Chapter Q1

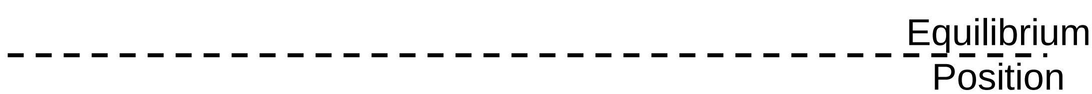
- Introduction to Quantum Mechanics
  - End of 19th Century only a few loose ends to wrap up.
  - Led to Relativity which you learned about last quarter
  - Led to Quantum Mechanics (1920's-30's and beyond)
    - Behavior of atomic and subatomic world
    - Newton's Laws don't hold
  - Foundation of most Physics Research that occurs these days.
- Basic Ideas behind Quantum Mechanics
  - Small Particles behave like waves
    - Not localized
    - Interference effects (…more soon)
    - Observations alter the system.
      - Basic quantities ***x, p***
      - In QM if you try to determine ***x*** you will effect ***p*** and visa versa.
    - Implies bound systems have quantized states (Atomic Spectra)
  - Waves (EM) behave like particles.
- We need to understand <u>Classical</u> wave first.

Classical Waves

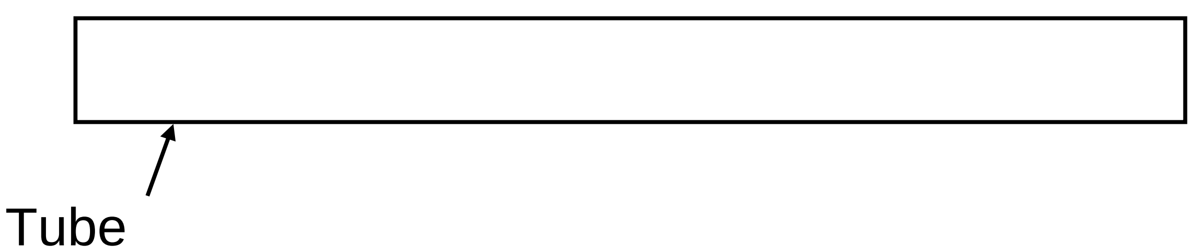
- Basics
  - Waves are a disturbance that travel in a medium.
    - e.g. water waves
    - Note: Particles in the medium are not traveling along with the wave.

Demo

  - We will look at 1-D wave to begin.
- Two types of waves
  - Tension Wave (sometime called transverse)
    - e.g. wave on a string.



