$\boxed{\text{KEY}}$

H133: 1094 Session 9

Write your name and answers on this sheet and hand it in at the end.
After the indicated time, move on to the next activity, even if you are not finished!

1. T7: MBoltz and EBoltz Problems [12 minutes]

Start up MBoltz and EBoltz from the H133 folder under the Start menu.

a. Do T7B.4. What should you choose for x1? For x2, does it matter if you choose 100 or 200? Why not? Argue that your result for the probability is reasonable.
Choose $x_1 = v/v_p = 1$. For x_2, we can use 100 or 200 or inf and get the same answer, since the contribution above $x=100$ is negligible. We get a probability of about $\boxed{0.57}$ which is reasonable since it is about 1/2 the area under the curve.

b. As a warm-up problem for using EBoltz, do T7B.6.
If we simply look at the graph for the interval $0 \le k_BT/\varepsilon \le 10$, we see $E_{avg}/k_BT > 0.9$ when $k_BT/\varepsilon \simeq 4.6$. So $T = \varepsilon\, 4.6/k_B = \dfrac{4.6(0.035\,eV)}{8.62\times10^{-5}\,eV/K} \doteq \boxed{1900\,K}$

c. Now for a more interesting exploration. The default energies for EBoltz are E = epsilon*n, as for the Einstein solid. What is the approximate average energy in the high-temperature limit? (Give it as a multiple of kT.) If we graph up to $k_BT/\varepsilon \le 1000$, we see $\boxed{E_{avg} \doteq k_BT}$.

Now change the energy to be E = epsilon*n^2 [you get n^2 by typing n^2]. What is the approximate average energy in the high-temperature limit?
Now E_{avg} approaches $\boxed{\frac{1}{2}k_BT}$

Now try E = epsilon * sqrt(n) [this is how you write a square root]. Once again, what is the approximate average energy in the high-temperature limit? [NOTE: Don't try kT/epsilon much greater than 10, or it will take forever.] Even with $0 \le k_BT/\varepsilon \le 10$, we see that the limit is about $\boxed{2k_BT}$

Have you discovered a pattern? What is it? Predict first and then test with E = epsilon*n^3.
If $E_n = \varepsilon n^\alpha$, then $E_{avg} \xrightarrow[T\to\infty]{} \doteq \frac{1}{\alpha}k_BT$. So should find $E_{avg} \doteq \frac{1}{3}kT$ for εn^3.
It works! (Back to $0 \le k_BT/\varepsilon \le 1000$)

2. T8: Entropy of Monatomic Gas and Entropy Changes [15 minutes]

a. Start with some basic applications of equations (T8.7) and (T8.8). Do T8T.1 and T8T.2, explaining your answers. $\Omega(U,V,N) \doteq \frac{1}{N!}\left(\frac{8mV^{2/3}U}{3Nh^2}\right)^{3N/2}$ for an ideal gas. In T8T.1, V and N are fixed. If T doubles, U doubles, so $\Omega_f/\Omega_i = (U_f/U_i)^{3N/2} = (T_f/T_i)^{3N/2} = 2^{1.5\times10^6} \Rightarrow$ (F)
T8T.2: Comparing argon and helium, only m is different and greater in argon, so argon has greater entropy. (B)
since same V, N, U (since both monatomic.

b. Do T8T.5. What kind of process is this? Show your work.
Constant c, temperature changing, no work process,
$$S = mc\ln\frac{T_f}{T_i} = (0.1\,kg)(4186\,J/K)\ln\frac{273+25}{273+5} \doteq 29.1\,K \quad (B)$$

c. Do T8B.8. [Hint: Phase changes are discussed in Section T8.4.]
Constant temperature phase change, so $\Delta S = \dfrac{\Delta Q}{T} = \dfrac{-mL}{T} = \dfrac{(-0.8\,kg)(333\times10^3\,J/kg)}{273\,K} \doteq \boxed{-980\,J/K}$
Entropy change is negative, as expected with decreasing internal energy.

d. The concept of a "suitable replacement process" (section T8.6) is important but sometimes tricky. You need to have the same initial and final macrostates and the process must be quasistatic. Use T8T.8 and T8T.9 to test your understanding. Only one answer each in T8T.8 and T8T.9 works; why do the others fail? In T8T.9, note that the ideal gas law applies to both the initial and final gases; your replacement process must be consistent.

T8T.8 B The original process adds heat once the waves settle down, so A is out. D doesn't add energy. You need to heat, so C is out.

T8T.9 D The slow adiabatic compression has $TV^{2/3} \doteq$ const, so if $V \to \frac{1}{2}V$, T less than doubles. $\Rightarrow$ need to heat up to 2T. A, B, C would have the wrong temperature.

3. Introduction to T9: Heat Engines [15 minutes]

A perfect engine, which converts thermal energy (heat) *completely* into work, would violate the 2nd Law of Thermodynamics. Flowing heat carries entropy from one object to another, but flowing work does not. (E.g., raising a weight or adiabatically compressing a gas doesn't change the entropy.) A real heat engine needs to expel the entropy the engine gets from the heat source (a hot reservoir at T_H), so there needs to be a cold reservoir at T_C.

According to the first law, $|W| = |Q_H| - |Q_C|$, where Q_H is the heat energy from the hot reservoir and Q_C is the heat energy sent to the cold reservoir. Then the efficiency e is given by

e = benefit/cost = $|W|/|Q_H| = 1 - |Q_C|/|Q_H| < (T_H - T_C)/T_H$ (the < is really "less than or equal to").

a. To get warmed up at applying the idea of efficiency, do T9T.3, being careful to identify $|W|$, $|Q_H|$, and $|Q_C|$ and then do T9T.5. Do you think the personal fan is practical?

T9T.3: In one second, $|W| = 300\,J$, $|Q_c| = 1200\,J$, so $|Q_H| = 1500\,J$. Then $e = \frac{300}{1500} = 0.2$ A

T9T.5: $e = \frac{T_H - T_C}{T_H} = \frac{273+37-(273+22)}{273+37} = \frac{15}{310} \doteq 5\%$, E Probably not, but more analysis of available work needed.

b. Is there a symmetry between heat energy and work? Answer T9B.1.

Examples: space heater, sliding block (from friction), refridgeration. Complete conversion of mechanical energy into heat inevitably increases the entropy, so OK. Complete conversion of heat into mechanical energy would violate the 2nd law; waste heat required.

c. Try an engineering application: the power plant proposed in T9S.8, making use of the basic efficiency relation above. Be careful with the temperatures given in centrigrade!

$T_H = 30° + 273 = 303K$. $T_C = 4 + 273 = 277\,K$. In one second, $|W| = 10^9\,J$.
$|Q_H| = \frac{|W|}{e} = 10^9/(26/303) = 1.1 \times 10^{10}\,J$. So $|Q_c| = |Q_H| - |W| = 10^{10}\,J = 10^4\,MJ$, which means the power is 10,000 MW.

d. Let's finish with some T9S questions that we'll discuss on Thursday. Give a one or two sentence answer to each of the following:

- T9S.2. Hydroelectric plants convert mechanical energy of falling water into mechanical energy of rotating turbines; only inefficiency from friction. Nuclear power plant is restricted to efficiency set by temperature of its hot and cold reservoirs.
- T9S.3. 100% efficient only if we consider the electrical energy coming into the heater converting to heat. But much less than 100% efficiency in conversion of thermal energy in power plant to electrical energy.
- T9S.4.
"You can't win" means you can't create energy from nothing (eg. create additional energy). "You can't break even" means that efficiencies are less than one; there is always some energy waste as heat in converting heat energy to mechanical energy.