Where do we draw the line? How can we take advantage of moving the line?
Thanks: colleagues at low resolution

- ANL: L. Platter
- Darmstadt: H.-W. Hammer, K. Hebeler, R. Roth et al., A. Schwenk
- IIT (Madras): S. Ramanan
- Iowa State: P. Maris, J. Vary
- Jülich: A. Nogga
- Michigan State: S. Bogner, A. Ekstrom
- LLNL: E. Jurgenson, N. Schunck
- ORNL / UofT: G. Hagen, W. Nazarewicz, T. Papenbrock, K. Wendt
- TRIUMF: S. Bacca, P. Navratil
- UNC: E. Anderson, J. Drut
- many others in NUCLEI, LENPIC, ...
Why should we care what happens to UV physics?

- Evolution of Hamiltonians and other operators
  - Where does UV physics go as we lower a cutoff?
  - When do many-body terms become important?
  - Flow to universal Hamiltonians: can we exploit it?

- Using the EFT cutoff (\( \Lambda \)) scale: Naturalness?
  - Bayesian priors for fitting LECs?
  - What is learned from regulator cutoff variation?

- Which is better: EFT at lower cutoff or SRG?
  - Is SRG decoupling the same as cutting off?
  - Does it matter how we cut off UV physics?

- UV basis extrapolation; e.g., for SRG-evolved potentials
  - Universal/dual aspects of UV vs. IR? What's different?

- Knock-out experiments: short-range correlations and all that
  - What role do the UV parts of wave functions play?
  - What factorization (separation) scale should we use?

Plan: random walk through these topics (mostly questions!)
What does changing a cutoff do in an EFT?

- (Local) field theory version in perturbation theory (diagrams)
  - Loops (sums over intermediate states) \( \frac{d}{d\Lambda_c} \left[ \begin{array}{c}
\end{array} \right] = 0 \)
    \[
    \int_{\Lambda_c} \frac{d^3 q}{(2\pi)^3} \frac{C_0 M C_0}{k^2 - q^2 + i\epsilon} C_0(\Lambda_c) \propto \frac{\Lambda_c}{2\pi^2} + \cdots
    \]
  - Momentum-dependent vertices \( \longrightarrow \) Taylor expansion in \( k^2 \)
  - Claim: \( V_{\text{low } k} \) RG and SRG decoupling work analogously

\[ V_{\text{low } k} \] SRG ("T" generator)
What does changing a cutoff do in an EFT?

- *(Local) field theory version in perturbation theory (diagrams)*
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    $$\int_{\Lambda_c}^{\Lambda_c} \frac{d^3q}{(2\pi)^3} \frac{C_0 M C_0}{k^2 - q^2 + i\epsilon}$$
    
    $$C_0(\Lambda_c) \propto \frac{\Lambda_c}{2\pi^2} + \cdots$$

- Momentum-dependent vertices $\Rightarrow$ Taylor expansion in $k^2$

- Claim: $V_{\text{low } k}$ RG and SRG decoupling work analogously

SRG ("BD" generator)  
SRG ("T" generator)
Approach to universality (fate of high-$q$ physics!)

Run NN to lower $\lambda$ via SRG $\implies \approx$ Universal low-$k$ $V_{\text{NN}}$

$q \gg \lambda$ (or $\Lambda$) intermediate states
$\implies$ replace with contact terms:
$C_0 \delta^3(\mathbf{x} - \mathbf{x}') + \cdots$

$[\text{cf. } \mathcal{L}_{\text{eff}} = \cdots + \frac{1}{2} C_0 (\psi^\dagger \psi)^2 + \cdots ]$

- Similar pattern with phenomenological potentials (e.g., AV18)

Factorization: $\Delta V_\lambda(k, k') = \int U_\lambda(k, q) V_\lambda(q, q') U_\lambda^\dagger(q', k')$ for $k, k' < \lambda$, $q, q' \gg \lambda$

