local potential must have a strongly repulsive core to be consistent the observed sign change of the phase shifts in the NN $^1S_0$ channel.

(c) Suppose we have a momentum dependent potential without a hard core as an alternative. Can we still get a sign change?

6. How can you generate an infinite positive scattering length with a square well potential? Is it possible for a potential to lead to an infinite positive scattering length without having a bound state?

7. *T*-matrix for a separable potential [adapted from Taylor, Scattering Theory]. A separable potential has the form
$$\widehat{V} = g|\eta\rangle\langle\eta| \,,$$
where we usually choose $|\eta\rangle$ to be a normalized vector given, for example, by its momentum space function $\eta(\mathbf{k}) \equiv \langle\mathbf{k}|\eta\rangle$ (note that we're not in partial waves here). Recall the Lippmann-Schwinger equation for the operator $T(z)$ described in the Scattering Theory 1 notes (long version):
$$\widehat{T}(z) = \widehat{V} + \widehat{V}\frac{1}{z - \widehat{H}_0}\widehat{T}(z) = \widehat{V} + \widehat{V}\frac{1}{z - \widehat{H}_0}\widehat{V} + \widehat{V}\frac{1}{z - \widehat{H}_0}\widehat{V}\frac{1}{z - \widehat{H}_0}\widehat{V} + \cdots \,.$$

(a) Show that $T(z)$ is given explicitly by

$$T(z) = \frac{g\,|\eta\rangle\langle\eta|}{1 - g\Delta(z)}\,,$$

where

$$\Delta(z) = \langle\eta|\frac{1}{z - H_0}|\eta\rangle = \int d^3k\,\frac{|\eta(\mathbf{k})|^2}{z - E_k}$$

with $E_k = k^2/2\mu$. [Hint: substitute the separable form for $\widehat{V}$ into the Born series for $T(z)$ and note the form of each term.]

(b) Show that the Born series for $T(z)$ is convergent for g small but divergent for g large.

(c) The poles of $T(z)$ as a function of complex energy z tells of about the bound states of the potential (see the end of the Scattering Theory 1 notes). Show that the separable potential has either one or no bound states.

8. Discussion questions for those who have had a quantum field theory course. (To be explained to those who haven't!)

(a) In what ways is nonrelativistic quantum mechanical scattering simpler than what you learn in relativistic quantum field theory?

(b) In what ways is the nonrelativistic quantum mechanical bound state problem simpler than solving for bound state in relativistic QFT? For example, you might consider solving for positronium in QED versus using an effective potential. [Hint: Look up the Bethe-Salpeter equation.]