

Write your name and answers on this sheet and hand it in at the end.

After the indicated time, move on to the next activity, even if you are not finished!

1. Q10: The Schrodinger Equation [20 min.]

In chapter Q10, we motivate the Schrodinger equation (Q10.12). Here we consider some basic problems from the chapter.

a. Answer two-minute question Q10T.4.

We derived the Schrodinger equation starting with the assumption that there was a fixed energy E . \Rightarrow **C**

b. How can we test if a given wave function $\psi(x)$ is a solution to the Schrodinger equation? [See Section Q10.4 and equation (Q10.14).]

$\psi_E(x)$ is a solution if when we plug it into $-\frac{\hbar^2}{2m} \frac{d^2 \psi_E}{dx^2} - [E - V(x)] \psi_E(x) = 0$, we get zero on the left side of the equation for all values of x .

c. Do problem Q10S.1, showing that $b = \sqrt{2m(E - V_0)}/\hbar$. [Note the hint.] Is b real?

If $\psi_E(x) = A \cos bx$, then $\frac{d\psi_E}{dx} = -Ab \sin bx$ and $\frac{d^2 \psi_E}{dx^2} = -Ab^2 \cos bx$
 So $-\frac{\hbar^2}{2m} \frac{d^2 \psi_E}{dx^2} - (E - V_0) \psi_E(x) = +\frac{\hbar^2}{2m} b^2 A \cos bx - (E - V_0) A \cos bx = 0$.
 This is zero for all x if $b^2 = \frac{2m}{\hbar^2} (E - V_0) > 0$ or $b = \sqrt{2m(E - V_0)}/\hbar$

d. Do problem Q10S.3. [Note the hint.] Make sure that your answer for b is a real number when $E < V_0$.

If $\psi_E(x) = A e^{\pm bx}$, $\frac{d\psi_E}{dx} = \pm b A e^{\pm bx}$ and $\frac{d^2 \psi_E}{dx^2} = b^2 A e^{\pm bx}$
 So $-\frac{\hbar^2}{2m} \frac{d^2 \psi_E}{dx^2} - (E - V_0) \psi_E(x) = -\frac{\hbar^2}{2m} b^2 A e^{\pm bx} - (E - V_0) A e^{\pm bx} = 0$
 or $b^2 = \frac{2m}{\hbar^2} (V_0 - E) > 0$ so $b = \sqrt{2m(V_0 - E)}/\hbar$, which is real since $b^2 > 0$.

e. Why must a physically acceptable wave function go to zero as x goes to plus or minus infinity?

If not, the total probability calculated from $\int_{-\infty}^{\infty} |\psi_E(x)|^2 dx$ would be infinite (instead of 1).

f. What happens in SchroSolver if you choose an energy that is not an energy eigenvalue? How can you use this to zero in on the correct eigenvalue? Try out your method.

On the right side (large positive x), the wave function either blows up or down. The correct eigenvalue lies between a guess that blows up and one that blows down (with the same number of bumps). So keep squeezing between these limits until the desired accuracy is reached.

2. Q11: Wavefunctionology [25 minutes]

Do the "Wavefunctionology Exercises" worksheet and hand it in together with this sheet. Below we give some suggestions and hints for each part.

Part 1.

- There is one error for two of the wave functions and two errors for each of the other two.
- To be systematic, consider each of possibilities B through F in turn.
- Each of the incorrect possibilities corresponds to one of the "Rules for Sketching Wavefunctions", which are labeled 1 through 6 (see handout). B goes with rule 1, C goes with rule 6, D goes with rule 3, E and F both go with rule 2, and G goes with rule 4 or rule 5.

Part 2.

- First decide how many bumps there should be.
- Then mark where the wave function is wavelike and where it is exponential-like.
- Finally, where will the amplitude be large and small?
- Draw a smooth wave function with these features.
- Use the PhET applet "Quantum Bound States" or SchroSolver to check your answer.

Part 3.

- Follow the same steps as in Part 2.
- Use the SchroSolver to check your answer.

Part 4.

- Follow the same steps as in Part 2.
- Use the SchroSolver to check your answer.

"Quantum Bound States" is easier to check.

Use "Ramp"
May be easier to use "Quantum Bound States" with "Asymmetric"
Potential Well.

Start with "Well" then right click at
 $x=0.0$ at the height of the desired potential.

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You probably want to increase the width of the well.
Energy with well width 2.1nm and middle height at 7.5eV is about 4.5535eV.
(Put a nearby value and "Find".)