$U_\lambda \rightarrow_{K \cdot Q} K(k) [\int Q(q) V_\lambda(q, q') Q(q')] K(k')$ with $K(k) \approx 1$!
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Run NN to lower \(\lambda\) via SRG \(\implies\) \(\approx\) Universal low-\(k\) \(V_{NN}\)

\(k' < \lambda\)

\[V_\lambda\]

\(q \gg \lambda\)

\[C_0 + \cdots\]

\(q \gg \lambda\) (or \(\Lambda\)) intermediate states \(\implies\) replace with contact terms:

\[C_0 \delta^3(x - x') + \cdots\]

(cf. \(L_{\text{eft}} = \cdots + \frac{1}{2} C_0 (\psi^\dagger \psi)^2 + \cdots\))

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Approach to universality (fate of high-\(q\) physics!)

Run NN to lower \(\lambda\) via SRG \(\implies\) \(\approx\) Universal low-\(k\) \(V_{NN}\)

\[
\begin{align*}
\text{Off-Diagonal } V_\lambda(k, 0) \\
\lambda = 1.5 \text{ fm}^{-1} \\
1 S_0
\end{align*}
\]

\(q \gg \lambda\) (or \(\Lambda\)) intermediate states
\(\implies\) replace with contact terms:
\[
C_0 \delta^3(\mathbf{x} - \mathbf{x}') + \cdots
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[cf. \(\mathcal{L}_{\text{eff}} = \cdots + \frac{1}{2} C_0 (\psi^\dagger \psi)^2 + \cdots\)]

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\[
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\]
NN $V_{\text{SRG}}$ universality from phase equivalent potentials

Diagonal elements collapse where phase equivalent [Dainton et al, 2014]
NN $V_{\text{SRG}}$ universality from phase equivalent potentials

Diagonal elements collapse where phase equivalent potentials collapse [Dainton et al, 2014]
Are inverse scattering potentials sufficient? [Dainton et al]

Create a separable potential that is phase equivalent to AV18:

For the diagonal elements, yes, this is sufficient!
Are inverse scattering potentials sufficient? [Dainton et al]

Create a separable potential that is phase equivalent to AV18:

But for off-diagonal, need half-on-shell T-matrix (HOST) equivalence
Are inverse scattering potentials sufficient? [Dainton et al]

Create a separable potential that is phase equivalent to AV18:

With HOST equivalence, even delta shell potential plus OPE is sufficient!
Use universality to probe decoupling

- What if not phase equivalent *everywhere*?
- Use $^1P_1$ as example (for a change :)
- Result: local decoupling!
Use universality to probe decoupling

- What if not phase equivalent everywhere?
- Use $^1P_1$ as example (for a change :)
- Result: local decoupling!
Is there 3NF universality?

- Evolve chiral NNLO EFT potentials in momentum plane wave basis to $\lambda = 1.5 \text{ fm}^{-1}$ [K. Hebeler, Phys. Rev. C85 (2012) 021002]

- In one 3-body partial wave, fix one Jacobi momentum $(p, q)$ and plot vs. the other one:

- Collapse of curves includes non-trivial structure
Is there 3NF universality?

- Evolve in discretized momentum-space hyperspherical harmonics basis to $\lambda = 1.4 \, \text{fm}^{-1}$ [K. Wendt, Phys. Rev. C87 (2013) 061001]

- Contour plot of *integrand* for 3NF expectation value in triton

- Local projections of 3NF also show flow toward universal form

- Can we exploit universality à la Wilson? Stay tuned!
What else can we say about the flow of NN\cdots N potentials?

