

What you should know about . . .

## Waves

### 1. Basic wave properties [Q1]

- Superposition principle holds if wave equation is *linear*
- $f = \omega/2\pi$ ,  $\lambda = 2\pi/k$ , wave speed  $v = f\lambda = \omega/k$
- What does wave velocity depend on? e.g., string: tension and mass per unit length
- What is resonance?
- Finding normal modes (as in homework): boundary conditions, counting wavelengths
- Fourier series: a periodic function can be *uniquely* decomposed into a sum of sines and cosines

### 2. Interference phenomena [Q2,Q4]

- Diffraction through or around object (slit, eyeball, etc.) with size  $a$ 
  - total diffraction if  $\lambda > a$ ; no diffraction if  $\lambda \ll a$
  - Construction using path difference to derive  $\sin \theta = \lambda/a$  formula for first minimum.
  - When can objects be resolved? (qualitative and quantitative)
- Applying two-slit interference formula
  - Construction using path difference to derive formula for “bright spots”:  $d \sin \theta = n\lambda$
- When is the small-angle approximation ( $\sin \theta \approx \tan \theta \approx \theta$ ) valid?

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## Particlelike Properties of Waves and Wavelike Properties of Particles

### 1. Photon model [Q3]

- photon energy/momentum related to frequency and wavelength
- Basic relations for photons:  $E = hf = hc/\lambda$ ;  $E = pc$
- Basic photon calculations (photons per second emitted, absorbed, visible photon wavelengths, etc.)

### 2. Photoelectric effect [Q3]

- predictions of wave vs. particle models [also Q5,Q6]
- analysis of experiment:  $K = hf - W = hc/\lambda - W$
- What is a work function?

### 3. de Broglie wavelength: what does it mean? [Q4,Q5]

- $p = h/\lambda$  and  $E = hf$  ( $E \neq hc/\lambda$  for massive particles!). Nonrelativistic free particle:  $E = p^2/2m$
- relationship between kinetic energy and wavelength (qualitative and quantitative)
  - nonrelativistic:  $\lambda = h/\sqrt{2Km} = hc/\sqrt{2Kmc^2}$ . Relativistic version:  $\lambda = hc/\sqrt{K(K + 2mc^2)}$
- diffraction and interference of particles
  - occurs when deBroglie  $\lambda$  comparable to slit size or other relevant size.
  - applications: electron interference experiments, resolving particles ( $\lambda \lesssim a$ )

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## Quantum Mechanics

### 1. Probability interpretation of wave function [Q6]

- Calculating the probability to find a particle in some region of space (given a wave function)
- Possible results of successive measurements of an observable (e.g., energy or position); “collapse”

### 2. The probability rule and application to spin wave functions [Q6]

- If you have quanta in a state  $|\psi\rangle$  and measure an observable  $O$  of the quanta which has possible eigenvalues  $O_n$ ,  $n = 1, 2, 3, \dots$ , then the probability that your measurement yields the specific value  $O_n$  for the observable is

$$Pr(O_n) = |\langle O_n | \psi \rangle|^2,$$

with  $|O_n\rangle$  being the eigenstate of the observable  $O$  with eigenvalue  $O_n$ .

### 3. Time Dependence of a stationary state [ $\propto e^{-iE_n t/\hbar}$ ] [Q6]

- When is a solution a “stationary state”? [Definite energy  $\implies$  energy eigenfunctions!]
- What happens if you combine two stationary states?

### 4. Quanton in a Box [Q7]

- derivation of energy eigenvalues  $E_n = \hbar^2 n^2 / 8mL^2$  and eigenfunctions  $\psi_E(x) = A \sin(n\pi x/L)$  in box

### 5. Bohr model assumptions $\implies$ energy levels $E_n = -ke^2/2a_0 n^2$ and radii $r_n = n^2 a_0$ in hydrogen [Q7]

- Bohr radius  $a_0 = \hbar^2 / 4\pi^2 m k e^2 \approx 0.053 \text{ nm} \implies E_n = -(13.6 \text{ eV})/n^2$ . Results for hydrogen-like atoms.

### 6. Photon spectral lines [Q8]

- Relationship to energy levels (differences!)
- Infinite well vs. oscillator vs. hydrogen atom (energy levels *and* spectra)
- Pauli exclusion principle and spin; fermions vs. bosons

### 7. Atoms [Q9]: Selection rules (e.g., $\Delta l = \pm 1$ ), predicting ground state configuration, periodic table of elements

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## Energy Eigenfunctions [Q10, Q11]

1. Schrödinger eq:  $-\frac{\hbar^2}{2m} \frac{d^2 \psi_E(x)}{dx^2} + V(x) \psi_E(x) = E \psi_E(x)$ . Is a given function a possible energy eigenfunction?
2.  $\psi_E(x)$  is wavelike (curving towards the axis) in a classically allowed region ( $V(x) < E$ )
3. The local *curvature* (2nd derivative!) is related to the local wavelength which is related to the local kinetic energy  $K(x) = E - V(x)$  (see equations on previous page and  $K = \hbar^2 / 2m\lambda^2$ ).
4.  $\psi_E(x)$  is exponential-like (curving away from the axis) in a classically forbidden region ( $V(x) > E$ )
5. The wavefunction must decay to zero at  $|x| \rightarrow \infty$  (normalizability!).
6. This causes quantization of energy: Only for specific values of  $E$  does the wavefunction curve in the classically forbidden region in exactly the way needed for the wavefunction to approach zero at  $|x| \rightarrow \infty$ .
7. The quanta spend more time in regions where they move slowly (small  $K(x)$ ) – wave function has larger amplitude there.
8. Tunneling: the ability of a wavefunction to leak through a finite barrier due to its exponential tail in the classically forbidden region.