

What you should know about . . .

Nuclear Physics

1. Nuclei and strong interaction [Q12,Q13]

- nuclear notation: Z protons and A total nucleons $\implies \frac{A}{Z}X$; size: $r = r_0 A^{1/3}$, with $r_0 \approx 1.2$ fm
- Strong interaction between color neutral objects (such as nucleons) is short range (< 2 fm)
- Binding energy: $E_b = (\sum_{\text{parts}} E_{\text{parts}}) - E_{\text{sys}}$
- Mass defect: $\Delta m = (\sum_{\text{parts}} m_{\text{parts}}) - m_{\text{sys}} = E_b/c^2 = Zm_{\text{H}} + Nm_{\text{n}} - m_{\text{atom}}$
- binding energy formula ($A > 20$): $E_b(A, Z) = a_I A - a_S A^{2/3} - a_C Z^2/A^{1/3} - a_A(A - 2Z)^2/A$
with $a_I = 15.56$ MeV, $a_S = 17.23$ MeV, $a_C = 0.697$ MeV, $a_A = 23.285$ MeV
- binding energy per nucleon, $e_b = E_b/A$ and binding energy vs. mass ($E = mc^2$)

2. Radioactive decay (beta, alpha, gamma) [Q13,Q14]

- Predicting stability and type of decay: energy level diagrams, binding energy formula, atomic mass
- beta: $\beta^- \implies n \longrightarrow p^+ + e^- + \bar{\nu}_e$ $\beta^+ \implies p^+ \longrightarrow n + e^+ + \nu_e$ EC $\implies p^+ + e^- \longrightarrow n + \nu_e$
- alpha: helium nuclei (2 protons, 2 neutrons)
- gamma: photons from decays of excited nuclear states
- activity $\equiv -dN/dt = \lambda N \implies$ solution: $N(t) = N_0 e^{-\lambda t} \implies$ half-life $t_{1/2} = \ln 2/\lambda$
- $Z_{\text{stable}}(A)$ for stability against β decay: $Z_{\text{stable}} = A \left(\frac{1.0084}{2+0.015 A^{2/3}} \right)$.
This formula gives the stable isotope for even-odd nuclei (A is odd) or even-even nuclei (both Z_{stable} and N are even). For odd-odd nuclei (A even, N, Z_{stable} odd) the neighboring even-even nuclei may have lower atomic mass, in which case the stable nuclei have $Z = Z_{\text{stable}} \pm 1$.
- Beta decay arises from weak interactions; it leaves A unchanged. β^- decay converts a neutron to a proton, decreasing N by 1 and increasing Z by one. Electron capture (EC) and β^+ decay convert a proton in a neutron, increasing N by 1 and decreasing Z by one.
- β^- decay and EC from i to f are energetically possible if the atomic masses satisfy the relation $m_i > m_f$.
 β^+ decay is energetically possible if $m_i > m_f + 2m_e$.
- Alpha decay is due to strong interactions. It arises from emission of a $\frac{4}{2}\text{He}$ nucleus, by tunneling through the Coulomb barrier. It changes A by 4. Parent and daughter nuclei of decay chains involving both α and β decays differ in mass number by multiples of 4.
- Alpha decay is energetically allowed if $dE_b/dA \leq 7.07$ MeV where 7.07 MeV is the binding energy per nucleon in a $\frac{4}{2}\text{He}$ nucleus. For β -stable nuclei, this happens for A larger than about 150.
- Fission describes the decay of a large nucleus into two roughly equal-sized fragments. It additionally produces neutrons, since the fragment nuclei have smaller A and prefer a lower N/Z ratio for stability. The extra neutrons can trigger further fission events, leading to a chain reaction.
- Decay rate $\lambda = dP_{\text{decay}}/dt = (1/N) dN_{\text{decay}}/dt$
The activity $dN_{\text{decay}}/dt = \lambda N$ of a radioactive sample is measured in becquerel (1 Bq = 1 decay per second) or curie (1 Ci = $3.7 \cdot 10^{10}$ Bq).
- Half-life $t_{1/2} = (\ln 2)/\lambda$
- Exponential decay law: $N(t) = N(0) e^{-\lambda t}$

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Laws of Thermodynamics

1. Zeroth Law of Thermodynamics: objects A and B are in thermal equilibrium if and only if $T_A = T_B$ [T1]
 - A thermometer is a device that quantifies temperature, calibrated to a standard temperature scale.
 - T (in K) = T (in °C) + 273.15 \implies Use T in kelvin in all formulas
 - $dU = mc dT$, where m is the mass of the substance and c is its specific heat.
2. First Law of Thermodynamics: $\Delta U = Q + W$ [T3]
 - $Q > 0$ if heat energy entering system; $W > 0$ if work energy entering system
3. Second Law of Thermodynamics: $\Delta S \geq 0$ [T5]
 - be able to explain in your own words why entropy increases