  - Compress ional Wave (sometimes call longitudinal)
    - e.g. sound wave

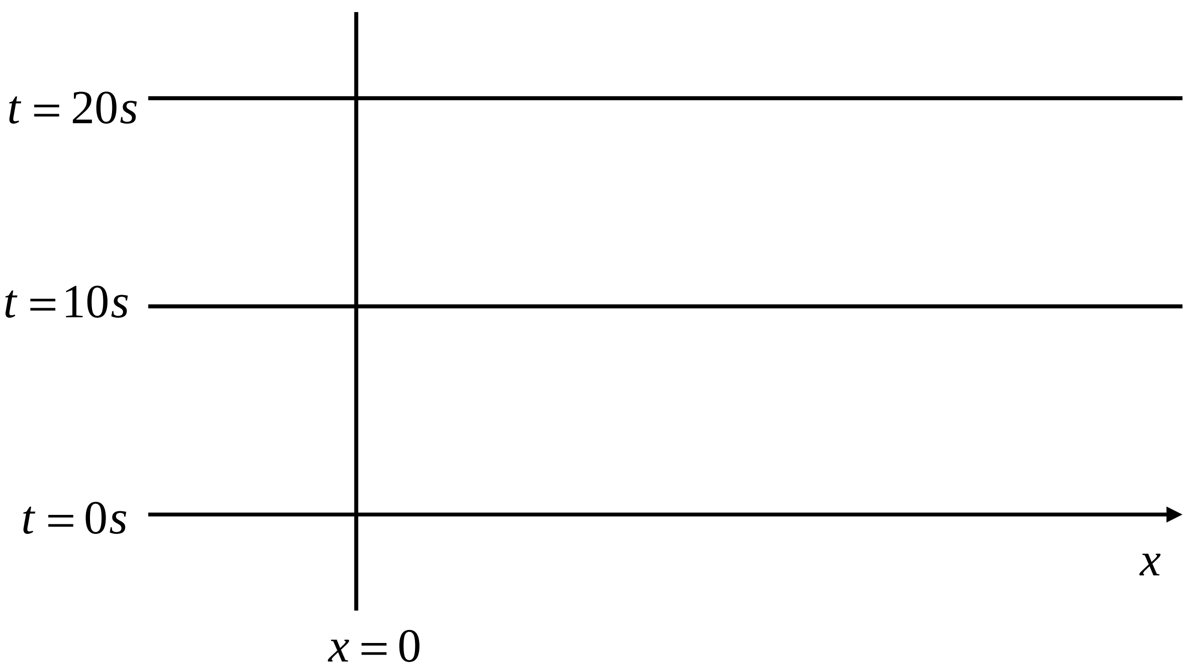


Mathematical Representation

- Function of position and time

$$f\left(x,t\right)$$

$$\text{e.g.} \quad f(x,t) = Ae^{-\frac{(x-bt)^2}{2\sigma^2}}$$



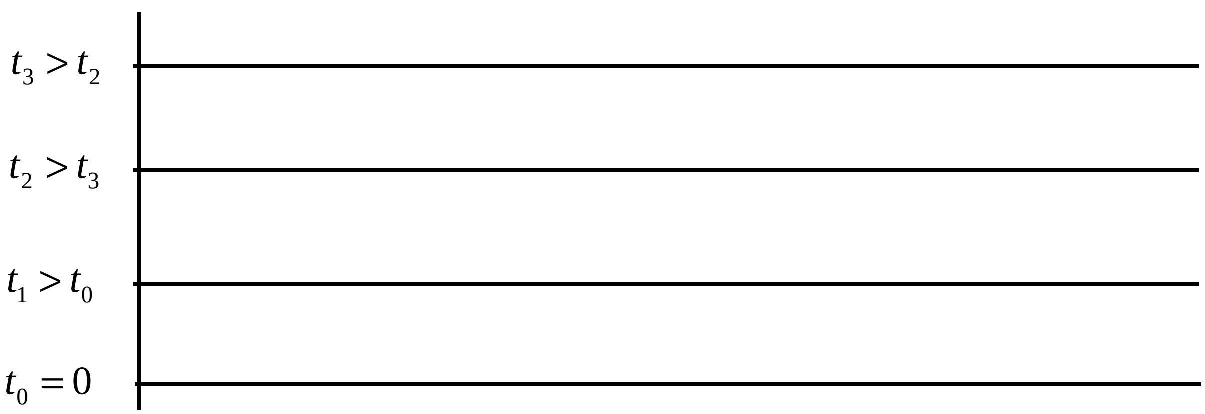
- Also common to think about functions with sines and cosines…more later.