- Can arise from counterterm for new UV cutoff dependence, e.g., changes in $\Lambda_c$ must be absorbed by 3-body coupling $D_0(\Lambda_c)$

$$
\frac{d}{d\Lambda_c} \left[ \begin{array}{c}
\alpha(C_0)^4 \ln(k/\Lambda_c) \\
D_0(\Lambda_c) \alpha(C_0)^4 \ln(a_0\Lambda_c)
\end{array} \right] = 0
$$

RG invariance dictates 3-body coupling flow \cite{Braaten & Nieto}

- General RG: 3NF from integrating out or decoupling high-$k$ states

\begin{itemize}
  \item $\pi, \rho, \omega$
  \item $\Delta, N^*$
  \item $c_1, c_3, c_4$
  \item $c_D$
  \item $c_E$
\end{itemize}

low $\downarrow$ resolution
What do we know about the growth of NN$\cdots$N potentials?

- Many interesting results have appeared, prompting questions . . .

Early results in lightest systems [Jurgenson et al. (2009)]:

How does this hierarchy evolve with $A$?
What do we know about the growth of NN···N potentials?

- Many interesting results have appeared, prompting questions . . .

Team Roth: 4-body depends on cutoff on $c_3$ term.

How do we determine consistent regulators in this case?
Does local versus non-local cutoff function matter?
What do we know about the growth of NN···N potentials?

- Many interesting results have appeared, prompting questions . . .

Ratio of 3NF to NN in neutron matter [Hebeler, rjf (2013)]

Density scales as you would expect (at least here :), but $\lambda$ scaling?
What do we know about the *growth* of NN·N potentials?

- Many interesting results have appeared, prompting questions . . .

**Nuclear matter scaling:** use NN results at saturation $\Rightarrow \langle V_3 \rangle / \langle V_2 \rangle$

- **Simple dimensional scaling** (e.g., $(k_F/\Lambda)^3$ or $(k_F/\lambda)^3$) doesn’t work

  but a different scaling . . . $\frac{\langle V_3 \rangle}{\langle V_2 \rangle} \frac{f_\pi^2 \Lambda}{\rho} \lambda^{1/3} \sim \mathcal{O}(1)$ [where did $\rho/f_\pi^2 \Lambda$ come from?]
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Current answer: not enough yet! But tools in place to make progress!
How big should different contributions be?

- Enable chiral EFT power counting \( \implies \) NDA and naturalness

\[
\mathcal{L}_{\text{eft}} = c_{lmn} \left( \frac{N^\dagger (\cdots) N}{f_\pi^2 \Lambda_\chi} \right)^l \left( \frac{\pi}{f_\pi} \right)^m \left( \frac{\partial^\mu, m_\pi}{\Lambda_\chi} \right)^n f_\pi^2 \Lambda_\chi^2 \quad f_\pi \sim 100 \text{ MeV}
\]

- Georgi (1993): \( f_\pi \) for strongly interacting fields; rest is \( \Lambda_\chi \)

- Cohen et al. (1997). Uncanonical scaled EFT action at \( \Lambda \):

\[
S_\Lambda = \frac{1}{g^2} \int d^4 x \; \widehat{\mathcal{L}}_\Lambda \left( \frac{\pi' \Lambda', N' \Lambda^{3/2}}{\Lambda}, \frac{\partial}{\Lambda} \right) \quad \text{“natural” if loops \( \leq \) trees}
\]

- NDA: that bound is saturated: \( g \sim 4\pi \) with \( \Lambda \sim \Lambda_\chi \)

- Rescale to canonical kinetic normalization \( \implies \) NDA

- Claim: should match choosing \( \Lambda \sim \Lambda_\chi \) scale \( \implies \) NDA estimates

- \( \Lambda_\chi \) is not itself an adjustable cutoff but a physics scale
- e.g., from non-Goldstone-boson exchange such \( m_\rho \)
- Need calculations for quantitative \( \Lambda_\chi \)

- Other refs: Dugan and Golden (1993), Friar (1997)
How big should different contributions be?

