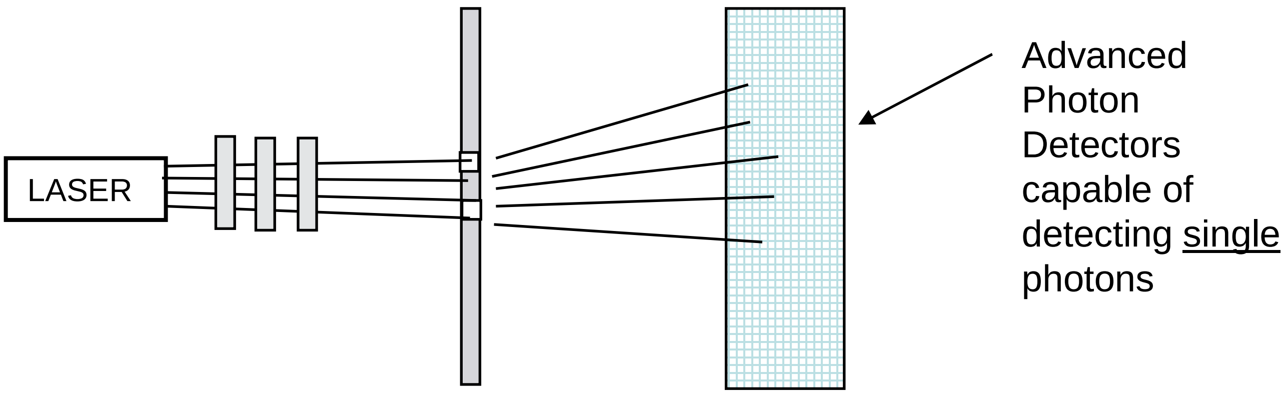
Chapter 5

Special Note: Professor Heinz will be lecturing on this chapter. He may not necessarily follow these notes. I provide them for completeness.

- We have seen that light can behave like a particle and we have seen particles can act like waves. The area of physics that studies these behavior is called <u>Quantum Mechanics.</u>
- We must adjust our thinking from classical particles and waves to one where our intuition will fail us. We can often get caught up in "That can't be!" because we are use to thinking in a "classical" sense.
- Moore likes to use the word "quanton", I find it a little cumbersome and I will use "quanta" (plural) or just "particle" but understand we mean something that can exhibit both classical "particle" and classical wave like behavior.
- In order to investigate some of the strange features of quantum mechanics let's create a simple "experiment" like we did with the photoelectric effect and see what observations we make.

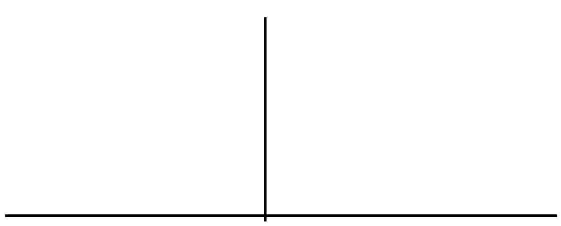
Experiment

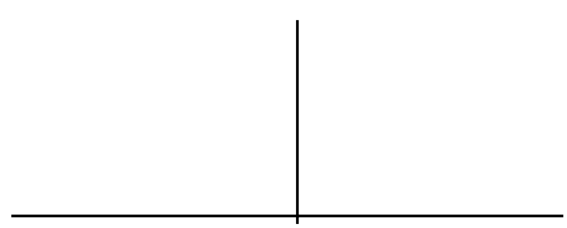
- Single-Quanton Interference from 2 Slits



- Filters setup so there is only 1/1000 chance there is a photon between the last filter and detector. So only 1 photon is moving through the experiment at once.
  - **γ can't influence each other**

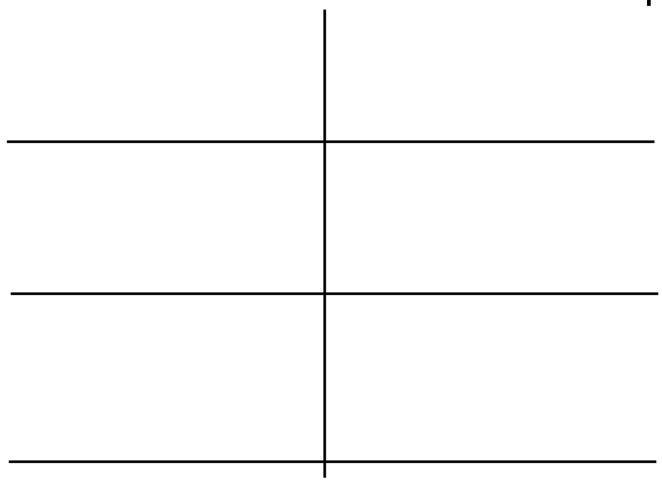
- Wave Model — Particle Model





Interference Experiment

- Now the actual experiment
  - Register discrete interactions of individual photons