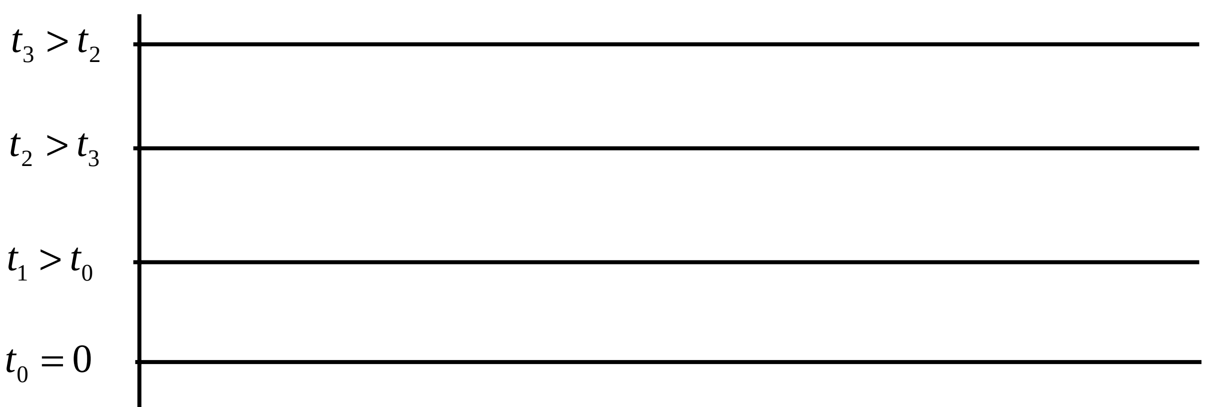
Superposition Principle

- ***If two waves described by $f_1(x,t)$ and $f_2(x,t)$ are moving in a medium, the combined wave is described by the algebraic sum of the two waves***

  $$f_{tot}(x,t) = f_1(x,t)+f_2(x,t)$$

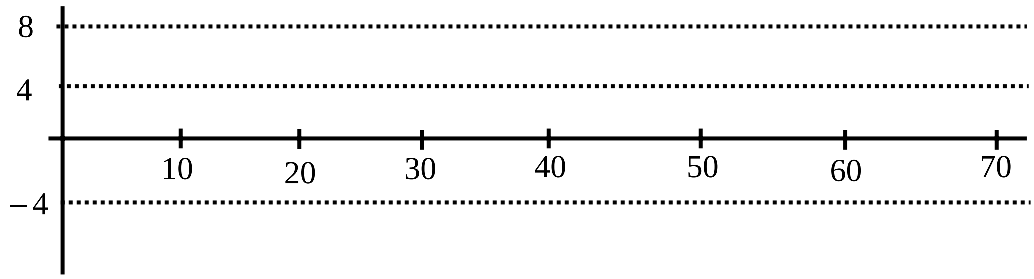
- This is a powerful statement even though it seems quite simple.
- It holds for many (but not all) waves. For what we study we will assume it holds.