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\[
\mathcal{L}_{\chi \text{eft}} = c_{lmn} \left( \frac{N^\dagger (\cdots) N}{f_\pi \Lambda_\chi} \right)^l \left( \frac{\pi}{f_\pi} \right)^m \left( \frac{\partial^\mu, m_\pi}{\Lambda_\chi} \right)^n f_\pi^2 \Lambda_\chi^2 \quad f_\pi \sim 100 \text{ MeV}
\]

- E.g., check NLO, NNLO constants from \( \mathcal{L}_{NN} \) [Epelbaum et al.]
  Take \( \Lambda_\chi \Rightarrow \) cutoff \( \Lambda: 500 \ldots 600 \text{ MeV}):

<table>
<thead>
<tr>
<th>( f_\pi^2 C_S )</th>
<th>(-1.079 \ldots -0.953)</th>
<th>( f_\pi^2 C_T )</th>
<th>( 0.002 \ldots 0.040)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_\pi^2 \Lambda_\chi^2 C_1 )</td>
<td>(3.143 \ldots 2.665)</td>
<td>(4 f_\pi^2 \Lambda_\chi^2 C_2)</td>
<td>(2.029 \ldots 2.251)</td>
</tr>
<tr>
<td>( f_\pi^2 \Lambda_\chi^2 C_3 )</td>
<td>(0.403 \ldots 0.281)</td>
<td>(4 f_\pi^2 \Lambda_\chi^2 C_4)</td>
<td>(-0.364 \ldots -0.428)</td>
</tr>
<tr>
<td>( 2 f_\pi^2 \Lambda_\chi^2 C_5 )</td>
<td>(2.846 \ldots 3.410)</td>
<td>(f_\pi^2 \Lambda_\chi^2 C_6)</td>
<td>(-0.728 \ldots -0.668)</td>
</tr>
<tr>
<td>( 4 f_\pi^2 \Lambda_\chi^2 C_7 )</td>
<td>(-1.929 \ldots -1.681)</td>
<td>(f_\pi^2 \Lambda_\chi^2 C_6)</td>
<td>()</td>
</tr>
</tbody>
</table>

- \( 1/3 \lesssim c_{lmn} \lesssim 3 \Rightarrow \) natural! \( \Rightarrow \) truncation error estimates
- If unnaturally large, signal of missing long-distance physics (e.g., \( \Delta \) in \( c_i \)'s) or over-fitting
- \( f_\pi^2 C_T \) unnaturally small \( \Rightarrow \) \( SU(4) \) spin-isospin symmetry
How big should different contributions be?

- Enable chiral EFT power counting $\Rightarrow$ NDA and naturalness

$$\mathcal{L}_{\chi \text{eft}} = c_{lmn} \left( \frac{N^\dagger (\cdots) N}{f_\pi^2 \Lambda_\chi} \right)^l \left( \frac{\pi}{f_\pi} \right)^m \left( \frac{\partial^\mu, m_\pi}{\Lambda_\chi} \right)^n f_\pi^2 \Lambda_\chi^2 \quad f_\pi \sim 100 \text{ MeV}$$

Applications to coefficients in relativistic and Skyrme density functionals

- Identify unnaturally large and small Skyrme coefficients
- Guide fitting attempts with generalized EDF’s?
How big should different contributions be?

- Enable chiral EFT power counting $\implies$ NDA and naturalness

$$\mathcal{L}_{\chi \text{eft}} = c_{lmn} \left( \frac{N^\dagger (\cdots) N}{f_\pi^2 \Lambda_\chi} \right)^l \left( \frac{\pi}{f_\pi} \right)^m \left( \frac{\partial^\mu, m_\pi}{\Lambda_\chi} \right)^n f_\pi^2 \Lambda_\chi^2 f_\pi \sim 100 \text{ MeV}$$

- Old chiral NDA analysis for EDFs: [Friar et al., rjf et al.]

$$c \left[ \frac{N^\dagger N}{f_\pi^2 \Lambda} \right]^l \left[ \frac{\nabla}{\Lambda} \right]^n f_\pi^2 \Lambda^2$$

$$\rho \longleftrightarrow N^\dagger N$$

$$\tau \longleftrightarrow \nabla N^\dagger \cdot \nabla N$$

$$J \longleftrightarrow N^\dagger \nabla N$$

- Density expansion?

$$1000 \geq \Lambda \geq 500 \implies \frac{1}{7} \leq \frac{\rho_0}{f_\pi^2 \Lambda} \leq \frac{1}{4}$$

- Also gradient expansion

- Applied to RMF, Skyrme EDFs

Density expansion?

