H133: 1094 Session 4 $\boxed{\text{KEY}}$

Write your name and answers on this sheet and hand it in at the end.

After the indicated time, move on to the next activity, even if you are not finished!

1. Quanton in a Box [12 min.]

Start up the PhET applet "Quantum Bound States". (Start->Programs->PhET, choose "Quantum Phenomena" from the left menu, and click on the Quantum Bound States icon). This applet shows energy levels and the corresponding wavefunctions and probability densities for the energy eigenvectors of different potentials. The "square well" potential that comes up initially is a finite-depth version of the "quanton in a box" potential.

a. What force would a Newtonian (i.e., classical) particle feel at each x position? Note that the wavefunctions don't go to zero at the "walls" of the well; could this happen classically? Explain.

A very strong force ($F=-dV/dx$) to the left at the right wall and to the right at the left wall, and zero everywhere else.
Classically the walls are turning points; penetrating would mean negative kinetic energy for a particle.

b. For what energies would a quanton be "unbound" (i.e., not confined to the well)?

For the initial simulation, it would be unbound for any $E > 10\,eV$.

c. Roughly predict the energy of the n=2 state (second-lowest energy) by estimating its wavelength from the simulation and using this to find the momentum and then the kinetic energy. (To show the wave function, click on the 2nd level, switch the Display to "Wave Function", and press "Pause" to stop time.) Compare to the value given by the simulation.

The wavelength is about 1nm $\Rightarrow p = h/\lambda$ or $pc \approx hc/\lambda \approx \frac{1240\,eV\cdot nm}{1\,nm} \approx 1240\,eV$

$E = \frac{p^2}{2m} = \frac{(pc)^2}{2mc^2} = \frac{(1240\,eV)^2}{2\cdot 511{,}000\,eV} \approx \boxed{1.5\,eV}$ The simulation says 1.2eV, so we slightly underestimated the wavelength.

d. Predict (and explain) what will happen to the energy levels if you make the box shallower. [Hint: Does the wave function penetrate the walls more or less?] Check your prediction by selecting "Configure Potential" and reducing the "Height". Try a new explanation if you were wrong the first time.

As the box is made shallower, the wave function penetrates more, so it gets longer. So p_n is smaller and thus E_n goes down. This agrees with the simulation.

e. Do two-minute problem Q8T.3. [See Equation (Q8.5b) for reference.]

$\lambda = \frac{8mc^2L^2}{hc(n_i^2-n_f^2)}$ Longest wavelength is $2\to1$, 3^{rd} longest is $4\to3$.

So $\frac{8mc^2L_A^2}{hc(4-1)} = \frac{8mc^2L_B^2}{hc(16-9)} \Rightarrow \frac{L_A^2}{L_B^2} = \frac{4-1}{16-9}$ or $\frac{L_A}{L_B} = \boxed{\sqrt{\frac{3}{7}}}$ $\boxed{C}$

2. Harmonic Oscillator [12 minutes]

Change the Potential Well using the pulldown menu on the upper right to Harmonic Oscillator. This has the potential energy $k*x^2/2$, which is the same as a spring.

a. For what energies would a quanton be "unbound" (i.e., not confined to the harmonic oscillator)?

All energies are bound.

b. Vibrational levels of molecules are described by a harmonic oscillator potential. Do problem Q8B.4.

Longest wavelength means smallest $n_i - n_f \Rightarrow n_i - n_f = 1$

$\hbar\omega = \frac{hc}{\lambda}\frac{1}{n_i-n_f} \rightarrow \frac{1240\,eV\text{-}nm}{4540\,nm}\cdot\frac{1}{1} \approx \boxed{0.27\,eV}$

c. Time dependence I. Based on Rule 6, what is the time dependence of the probability density for the state with E2? (Find the time dependence of the wave function psi2(x,t) for a state composed only of eigenvector |E2> and then calculate the probability density.) Does this agree with the simulation?