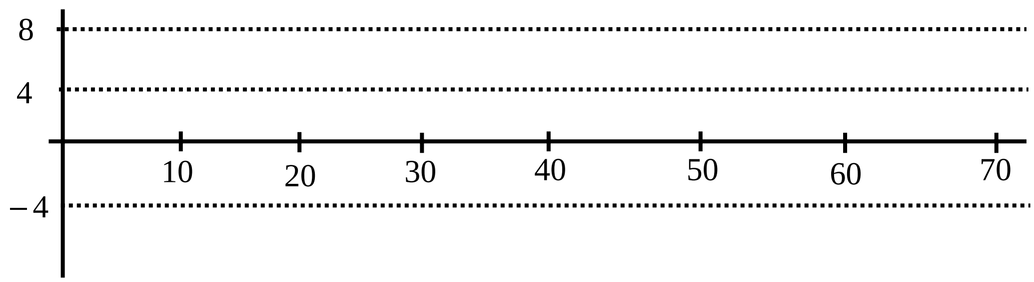


Example

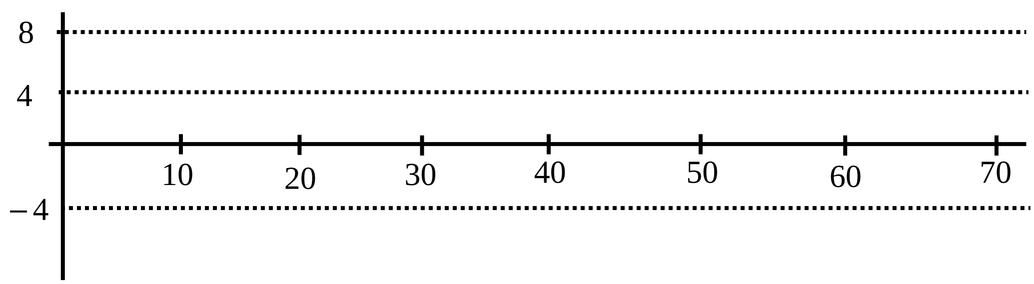
- Q1B.1



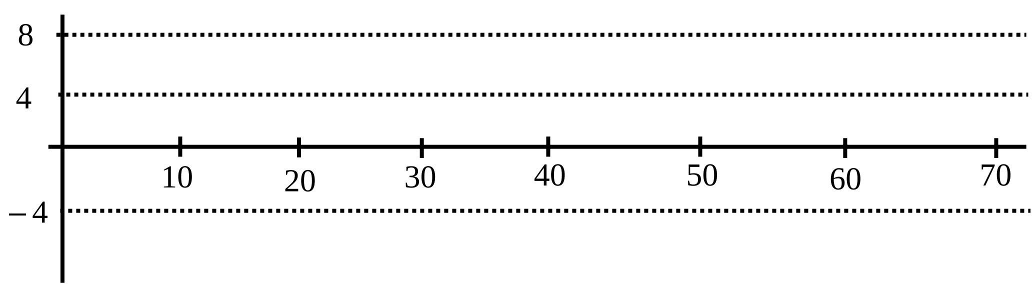
$t = 0$



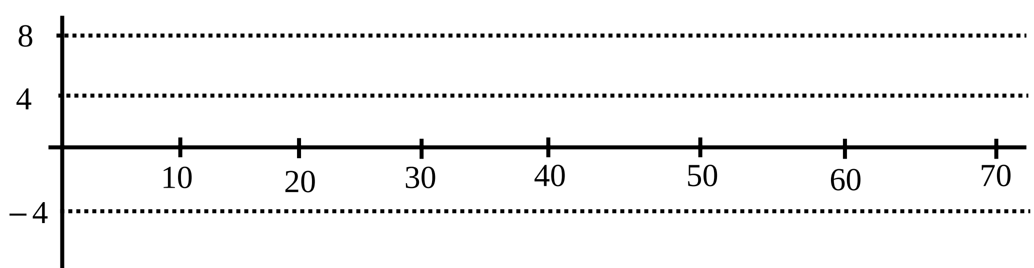
$t = 2s$



$t = 3s$



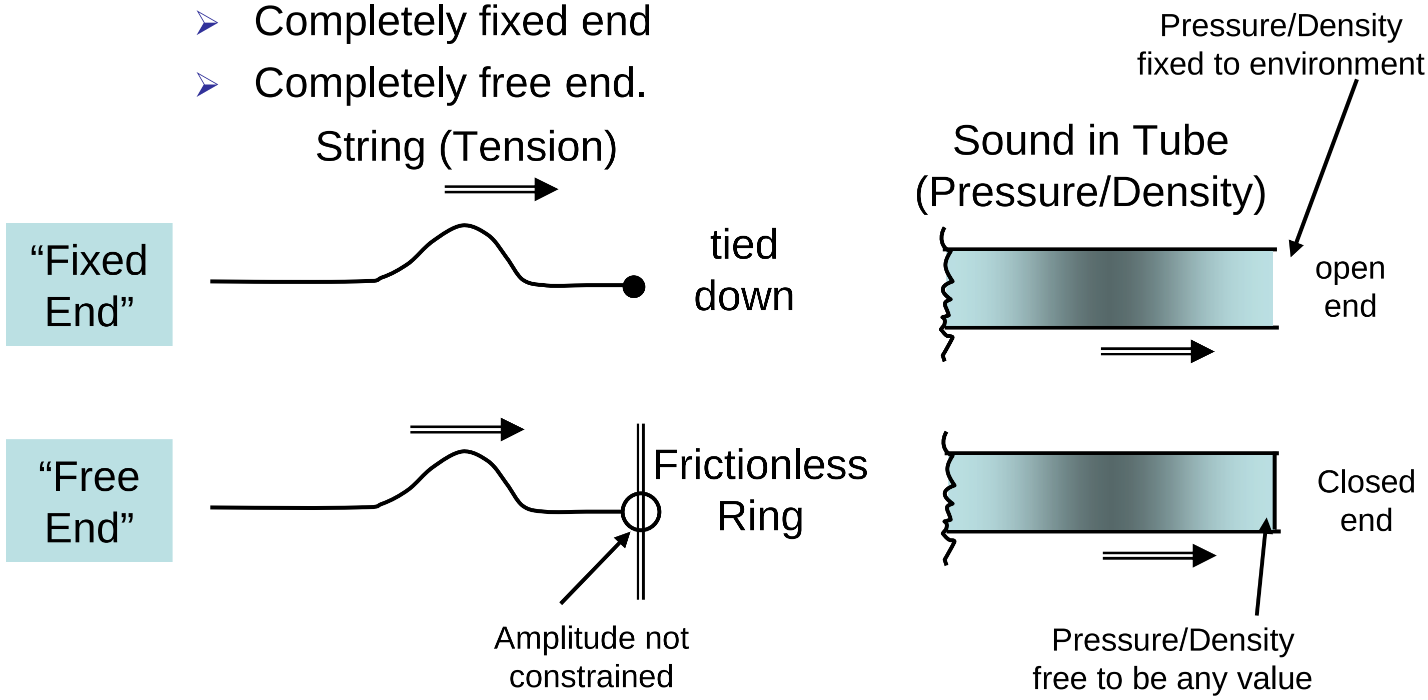
$t = 4s$



$t = 6s$

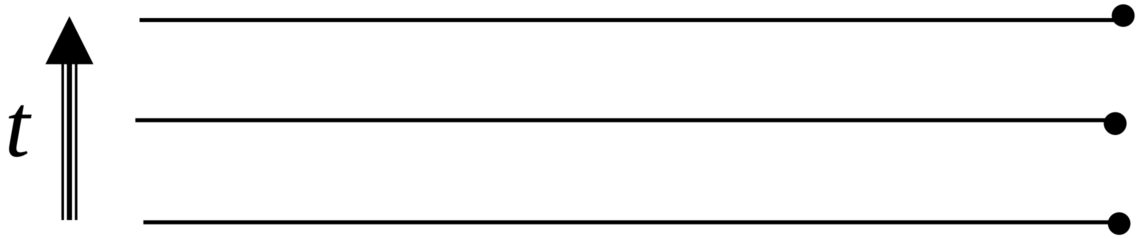
Reflection

- One of the more interesting features of waves is what occurs when they encounter a boundary.
  - Examples:
    - A rope tied off at an end
    - Waves in a pool
    - Density changes of a string/rope.
  - The issue of what happens to waves at a boundary will also be important when we look at waves of particles in QM.