<table>
<thead>
<tr>
<th>energy/particle (MeV)</th>
<th>power of density</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>2</td>
</tr>
<tr>
<td>500</td>
<td>3</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>50</td>
<td>5</td>
</tr>
</tbody>
</table>

- $\times$ natural $(\Lambda=600 \text{ MeV})$
- $\diamond$ Skyrme $\rho^\text{n}$
- $\bigcirc$ RMFT-II $\rho^\text{n} \text{ net}$
- $\square$ RMFT-I $\rho^\text{n} \text{ net}$

$$k_F = 1.35 \text{ fm}^{-1}$$
How big *should* different contributions be?

- Enable chiral EFT power counting $\implies$ NDA and naturalness

\[ \mathcal{L}_{\chi\text{eft}} = c_{lmn} \left( \frac{N^\dagger \cdots N}{f_\pi^2 \Lambda_{\chi}} \right)^l \left( \frac{\pi}{f_\pi} \right)^m \left( \frac{\partial \mu, m_\pi}{\Lambda_{\chi}} \right)^n f_\pi^2 \Lambda_{\chi}^2 \quad f_\pi \sim 100 \text{ MeV} \]

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What is the breakdown scale? Not clear for $\chi$EFT! How do we analyze?
Error ("Lepage") plots revisited  [Lepage (1997); Steele, rjf (1999)]

• What is the evidence that the EFT is working as it should and we’re not just fitting (or over-fitting) elephants with many parameters?

- Slope of error curve with energy should increase with EFT order
- Breakdown scale ($\Lambda_\chi$) where error curves intersect or where error stops improving (stabilized prediction)
  - Can we apply to observables other than phase shifts?
  - Investigations with toy models in progress [S. Wesolowski]
- What about error bands from regulator cutoff $\Lambda$ variations?
How should we fit the LECs? Constrained curve fitting

- A new era for fitting and testing chiral Hamiltonians [see A. Ekstrom]
- Deficiencies revealed; more advanced interactions coming

Practical/theory motivations for Bayesian priors [Lepage (2001)]:
- Constraints consistent with Lepage plots (can be tricky)
- Would like to be independent of where we stop fitting ($E$, order)
- Want the theory error at each order incorporated appropriately
- Do not want constants to play off each other

Bayesian fits in 30 seconds. Suppose we have parameters $a = \{a_0, a_1, \cdots, a_M\}$, a data set $d = \{d_1, d_2, \cdots, d_N\}$, and a theory $f$.
- Goal: what $a$ to use (with error) given a data set $d \implies pr(a|d, f)$
- Known: given $a$, what is the chance we get $d \implies pr(d|a, f)$

Joint probability $pr(d, a)$ can be decomposed into conditional probabilities two ways (and so are equal):

$$pr(a|d, f)pr(d|f) = pr(d|a, f)pr(a|f)$$

e.g., $pr(d|a, f) \propto \prod_{k=1}^{N} e^{-x^2/2}$

Now just put $pr(d|f)$ on the other side. The “priors” are $pr(a|f)$. 
“Prior” work by Schindler/Phillips: naturalness as a prior

- “Bayesian Methods for Parameter Estimation in Effective Field Theories”
- Test application to chiral perturbation theory
- $M$ coefficients naturalness values in normal distribution

$$pr(a|M, R) = \left( \prod_{i=0}^{M} \frac{1}{\sqrt{2\pi R}} \right) e^{-\frac{1}{2} \sum_{i=0}^{M} a_i^2 / R^2} \implies R \text{ is width}$$

- In progress: revisit by S. Wesolowski, D. Phillips, rjf for NN···N

- Is normal distribution for natural $a = \{a_i\}$ appropriate given we expect $1/n < a_i < n$?
- Maybe log normal distribution instead for $|a_i|

$$f(x; \mu, \sigma) = \frac{1}{x \sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}, \quad x > 0.$$  