Name: KEY

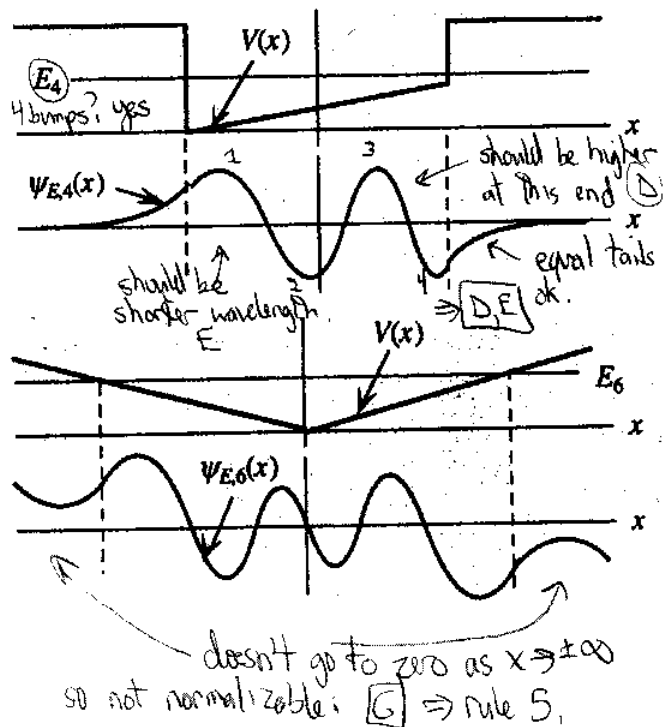
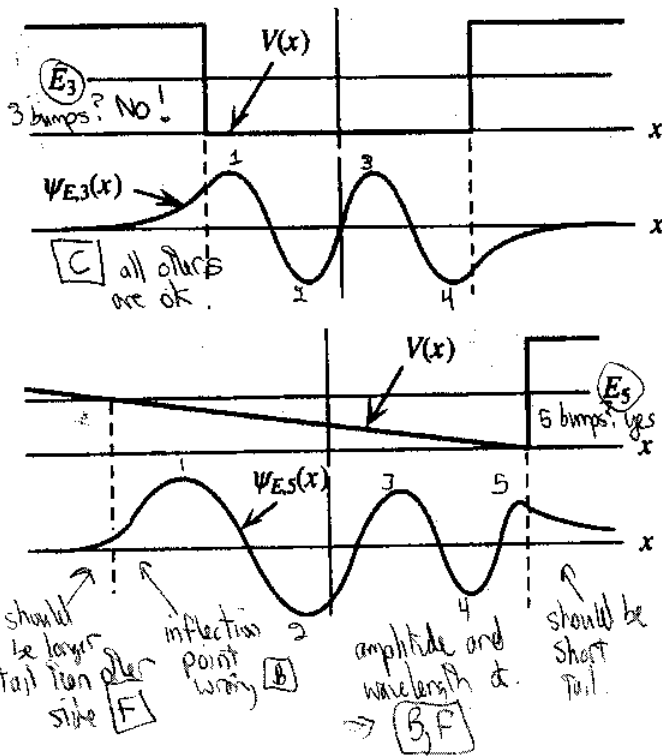
Wavefunctionology Exercises

These exercises are adapted from chapter Q11 of Tom Moore's quantum mechanics book in the "Six Ideas that Shaped Physics" series.

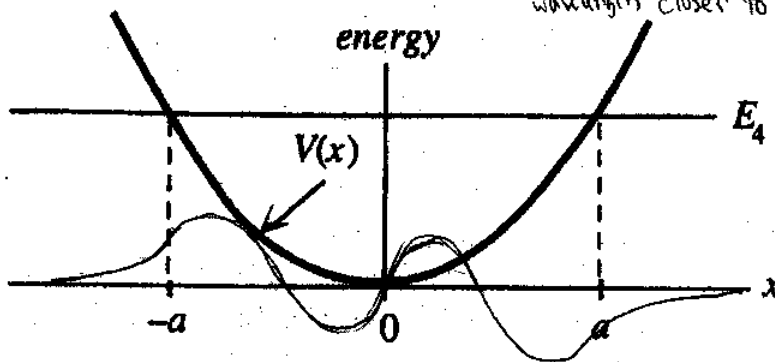
- Each of the following potential energy graphs with energy E_i marked has a wave function supposedly corresponding to the i^{th} -lowest possible energy. What (if anything) is wrong with the wave function as drawn. In some cases, multiple things may be wrong; indicate them all.

You can choose from the following possible responses. The wave function is ...

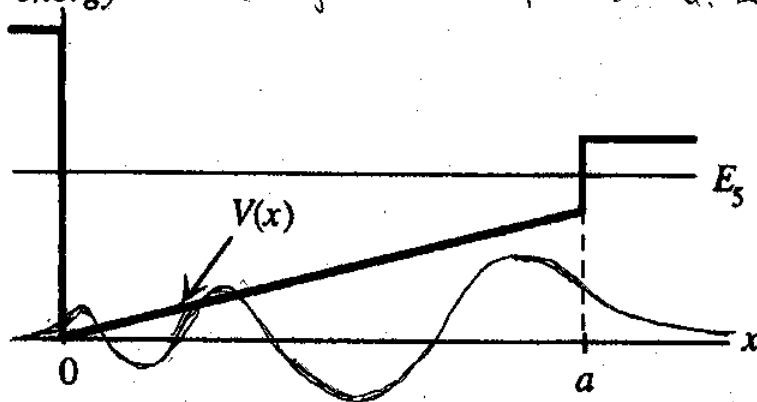
- correctly drawn (more or less).
- incorrect because it curves toward the axis in a forbidden region or it curves away from the axis in an allowed region.
- incorrect because its wavy part doesn't have the correct number of bumps (or you could count nodes instead).
- incorrect because the amplitude of its wavy part is wrong.
- incorrect because the wavelength of its wavy part is wrong.
- incorrect because one of the exponential tails is the wrong length.
- incorrect for other reasons (specify).



2. Sketch on the x -axis the energy eigenfunction (standing wave solution) corresponding to the fourth-lowest bound state energy for a particle whose potential energy is shown by the graph below. $E_4 \Rightarrow 4$ bumps. Inflection points at $\pm a$. Larger amplitude and wavelength closer to $\pm a$.



3. Sketch on the x -axis the energy eigenfunction (standing wave solution) corresponding to the fifth-lowest bound state energy for a particle whose potential energy is shown by the graph below. $E_5 \Rightarrow 5$ bumps. Increasing amplitude and wavelength toward x right. Inflection points at $\pm a$. Larger tail on right.



4. Sketch on the x -axis the energy eigenfunction (standing wave solution) corresponding to the fourth-lowest bound state energy for a particle whose potential energy is shown by the graph below. E_4 so 4 bumps. Inflection points at $\pm a$, same tails, like 2 but larger amplitude and wavelength in the middle.

