H133: 1094 Session 8 KEY

Write your name and answers on this sheet and hand it in at the end.
After the indicated time, move on to the next activity, even if you are not finished!

1. T5 and T6 Group Problems [15 minutes]

a. Answer T5S.1 in a couple of sentences. The small amount of order created in cleaning up your room is much more than compensated by the microscopic disorder created as your body burns food to move your muscles. The heat generated escapes into the environment, with a large increase in entropy.

b. Do T6T.2. Use equation (T6.6) and the fact that "normal" objects have positive temperatures to justify your answer. Normal objects have $T > 0$, so from $\frac{1}{T} = \frac{dS}{dU}$, the slope must be > 0. This means you can go almost up to the peak in S (at $\frac{1}{2}U_{max}$), but no higher. $\Rightarrow$ $\boxed{C}$

c. Do T6B.2. A simple application of (T6.6).
We're given $\Delta S = 338.1k_B - 305.2k_B = 32.9k_B$ while $\Delta U = 1\varepsilon$. So $k_BT/\varepsilon \doteq \frac{1}{\varepsilon}\frac{\Delta U}{\Delta S} = \frac{1}{\varepsilon}\frac{1\varepsilon\, k_B}{32.9k_B} \doteq \boxed{0.0304}$
With $\varepsilon = 1.0\,\text{eV}$, $T = 0.0304\,\text{eV} \times \frac{1.6\times10^{-19}\,\text{J}}{\text{eV}} \cdot \frac{1}{1.38\times10^{-23}\,\text{J/K}} \doteq \boxed{352\,\text{K}}$

d. Do T6T.4 and T6T.5. These apply the Boltzmann factor formula as in equation (T6.20).
T6T.4 $\frac{Pr(E_1)}{Pr(E_0)} = \frac{1}{4} = e^{-\Delta E/k_BT} \Rightarrow \ln\frac{1}{4} = -\frac{\Delta E}{k_BT}$ or $\boxed{\frac{\Delta E}{k_BT} = \ln 4}$ C
T6T.5 $\frac{1}{4} = e^{-\Delta E/k_BT_2}$, T_2 up means exponent down so ratio gets larger. $\boxed{A}$

2. The Equilib Simulation [10 minutes]

Download Equilib.exe from the H133 page to your desktop. You have two Einstein solids, labeled A and B. When you first start, there are 400 oscillators in each (so 400/3 = 133 or so atoms each). The total energy is U = 2000 and $U_A = 0$ while $U_B = 2000$. With every "step", each oscillator has a chance to exchange one energy unit with one of its neighbors, selected at random. The two solids can exchange energy through the oscillators along their boundary.

a. Press "reset" and then "evolve" to get started. The graph shows you the energy in solid A. What is the equilibrium energy for A? *Why does it tend toward equilibrium?* How many steps (roughly) does it take to get to equilibrium? Determine roughly the size of fluctuations about equilibrium. That is, about how far away (in energy) does it get away from equilibrium as time goes on?
Equilibrium energy is 1000. It tends toward equilibrium because the macropartitions near $U_A = 1000$ have by far the most microstates. It takes about 30×10^3 steps to get to equilibrium. The fluctuations are about ± 100.

b. Now repeat the last part after switching the number of oscillators for each solid to 100. What is the size of the fluctuations now? They grow to about ± 200.

c. Let's test your observations against the theoretical result of problem T5A.1, which estimates how much the energy of a system (in this case, solid A) should fluctuate. The key result is equation (T5.11), which says that the magnitude of the fluctuations should decrease like one over the square-root of the number of oscillators. For the Einstein solid, what should the value of "a" be? Is equation (T5.11) consistent with your results from the last two parts? BONUS: Use (T5.11) and (T5.10) to make an *absolute* prediction of the fluctuations. "a" should be 3. Since we divided N by 4, the fluctuations should have grown by $\sqrt{4}$ or 2, which is consistent.

3. Nuclear Magnetic Resonance (NMR) PhET Simulation [10 minutes]

Start up the PhET applet "Simplified MRI" *from the H133 webpage* (under "Quantum Phenomena"). In the "Simplified NMR" section, there is an array of magnetic moments (from the protons in hydrogen initially) which can point up or down (spin-up or spin-down) with respect to a constant magnetic field (controlled at the right).

a. The energy display on the right shows how many are in the lower and upper energy levels (which are split by the magnetic field; vary the field to verify this). Set the magnetic field to 2 tesla. Record the ratio of up spins to down spins (since the spins are always flipping, watch for a while and take the most frequent ratio). Write an equation for this ratio as a Boltzmann factor that involves the energy difference between the level and the temperature.

The ratio is about 1 to 4.

$\frac{Prob(up)}{Prob(down)} = e^{-\Delta E/k_BT} = \frac{1}{4}$ where ΔE is the energy difference.

b. Give it some power and adjust the frequency until the spins are in resonance. Describe what happens. Convert the resonant frequency to a photon energy in Joules and use your formula for the last part to calculate the approximate temperature of the system. [Hint: it is pretty cold!] BONUS: Verify your temperature by changing the field and getting a new photon energy.

Resonance is at about 86 MHz $= 86\times10^6$ Hz. This means $\Delta E = hf = 6.63\times10^{-34}\,J.s \cdot 86\times10^{6}\,s^{-1} \doteq \boxed{5.7\times10^{-26}\,J}$

(We can verify this by noting that $\mu_{proton} = 1.4\times10^{-26}\,J/T$, so that $\Delta E = 2\mu B = 5.6\times10^{-26}\,J$.)

So $\frac{\Delta E}{k_BT} = \ln 4$ or $T = \frac{\Delta E}{k_B \ln 4} \doteq \frac{5.7\times10^{-26}\,J}{1.38\times10^{-23}\,J/K \cdot \ln 4} \doteq 3\times10^{-3}\,K \doteq \boxed{3\,mK}$

4. T5S.3 with the StatMech Program [10 Minutes]

Download the (new) StatMech program from the H133 webpage. N is the number of atoms and U the total internal energy. Set it up for $N_A = N_B = 100$ and $U = 200e$.

a. How many times more likely is the system to be found in the center macropartition than in the extreme macropartition where $U_A = 0$ and $U_B = 200e$?

About 0.9×10^{48}

b. What is roughly the range of values that U_A is likely to have more than 99% of the time? [Use graph.]

$70e \le U_A \le 130e$

c. If U_A were initially to have the extreme value 0, how many times more likely is it to move to the next macropartition nearer the center than to remain in the extreme one?

Prob(next)/Prob(extreme) $\doteq 120$

d. Run again and answer these same three questions for $N_A = N_B = 1000$ and $U = 200e$. What is the effect of increasing just the system size by a factor of 10? a) 3.6×10^{57} b) $73e \le U_A \le 127e$ c) 187

The spread of values gets narrower and the extremes get depopulated more rapidly.

e. Now try $N_A = N_B = 100$ and $U = 2000e$. What is the effect of increasing just the energy available to the system by a factor of 10? a) 0.9×10^{204}, b) $82 \le U_A \le 118$ c) 0.7×10^{13}

Same effects but much greater than just increasing the size alone.