Consequences of the Uncertainty Principle

The deBroglie relation $\lambda = h/p$ and the uncertainty principle $\Delta p_x \Delta x \geq \hbar/2$, which reflect the wavelike nature of matter, imply that a *localized* particle (in an atom or a nucleus or passing through a slit, etc.) cannot have zero kinetic energy.

Three ways to see this (there are more!):

1. From the uncertainty principle, if a particle is confined to Δx, the momentum will be at least $\Delta p_x = \hbar/(2\Delta x)$, where $\hbar = h/2\pi$.

2. If a particle with initial momentum $p_x = p$ and $p_y = 0$ passes through a slit of width d, it will diffract, which means it spreads out in the y direction. So localizing in the y direction makes $p_y > 0$. Estimate: The angle to the first minimum is given by $\lambda = d \sin\theta$ and $\sin\theta \approx p_y/p$. The deBroglie relation $\lambda = h/p$ gives the uncertainty principle result with $\delta y \approx d$ and $\delta p_y \approx p_y$.

3. The wavelength of a particle cannot be much larger than the size of the region of localization. From $p = h/\lambda$, this means that p has a minimum size and therefore the magnitude of the velocity and kinetic energy have minimum values.

Comments:

- These are rough (order-of-magnitude) estimates, which may differ by factors like 2π.

- The value $\hbar/2$ is the *best case* possible; the product of uncertainties can be much greater.

- Δx is usually less than the actual size of the localization region. For example, if a particle is equally likely to be anywhere from 0 to L on the x-axis, Δx is not L but $\Delta x = L/\sqrt{12}$. We can show this using the precise definition $\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$. The average value of x is $\langle x\rangle = (1/L)\int_0^L x\,dx = L/2$. The average value of x^2 is $\langle x^2\rangle = (1/L)\int_0^L x^2\,dx = L^2/3$. Combining these gives the desired result.

- To find the nonrelativistic kinetic energy in three dimensions from $p^2/2m$, we should remember that $p^2 = p_x^2 + p_y^2 + p_z^2$, so if the particle is localized in all three directions, there will be uncertainties in each of p_x, p_y, and p_z. (So in a cube, there is an overall factor of three times the $(\Delta p_x)^2$ term.)