- When waves encounter a boundary they are partly or entirely reflected from the boundary.
- Let's start by looking at two extreme cases for both tension and pressure waves:
  - Completely fixed end
  - Completely free end.

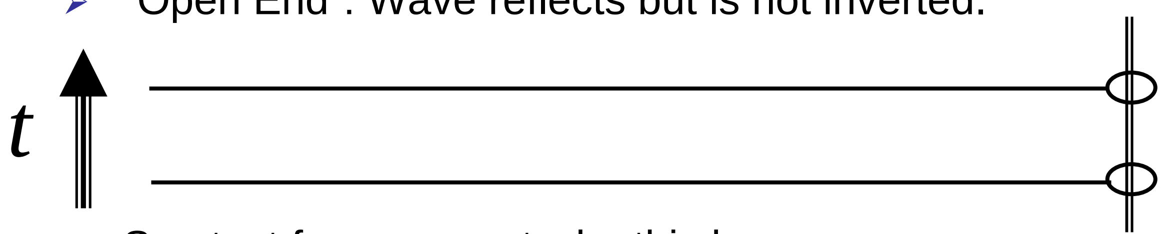


Reflections

- What happens when the waves strike these boundaries?
  - “Fixed Ends”: Wave reflects but is inverted



  - “Open End”: Wave reflects but is not inverted.



  - See text for argument why this happens
- Intermediate Case (Most Cases)

  (1) Light – to – heavy (2) Heavy – to – Light

What is the analogy for sound waves?