  - Many *individual* photons seem to form interference pattern. (Prob. of location governed by wave-like prop.)
  - Interaction at detector is “particle-like”
  - Trajectories of individual photons are random
  - Interference pattern becomes recognizable after many photons have gone through the experiment.
  - Probability of location is governed by wave-like properties.
    - This is a statistical statement
  - Because 999/1000 photons go through the experiment alone we cannot claim the pattern is due to interaction between photons.

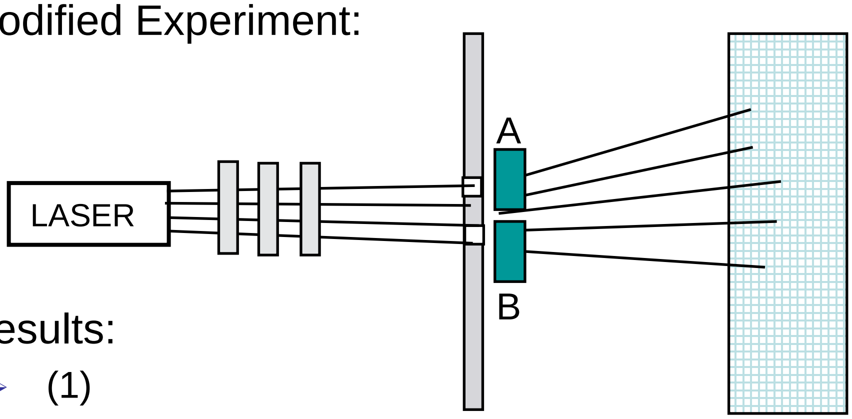
Interference Experiment

- If we change the slit separation the interference pattern ____________.
- If we remove/cover one of the slits the interference pattern _________.
- What do we say about this?
  -
- The classical particle model cannot handle this!
  -
- A classical wave model can explain the "knowing" about the two slits but it cannot explain that we see discrete "hit points" for each photon on the detector.

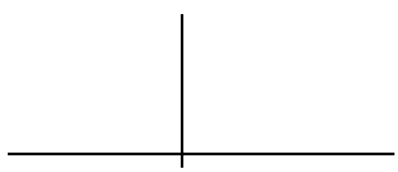
Again we are faced with the fact that the particle and wave model cannot both explain what we observe in experiment. In addition, as we look a the experiment in detail, we begin to see some of the strange features of quantum mechanics. Specifically, individual photon seem to know about both slits but hit the detector in one specific spot.

Trajectories

- As we struggle with the particle “knowing” about the two slits, a natural question to ask would be whether we can tell *which* slit a photon pass through.
- Modified Experiment:



- Results:
  - (1)
    - .
  - (2)



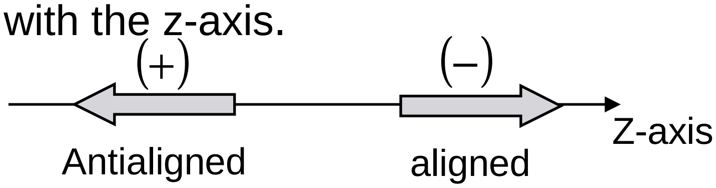
- In QM the <u>act</u> of making a measurement can change the out-come of the experiment. This is very different than Newtonian perception that we are outside observers……<u>AMAZING!</u>

Spin

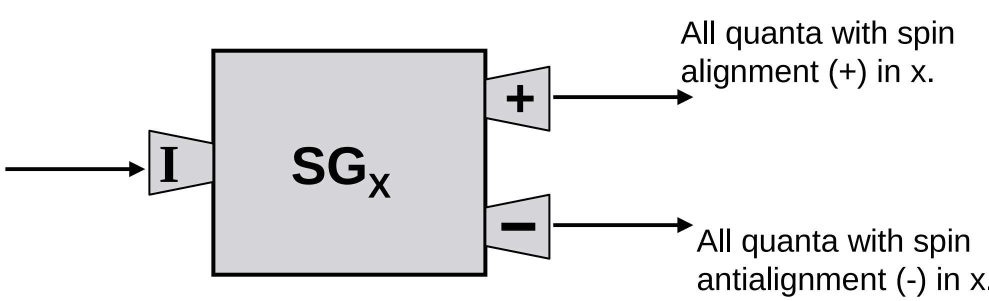
- Another area that we can learn about some of the strange effects of how observations can impact the outcome of an experiment has to do with "spin".

