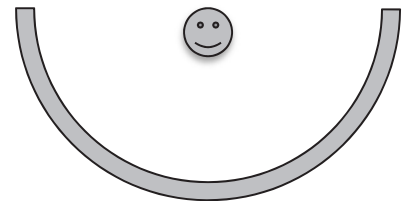


Integrals

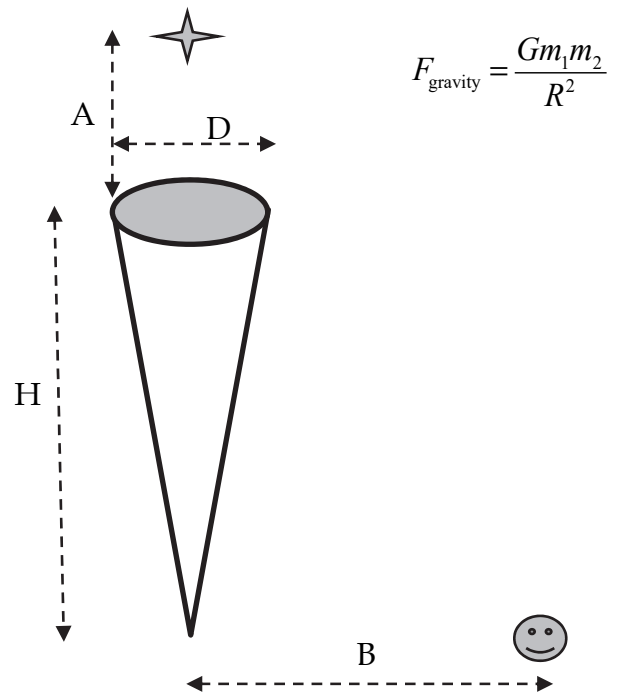
(1) Planet Imogene [Herdman] consists of a thin cylinder of length L and mass M . A moon is oriented along the axis of planet Imogene as shown, with mass m and a distance Z away from the near end of the planet. What is the gravitational acceleration that planet Imogene exerts on the moon?



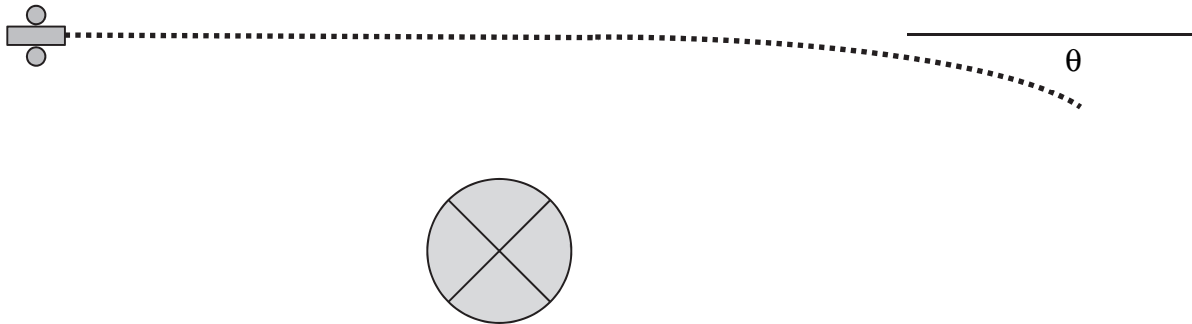
(2) Planet Uriah [the Hittite] consists of a thin semicircular line of radius R and linear mass density (kg/m) of δ . A moon is oriented at the center of the semicircle. What is the gravitational acceleration that planet Uriah exerts on the moon?



(3) Planet Vanilla [ice cream] is a solid cone of uniform mass density ρ (kg/m³). What is the gravitational acceleration that planet Vanilla exerts on the star? What is the x -component of the gravitational force that planet Vanilla exerts on the moon? Assume that D is much smaller than any of A , B , or H .



(4) A space probe passes by spherical planet Xerxes [king of Persia] as shown, passing a closest distance of B from the planet. As it does so, gravity from planet Xerxes (mass M) slightly deflects the path of the space probe. Determine the direction (angle) of the velocity of the space probe after it has finished passing the planet (a long distance past the planet). Ignore the horizontal (in the picture) force of gravity, in effect assuming that gravity does not change the horizontal velocity v_x but only the vertical velocity of the probe as the probe travels by the planet in a nearly straight line.



$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

$$\int \frac{x dx}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2}$$

$$\int \frac{x dx}{(x^2 + a^2)^{3/2}} = \frac{-1}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x^2 dx}{ax + b} = \frac{(ax + b)^2}{2a^3} - \frac{2b(ax + b)}{a^3} + \frac{b^2}{a^3} \ln(ax + b)$$

$$\int \frac{x^2 dx}{(ax + b)^2} = \frac{ax + b}{a^3} - \frac{b^2}{a^3(ax + b)} - \frac{2b}{a^3} \ln(ax + b)$$

$$\int \frac{x^2 dx}{\sqrt{x^2 + a^2}} = \frac{x\sqrt{x^2 + a^2}}{2} - \frac{a^2}{2} \ln(x + \sqrt{x^2 + a^2})$$

$$\int \frac{x^3 dx}{(x^2 + a^2)^{3/2}} = \sqrt{x^2 + a^2} + \frac{a^2}{\sqrt{x^2 + a^2}}$$

$$\int \frac{dx}{\sqrt{(x+b)^2 + a^2}} = \ln((x+b) + \sqrt{(x+b)^2 + a^2})$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 \sqrt{x^2 + a^2}}$$

$$\int \frac{x dx}{(x^2 + a^2)^{3/2}} = \frac{-1}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x dx}{ax + b} = \frac{x}{a} - \frac{b}{a^2} + \ln(ax + b)$$

$$\int \frac{x dx}{(ax + b)^2} = \frac{b}{a^2(ax + b)} + \frac{1}{a^2} \ln(ax + b)$$

$$\int \frac{x^2 dx}{(x^2 + a^2)^{3/2}} = \frac{-x}{\sqrt{x^2 + a^2}} + \ln(x + \sqrt{x^2 + a^2})$$