Sine Waves

- When discussing fixed (or free) boundaries where we have reflections we can set up an interesting phenomena called **standing waves.**

A sine wave traveling in the x direction has the form

$$f(x,t)=A\sin(\underbrace{kx-\omega t+\phi}_{\text{"phase"}})$$

"Phase Constant" : Make 0 by redefining x=0 or t=0

Where A is the amplitude, k is the wave number and ω is the angular frequency.

$$k=\frac{2\pi}{\lambda} \qquad \omega=\frac{2\pi}{T}=2\pi f \qquad T=\frac{1}{f}$$

By following a peak in the wave we can measure the velocity of this traveling wave. The peak is defined by having the phase = π/2.

$$\frac{\pi}{2}=kx_{peak}-\omega t$$

Solving for x_{peak} we find

$$x_{peak}=\frac{\pi}{2k}+\frac{\omega}{k}t$$

$$v_{peak}=\frac{dx_{peak}}{dt}=0+\frac{\omega}{k}=\frac{\omega}{k}$$

$$v_{peak}=\frac{\omega}{k}=\frac{2\pi/T}{2\pi/\lambda}=\frac{\lambda}{T}=\lambda f$$

Example

A sine wave traveling in the +x direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and frequency of 8.00 Hz. The vertical displacement at t=0, x=0 is 15.0 cm.

a) $k = ?$, $T = ?$, $\omega = ?$, $v = ?$

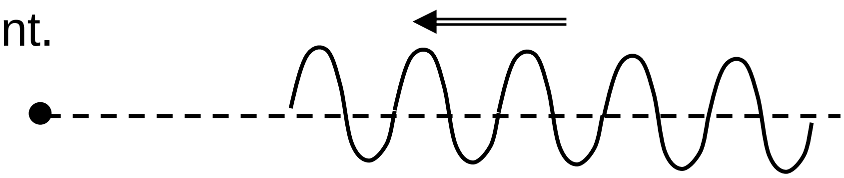
b) What is the phase constant?

Standing Waves

- What about a wave traveling in the –x direction?

$$f(x,t) = A\sin(kx + \omega t)$$

How does all this connect to "standing waves"? Imagine a wave traveling in the negative x direction toward a fixed point.



When the wave is reflected an inverted wave traveling the +x direction. According to the superposition principle we add the two functions

$$f(x,t) = A\sin(kx + \omega t)\ ?\ A\sin(kx - \omega t)$$

What should the sign "?" be? A "+" or a "–" ? Think about at the **fixed** boundary (x=0), we think the wave should be inverted:

$$f(x=0,t) = A\sin(\omega t)\ ?\ A\sin(-\omega t)$$
$$= A\sin(\omega t)\ ?\ \left(-A\sin(\omega t)\right) = 0$$

Since the first term represents the incoming + wave and the second term represents the outgoing inverted wave the sign "?" must be positive.

Standing Waves

- Now let's change the form of the superposition of the two waves so that it is in a more revealing form:

$$
\begin{aligned}
f(x,t) &= A\sin(kx+\omega t) + A\sin(kx-\omega t) \\
&= A\{\sin kx\cos\omega t + \cos kx\sin\omega t + \sin kx\cos\omega t - \cos kx\sin\omega t\} \\
&= 2A\sin kx\cos\omega t
\end{aligned}
$$

  Note the structure of this relationship the position and the time dependence has been factorized.

$$
f(x,t) = A(x)B(t)
$$

- At fixed $x=x_o$ we have $f(t)=A(x_o)\cos(\omega t)$, so we have something that is oscillating with a frequency ω and a maximum amplitude $A(x_o)$.
- At fixed $t=t_o$ we have $f(x)=\sin(kx)(B(t_o))$, so we have a wave with wavenumber k and max. amplitude $B(t_o)$.

Standing Waves

- If we consider a string of length L fixed at both ends (x=0, x=L):

$$\sin(k0) = 0$$
$$\sin(kL) = 0 \Rightarrow kL = n\pi \quad n=1,2,3,\ldots$$

- So only certain wave numbers (freq.) will allow the existence of a standing wave.

$$k = n(\pi/L) \text{ or } L = n(\lambda/2) \quad n=1,2,3,\ldots$$

- This implies we must fit an ***exact*** number of *half* wavelengths between the fixed ends.