> **Electrons (and other quanta) have a vector property called spin. We will see later it acts like an angular momentum. However, for now think of it as an intrinsic quantity like electric charge or mass, except it is a *vector* quantity.**

- Experimentally we can try to determine the component of the spin (vector) along a particular direction, say the z-axis.
  - Results: We find that the spin is either totally aligned or antialigned with the z-axis.

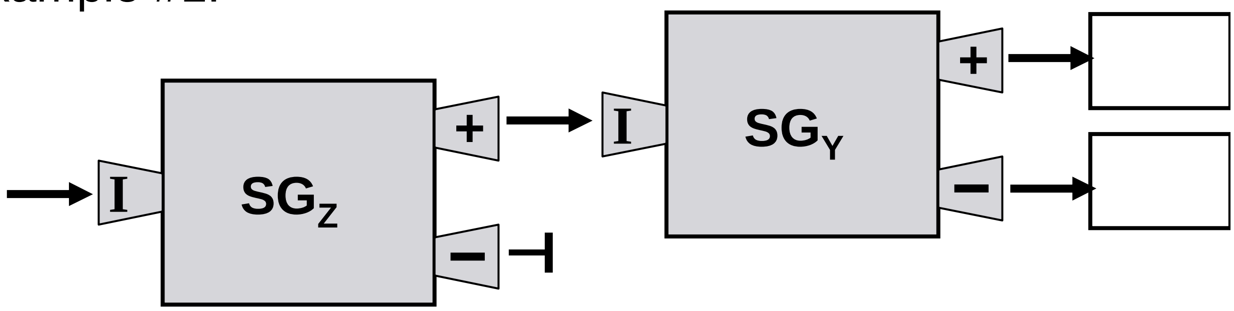


- "Stern-Gerlach: Device (For now "black-box")

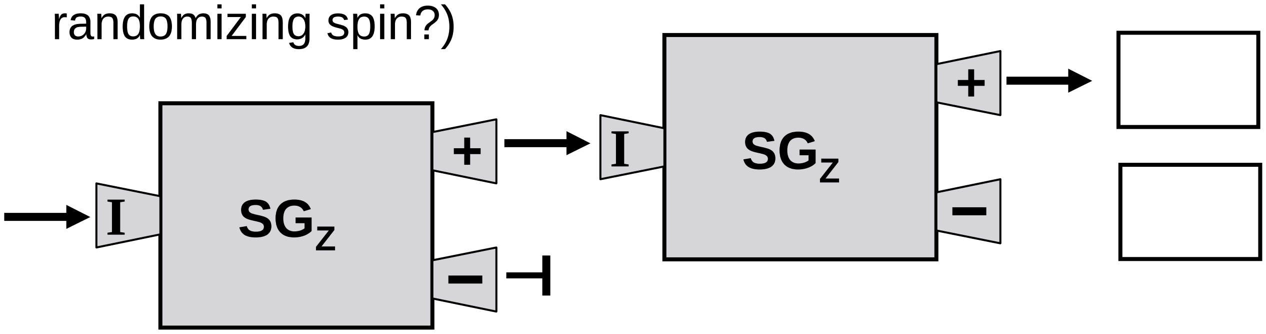


Fun with SG Devices

- We can obtain some insight (and confusion?) about QM by stringing SG devices together in different ways.
- Example #1:



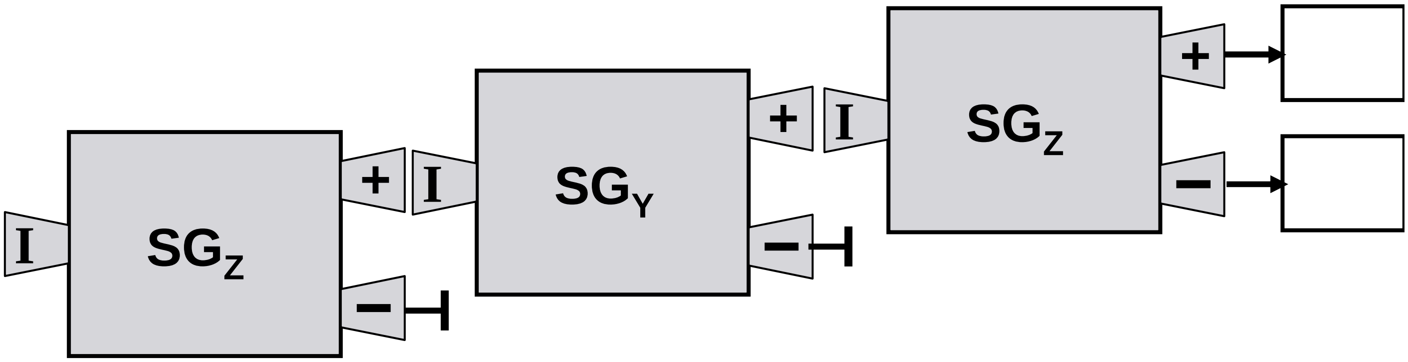
- Results:
  - (1)
  - (2)
  - (3)
- Example #2 (Reality check…are the SG devices just randomizing spin?)



Nope…they work

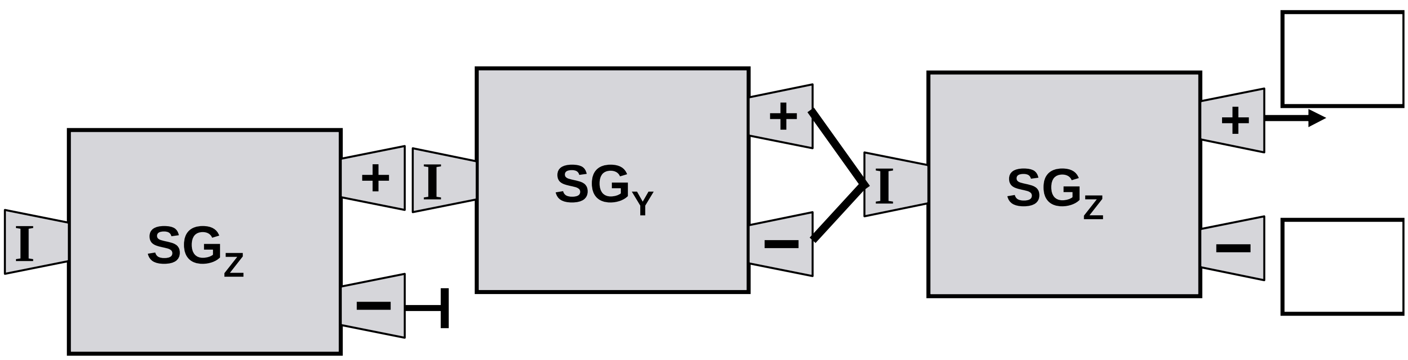
SG Examples

- Example #3



  - (1)
  - (2)
  - (3)

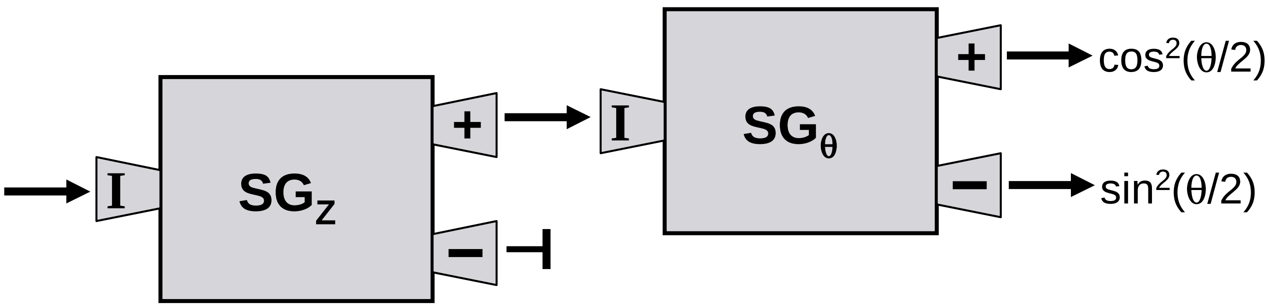
- Example #4 (Mind Blower!)



  - (1)

More SG Experiments

- Experiment #5 (Think about a SG that is aligned for an angle θ.