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Ideal Gases [T2, T3]

1. Ideal gas law: $PV = Nk_B T$, where N is the number of molecules and $k_B = 1.38 \times 10^{-23}$ J/K = 8.62×10^{-5} eV/K.
 - n = number of moles = N/N_A , where $N_A = 6.02 \times 10^{23}$.
2. “Equipartition theorem” [T2]
 - each degree of freedom gets $\frac{1}{2}k_B T$ on average \implies translational $K = \frac{1}{2}m[v^2]_{\text{avg}} = \frac{3}{2}k_B T$
 - $v_{\text{rms}} \equiv ([v^2]_{\text{avg}})^{1/2} = \sqrt{3k_B T/m}$
 - monatomic gas: $U = \frac{3}{2}Nk_B T$; diatomic gas: $U = \frac{5}{2}Nk_B T$
 - mode is fully unfrozen when $k_B T > 2\Delta E$ (c.f. Boltzmann factor $e^{-\Delta E/k_B T}$)
3. Gas processes [T3]
 - interpreting a PV diagram \implies ideal gas law applies at each point on curve
 - work from $W = -\int P dV \implies$ area under curve
 - net work from closed cycle is enclosed area
 - find T or N from ideal gas law and P, V
 - find ΔU from ΔT , then Q from First Law
 - isochoric: $V = \text{constant} \implies W = 0$
 - isobaric: $P = \text{constant} \implies W = -P(V_f - V_i)$
 - isothermal: $T = \text{constant} \implies W = -\int P dV = -(Nk_B T) \int dV/V = -Nk_B T \ln(V_f/V_i)$
 - adiabatic: $Q = 0 \implies PV^\gamma = \text{constant}$
 - $\gamma = 5/3 = 1.67$ for monatomic, $7/5 = 1.4$ for diatomic, 1.33 for polyatomic gas

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Macrostates, Microstates, and Entropy

1. Definitions: [T4]

- *macrostate* specified by macroscopic variables (e.g., 3 of P , V , N , or T for ideal gas)
- *microstate* specified by quantum state of *every* molecule
- *multiplicity* Ω is number of microstates consistent with a macrostate (e.g., same U , N)
- macropartition table:
 - know how to construct one given $\Omega(U, N)$
 - using $U = U_A + U_B = \text{constant}$; $\Omega_{AB} = \Omega_A \times \Omega_B$
 - predicting the most probable macropartition
- fundamental assumption: each accessible microstate is equally probable
 \implies relative probabilities of macropartitions equals ratio of (total) multiplicities

2. Einstein Solid with oscillator energy $\varepsilon = \hbar\omega$ [T4]

- $\Omega(N, U) = \frac{(3N + U/\varepsilon - 1)!}{(3N - 1)!(U/\varepsilon)!} \quad U = \sum_i^{3N} n_i \varepsilon \quad U = 3Nk_B T$

3. Temperature and Entropy $S = k_b \ln \Omega$ [T5, T6]

- $S_{AB} = S_A + S_B$
- given S , find multiplicity $\Omega = e^{S/k_b}$
- $dS/dU = 1/T$ defines temperature
- Boltzmann factor: $\text{Pr}(E) = \frac{1}{Z} e^{-E/k_B T} = e^{-E/k_B T} / \sum_{\text{all states}} e^{E_i/k_B T}$
- Movie test for reversible vs. irreversible processes
- explain why heat spontaneously flows from hot to cold bodies

4. Maxwell-Boltzmann Distribution and Average Energy [T7]

- $\text{Pr}(\text{speed within } dv \text{ centered on } v) = \frac{4}{\pi^{1/2}} \left(\frac{v}{v_P}\right)^2 e^{-(v/v_P)^2} \quad \text{where } v_P \equiv (2k_B T/m)^{1/2}$
- Average energy: $E_{\text{avg}} = \sum E_n \left(\frac{e^{-E_n/k_B T}}{Z}\right) = \frac{\sum E_n e^{-E_n/k_B T}}{\sum e^{-E_n/k_B T}}$

5. Calculating Entropy Changes [T8]

- $\Delta S = \int dQ/T$ (N constant, quasistatic volume change)
- $\Delta S = 0$ for (quasistatic) *adiabatic* volume changes
- isothermal $\implies \Delta S = Q/T$ (Q given or find from $Q = -W$)
- phase change: $\Delta S = \pm mL/T$ with L the latent heat
- If no work, $dQ = dU = mc dT \implies \Delta S = mc \ln(T_f/T_i)$
- Non-quasistatic processes \implies find ΔS from replacement process