- We can also express this in terms of frequency

$$f = \omega/(2\pi) = (\omega/k)(k/2\pi) = n\nu/(2L)$$

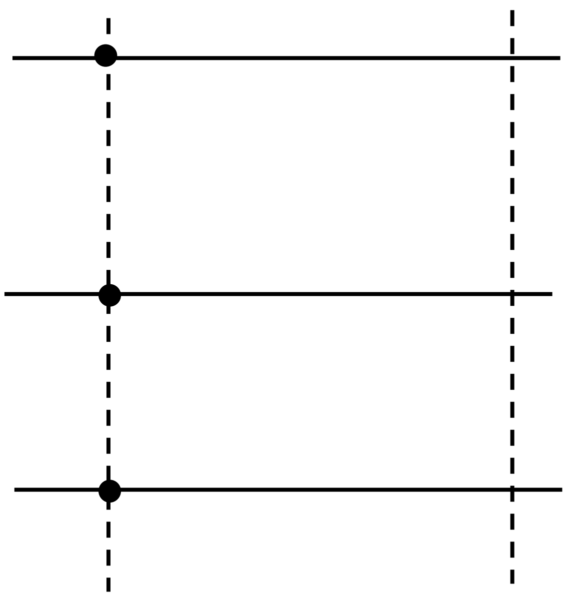
- This concept that only *certain* frequencies of waves work, it important. We say this is *quantized*…something that is important when we get to <u>quantum mechanics.</u>

Standing Waves

- We just considered the case when the string is fixed at two ends. We can consider two other cases.
- (A) Fixed one end x=0 and totally free at x=L

$$\sin(k0)=0$$
$$\sin(kL) = 1 \Rightarrow kL=n(\pi/2) \quad n=1,3,5,\ldots$$
$$\Rightarrow \quad L=n(\lambda/4) \quad \text{and} \quad f = n\nu/(4L)$$

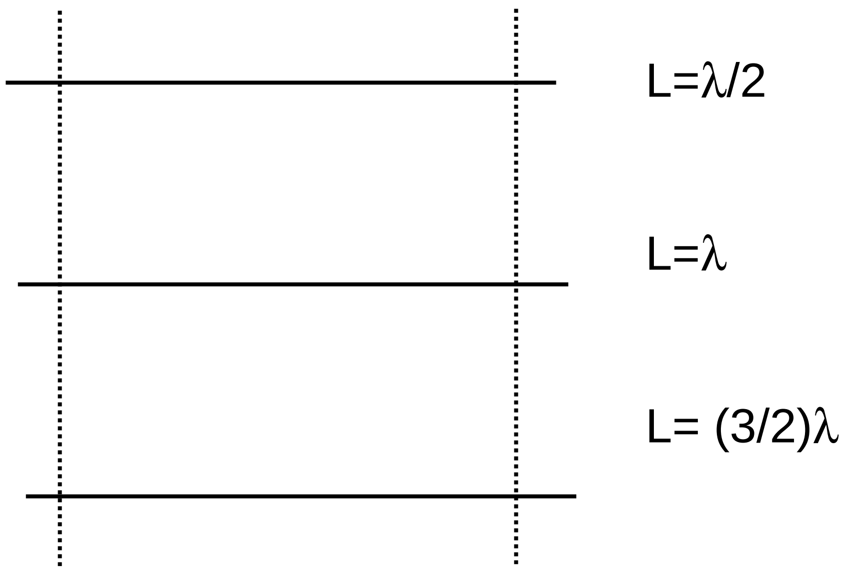
- We must fit an *odd* number of ¼ wavelengths.



- Notice that for the case of 1 free end the fundamental frequency is ½ the fundamental frequency if <u>both</u> ends were fixed.

Standing Waves

- (B) We could also include the case where both ends are free.



$$L = n\,(\lambda/2) \qquad n=1,2,3,\ldots$$

- This has the same conditions as when both ends are fixed in place.

- Please Read Sections 1.6 and 1.7 you will be responsible for them.