How does this prior relate to weighting by the order of expansion?
“Prior” work by Schindler/Phillips: naturalness as a prior

- Schindler/Phillips toy problem: find $M$ lowest-order coefficients in expansion of $g(x) = \left(\frac{1}{2} + \tan\left(\frac{\pi}{2} x\right)\right)^2 = \sum_{i=0}^{\infty} a_i x^i$
  - $\approx 0.25 + 1.57x + 2.47x^2 + 1.29x^3 + \ldots$

by ordinary $\chi^2$ fitting and using Bayesian priors on the “naturalness” of coefficients.

- Coefficients are of order unity: $1/4 < a_i < 4$
- Limited measurements and experimental noise
- Goal: determine $a_0$ and $a_1$

### Usual $\chi^2$ fit

<table>
<thead>
<tr>
<th>$M$</th>
<th>$\chi^2$</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.49</td>
<td>0.22±0.02</td>
<td>2.47±0.11</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.85</td>
<td>0.29±0.02</td>
<td>1.04±0.40</td>
<td>4.91±1.31</td>
</tr>
<tr>
<td>3</td>
<td>0.85</td>
<td>0.26±0.04</td>
<td>2.00±1.12</td>
<td>-2.55±8.27</td>
</tr>
<tr>
<td>4</td>
<td>0.60</td>
<td>0.18±0.07</td>
<td>5.74±2.81</td>
<td>-50.4±34.0</td>
</tr>
<tr>
<td>5</td>
<td>0.57</td>
<td>0.28±0.14</td>
<td>0.24±7.08</td>
<td>46.9±120.0</td>
</tr>
</tbody>
</table>

### With natural prior

<table>
<thead>
<tr>
<th>$M$</th>
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<th>$a_2$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0.23±0.14</td>
<td>2.42±0.11</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.27±0.03</td>
<td>1.50±0.35</td>
<td>3.21±1.21</td>
</tr>
<tr>
<td>3</td>
<td>0.27±0.03</td>
<td>1.54±0.33</td>
<td>2.80±1.19</td>
</tr>
<tr>
<td>4</td>
<td>0.27±0.03</td>
<td>1.54±0.35</td>
<td>2.76±1.18</td>
</tr>
<tr>
<td>5</td>
<td>0.28±0.05</td>
<td>1.57±0.21</td>
<td>2.79±1.11</td>
</tr>
</tbody>
</table>

$\implies$ marginalize over $M$ and log normal parameters

Controlled fitting protocol needed for consistent “running” of EFT
“Prior” work by Schindler/Phillips: naturalness as a prior

- Schindler/Phillips toy problem: find $M$ lowest-order coefficients in expansion of

$$g(x) = \left(\frac{1}{2} + \tan\left(\frac{\pi}{2} x\right)\right)^2 = \sum_{i=0}^{\infty} a_i x^i$$

$$\approx 0.25 + 1.57x + 2.47x^2 + 1.29x^3 + \cdots$$

by ordinary "$\chi^2$" fitting and using Bayesian priors on the "naturalness" of coefficients.

- Coefficients are of order unity: $1/4 < a_i < 4$

- Limited measurements and experimental noise

- Goal: determine $a_0$ and $a_1$

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Controlled fitting protocol needed for consistent “running” of EFT
Is there a motivation for lower EFT cutoffs?