- Results:
  - (1) This is a generalization.
  - (2) It works for the cases we already did
    - $\theta = 90^o$ ($SG_{90} = SG_Y$)
      - (+) $\cos^2(90/2)=(1/2^{1/2})^2 = ½$ (50%)
      - (–) $\sin^2(90/2)= (1/2^{1/2})^2 = ½$ (50%)
    - $\theta=0^o$ ($SG_o = SG_z$)
      - (+) $\cos^2(0/2)=(1)^2 = 1$ (100%)
      - (–) $\sin^2(0/2) =(0)^2 = 0$ (0%)
- We will determine a mathematical model that explains these observations.

Complex Numbers

- QM uses complex numbers so let's review some of the properties in preparation for chapter 6.
- Complex Number (c)

$$c = a + ib$$
$$i \equiv \sqrt{-1}$$
$$i^2 \equiv -1 \Rightarrow (ib)^2 = -b^2$$

a "real part" ib "imaginary part" a, b are real numbers

- Addition:

$$c_1 + c_2 \equiv (a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + i(b_1 + b_2)$$

- Multiplication:

$$c_1 c_2 \equiv (a_1 + ib_1)(a_2 + ib_2)$$
$$= a_1 a_2 - b_1 b_2 + i(a_2 b_1 + a_1 b_2)$$

- Complex Conjugate

$$c = (a_1 + ib_1)$$
$$c^* = a_1 - ib_1$$

- Absolute Square $|c|^2$

$$|c|^2 \equiv cc^* = (a + ib)(a - ib)$$
$$= a^2 + b^2 \quad (\text{Always Real and} > 0)$$

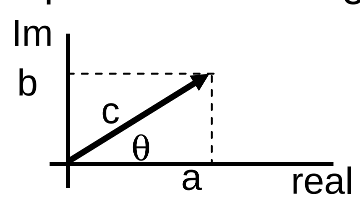
Complex Numbers

- Properties:

$$\left(c^*\right)^* = c$$
$$(c_1 + c_2)^* = c_1^* + c_2^*$$
$$(c_1 c_2)^* = c_1^* c_2^*$$
$$c^* = \left\{ \begin{matrix} c & \text{if c is purely real} \\ -c & \text{if c is purely imaginary} \end{matrix} \right\}$$

- More Notation:
  - Complex numbers can be expressed with angles

$$c = |c|(\cos\theta + i\sin\theta)$$



  - Even more useful is the following “simplification”

$$e^{i\theta} \equiv \cos\theta + i\sin\theta \qquad c = |c|e^{i\theta}$$

- Properties

$$e^{i0} = 1$$
$$e^{i\theta_1}e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$$
$$\frac{d}{d\theta}\left(e^{i\theta}\right) = ie^{i\theta}$$

Just as we would expect for exponentials.

Complex Numbers

- More Properties (θ is real)

$$\left|e^{i\theta}\right|^2 = 1$$

$$e^{-i\theta} = e^{i(-\theta)} = \cos\theta - i\sin\theta = \left(e^{i\theta}\right)^*$$

$$e^{i\theta} + e^{-i\theta} = 2\cos\theta$$

$$e^{i\theta} - e^{-i\theta} = 2\sin\theta$$

These don't have analogies with normal (real) exp.

- Vectors in QM
  - QM uses vectors of complex numbers to represent the state of the quanta

$$|\psi\rangle = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{bmatrix}$$

Ψ_i are complex numbers

  - Complex Conjugate

$$\langle\psi| = \begin{bmatrix} \psi_1^* \\ \psi_2^* \\ \vdots \\ \psi_n^* \end{bmatrix} \qquad \langle\psi| = \left[\psi_1^* \psi_2^* \ldots \psi_n^*\right]$$

  - Definition of Inner Product

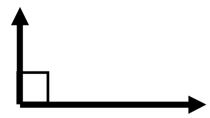
$$|u\rangle = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \qquad |w\rangle = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

Generalized Vector Dot Product: Complex Scalar

$$\langle u | w\rangle \equiv u_1^* w_1 + u_2^* w_2 + \ldots + u_n^* w_n$$

Complex Numbers

- Results of Inner Product
  - (1) If <u|w> = 0 then |u> and |w> are said to be "orthogonal".



$$\vec{A}\cdot\vec{B}=0$$

  - (2) If <u|u>=1 then |u> is said to be "normalized"

$$\vec{A}\cdot\vec{A}=1 \qquad \text{"Unit Vector"}$$

- All this notation will be helpful in the next chapter.