$\psi_2(x,t) = e^{-iE_2t/\hbar}\psi_2(x)$ so $|\psi_2(x,t)|^2 = |\psi_2(x)|^2$ so time independent. It agrees.

d. Time dependence II. Now use "Superposition State" to make a state with equal parts E0 and E2 (so c0 and c2 should be equal). Choose "Normalize" and then "Apply". Measure the period of the probability density using the clock in the lower left. *Bonus:* How is the result related to the energies E0 and E2?

$\boxed{T \doteq 1.9\,fs}$, $\Psi(x,t) = \psi_0(x) e^{-iE_0 t/\hbar} + \psi_2(x) e^{-iE_2 t/\hbar}$

so $|\Psi(x,t)|^2 = |\psi_0(x)|^2 + |\psi_2(x)|^2 + 2\psi_0(x)\psi_2(x) \cos[(E_2 - E_0)t/\hbar]$. So $(E_2 - E_0)T/\hbar = 2\pi$

(real)

or $T = \frac{h}{E_2 - E_0} = \frac{4.136 \times 10^{-15}\,eV \cdot s}{(2.80 - 0.56)\,eV} \doteq 1.85\,fs$ ✓

3. Models of the Hydrogen Atom [10 minutes]

Start up the PhET applet "Models of the Hydrogen Atom". This applet simulates an experiment in which you shoot photons of many different wavelengths ("White") or a single wavelength ("Monochromatic") at a box with hydrogen gas. With the switch on "Experiment", you can see what actually happens outside the box. With the switch on "Prediction", you can compare various models ranging from more classical to more quantum mechanical to see what they predict.

a. Turn on the electron gun and "Show Spectrometer". Take a look first at "Experiment", then switch to "Prediction". Try each of the models. Note that some are in clear contradiction to the experiment (e.g., the Billiard Ball model predicts many photons bouncing backwards). *For the last three, "Show electron energy diagram".*

b. For the "Classical Solar System" model, there is quickly a "kaboom". What is happening and why? (Click on the icon again to replay it.) How is this avoided in quantum mechanical models?

The electron is accelerating (moving in circle) so radiates continuously, losing energy quickly and spiraling into the center. In quantum models, the probability density for an energy eigenstate is time independent so no radiation.

c. How does the deBroglie model compare to the Bohr model? (E.g., similarities and differences.)

Both have well-defined circular orbits and the same energy diagram. The Bohr model has the electron circulating as a particle while deBroglie has it modeled as a wave closed in a circle.

d. Which of the models will (approximately) reproduce the known hydrogen spectrum?

Bohr, de Broglie, and Schrödinger.

4. Discharge Lamps [12 minutes]

Start up the PhET applet "Neon Lights and Other Discharge Lamps". This simulates the type of light we saw in class. You can show "One Atom" of gas or "Multiple Atoms". Start with "One Atom".

a. Switch from "Single" Electron Production to "Continuous" and click on the Spectrometer. What is the source of energy to move an atom from the ground state to an excited state?

The kinetic energy of the electrons accelerated between the cathode and anode is transferred by collision to the atoms.

b. The Spectrometer shows only one wavelength of light. Why? What do you need to change to get more wavelengths? (Try it!) What if the energy of collision is less than the n=2 level (marked with a circled number 1)? There is only enough energy initially to excite the n=2 level, so only one transition is possible. Increase the electron energy by increasing the voltage to get more wavelengths. If less than the n=2 level, then no photons are emitted.

c. Switch to "Multiple Atoms" and click on the Spectrometer again. Identify the wavelength of the red line and show that the Bohr model quantitatively predicts it. [Hint: see Q8X.5 on pg.156.]

Looks like 655 nm on the Spectrometer

The 3→2 transition predicts from (Q8.7) that $\lambda(3 \to 2) \doteq (91.3\,nm)\left(\frac{3^2 2^2}{3^2 - 2^2}\right) = 657\,nm$ ✓

d. Take a look at other atoms besides hydrogen. Why do they have more visible lines in their spectra?

More energy levels spaced closely enough so transitions are in the visible.