- Recent examples of calculations with soft EFT interactions
  - Nuclear matter calculations with soft *smooth cutoff* EFT potential [Corraggio et al., arXiv:1402.0965]
  - Lattice chiral EFT: coarse lattices $\rightarrow$ low $\Lambda$ cutoff $\rightarrow$ but many successes [see D. Lee]

- How is an EFT at two different scales related to an RG running via SRG or $V_{\text{lowk}}$?
  - First, distinguish breakdown $\Lambda_\chi$ from regulator $\Lambda$
  - For matching, choose $\Lambda \sim \Lambda_\chi$ for Weinberg counting

- Integrating out momenta in a local EFT (à la Georgi)
  - Integrate out momenta $\rightarrow$ non-local action
  - Derivative expansion and drop higher terms $\rightarrow$ back to local
  - Requires sufficient scale separation or error grows from dropped terms
  - cf. SRG $\rightarrow$ error is unchanged with softening
  - But what is happening if we instead refit the EFT?

- Which is better *in practice*? We need more comparisons!
Does it matter how we cutoff UV physics?

- Perhaps not in principle, but certainly in practice!
- What form does the T-generator SRG cutoff take?
  - Decoupling (roughly) imposes off-diagonal form for $V_\lambda(k, q)$
    
    \[
    V_\lambda(k, q) \xrightarrow{q \gg k} V_\lambda(0, q) \sim V_\infty(0, q) e^{-(q^4/\lambda^4)}
    \]
  - Test with a simple variational ansatz (from $k$-space S-eqn)
    
    \[
    u(k) = \frac{1}{(k^2 + \gamma^2)(k^2 + \mu^2)} e^{-(k^4/\lambda^4)} \quad w(k) = \frac{ak^2}{(k^2 + \gamma^2)(k^2 + \nu^2)^2} e^{-(k^4/\lambda^4)}
    \]

- error in deuteron energy for different initial potentials
- small $\lambda$ works pretty well
- $V_{\text{low } k}$ works even better!
What if we “lower” cutoff by a truncated oscillator basis?

[Work by S. Bogner, S. Koenig, S. More, T. Papenbrock, rjf ...]

- S. Coon: Finite oscillator basis imposes both IR and UV cutoffs
- Nature of UV vs. IR cutoff in light of dual nature of HO
  - Low-momentum (IR) spectrum is the same as hard-wall at
    \[ L_\Delta = \sqrt{2(N_{\text{max}} + 3/2 + \Delta) b_{\text{osc}}} \quad \text{with } b_{\text{osc}} \equiv \sqrt{\hbar/(\mu \Omega)} \]
    with \( \Delta = 2 \) [see T. Papenbrock]
  - Duality \( \Longrightarrow \) short distance (UV) same as hard wall in momentum with \( b_{\text{osc}} \rightarrow \hbar/b_{\text{osc}} \) in \( L_2 \Longrightarrow \) we expect
    \[ \Lambda_\Delta = \sqrt{2(N_{\text{max}} + 3/2 + \Delta) \hbar/b_{\text{osc}}} \quad \text{with } \Delta = 2 \]
- Analytic result for separable potential with hard cutoff \( \Lambda \):
  \[ V_{\lambda}(k, k') = g f_{\lambda}(k) f_{\lambda}(k') \quad \text{with } f_{\lambda}(k) = e^{-(k/\lambda)^n} \quad \Longrightarrow \Delta E \xrightarrow{\Lambda \gg \lambda} C \int_{\Lambda}^{\infty} dk f_{\lambda}^2(k) \]
- Expect asymptotic form of energy correction for SRG or smooth \( V_{\text{low } k} \) to (roughly) follow this form (with additional \( \Lambda \) dependence)
Examples for deuteron and RG-evolved potentials

[Thanks to K. Wendt for generating deuteron energies in IR-converged spaces]

\[ e^{-(k/\Lambda)^{2n}} H(k, k') e^{-(k'/\Lambda)^{2n}} \]

\[ N^3 \text{LO (500 MeV)} \]

\[ \text{SRG } \lambda = 2.0 \text{ fm}^{-1} \]

\[ \Delta E_d / E_d \] for different cutoff forms; hard wall is \( n = \infty \)
Examples for deuteron and RG-evolved potentials

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N^3LO (500 MeV)
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\[ \Delta E_d / E_d \] for \( \Lambda_0 \); looks like \( n = \infty \) but noticeable scatter
Examples for deuteron and RG-evolved potentials

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\[ \Delta E_d / E_d \text{ for } \Lambda_2; \text{ looks like } n = \infty \text{ and no scatter} \]
Examples for deuteron and RG-evolved potentials

[Thanks to K. Wendt for generating deuteron energies in IR-converged spaces]

For $\Lambda > \lambda$, $\Delta E_d / E_d \propto g(\Lambda)e^{-2(\Lambda/\lambda)^4}$ (??)
Examples for deuteron and RG-evolved potentials

[Thanks to K. Wendt for generating deuteron energies in IR-converged spaces]

For $\Lambda \lesssim \lambda$, $\Delta E_d / E_d \propto e^{-4(\Lambda / \lambda)^2}$ (roughly), as used empirically.
SRG/Vlowk wave functions versus “measured” SRCs

- Universal aspects of UV and IR truncations?
  - IR dictated by asymptotic many-body wave function
    \[ \implies \text{break-up channels} \implies \text{depends only on observables} \implies \text{independent of RG running (and initial potential)} \]
  - UV depends on potential; e.g., changes with RG running because UV potential and wave function do

- But expect similar (scaled) \( \Delta E \) for \( A > 2 \)

- Similar to discussions of short-range correlation physics
  - Frankfurt/Strikman arguments on asymptotic \( k \)-space wf
  - E.g., T. Neff et al. 2-body \( S = 0, T = 1 \) densities:
    - Two-body densities in coordinate space for \( A = 2, 3, 4 \):
      \[ S = 0, T = 1 \]
      \[ C_{0,1} \rho_{0,1} (r) \text{ [fm}^{-3}\text{]} \]
      \[ \begin{array}{c}
        \alpha \\
        t \\
        h \\
        \alpha^* 
      \end{array} \]

- Two-body densities in momentum space for \( A = 2, 3, 4 \):
  - \[ S = 1, T = 0 \]
  - \[ C_{0,1} n_{0,1} (k) \text{ [fm}^{-3}\text{]} \]
  - \[ \begin{array}{c}
        \alpha \\
        t \\
        h \\
        \alpha^* 
      \end{array} \]
Is any of this UV physics “measurable”? [see rjf, 1309.5771]

- Relevant to knock-out experiments of various types
- Issues of scale and scheme dependence (RG invariants?)
  - We have (implicitly or explicitly) established a separation or factorization scale when we calculate observables
  - If sufficient separation of scales, then impulse approximation can be good, and no ambiguities.
  - Generally scale dependent, e.g. parton vs. momentum distributions:

Which scale to use for experiment? Clear for QCD (gauge theory) but EFT?
Start with simplest problem: deuteron electrodisintegration

- In progress by S. More, K. Hebeler, rjf

- Build on Yang and Phillips EFT calculations, but beyond the EFT ("high-resolution probes of low-resolution nuclei")

- Old field redefinition arguments of Hammer, rjf; also with $U_\lambda(k, q)$

- Understand mixing of structure, FSI, and currents (can’t isolate!)

- Can we make money on factorization?
Additional comments (prejudices) on UV physics

- The fate of UV physics cuts across and unites many topics
- Calculational methods with microscopic forces are maturing
  - Deficiencies of current Hamiltonians clearly revealed
  - Opportunities: revisit old EFT technology while inventing new
  - Structure component ahead of reactions but RG can shift between; treating one in isolation can be dangerous
- Knock-out experiments need to be understood better
  - EFT and RG provide tools to do this
  - Different factorization scale for expt. analysis and calculation?
- Don’t be too narrow with "ab initio" for microscopic NN⋯N forces
  - Use sounds provincial in light of QCD
  - Low-energy paradigm: tower of effective theories (or turtles)
- Where should we think about the next rung on the EFT tower?
  - pionless EFT for halo nuclei
  - low-lying excitations in deformed nuclei [see T. Papenbrock]
  - DFT? [e.g., J. Dobaczewski et al.; revisit Landau-Migdal?]