

Experiment III – Electric Flux

When a charge distribution is symmetrical, we can use Gauss' Law, a special law for electric fields. The Gauss' Law method of determining the electric field depends on the idea of *electric flux*. We can think of electric flux as the number of electric field lines passing through some surface area. For Gauss' Law we will use a closed surface like a sphere or a box or a cylinder. The concept of flux applies not only to electric fields but also to other physical situations.

Activity 1 At your lab bench there are two boards with nails passing through them. Suppose that each nail represents an electric field line. The density (closeness) of the nails indicates the strength of the electric field – the stronger the field, the more densely packed the nails. The electric flux through the loop is proportional to the number of nails passing through the loop.

Use the metal loop and the nail boards to determine what affects the magnitude of the electric flux through the loop. Record your observations below. Be sure to use both nail boards, to consider all orientations of the loop, and to consider all shapes of the loop (it is flexible).

Electric flux is defined as $\Phi = \vec{E} \cdot \vec{A}$, where \vec{A} is a vector that is perpendicular to the surface (e.g. loop) and has a magnitude that is equal to the surface area. In the case of a closed surface (e.g., a sphere), small elements of area, $d\vec{A}$, point outward. Are your observations consistent with this definition? If the electric field is not the same at all points on the surface of the loop it is necessary to revise this definition, and integrate over the surface area inside the loop. The general equation for electric flux is $\Phi = \int \vec{E} \cdot d\vec{A}$.

With the definitions above we must be careful when determining the sign of a dot product that involves the direction of the electric field at a point with respect to an element $d\vec{A}$ of the closed surface. Since $d\vec{A}$ points outward, an electric field pointing outward would contribute a positive flux. Similarly, an electric field pointing inward would contribute a negative flux.

Activity 2

In this section you will draw the electric field lines of several simple charge configurations. While you will be able to draw only a 2-dimensional picture, keep in mind that these problems are inherently three dimensional. For each case:

1. Draw the electric field lines.
2. Draw the Gaussian surface for the flux calculation.
3. State whether the flux is positive, negative, or zero.

#1: A charge $+q$ when the surface is a sphere centered on the charge:

#2: A charge $-q$ when the surface is a sphere centered on the charge:

#3: A charge $+2q$ when a surface is a sphere centered on the charge.

How does this flux compare to the flux in Case #1? (Explain)

#4: A $+q$ charge when the surface is a sphere offset from the charge, but the charge is still inside the sphere.

How does this flux compare to the flux in Case #1? (Explain)

#5: A charge $+q$ when the surface is a sphere offset from the charge so that the charge is outside the sphere.

#6: A charge $+q$ when an irregular surface surrounds the charge.

How does this flux compare to the flux in Case #1? (Explain)

#7: An electric dipole made with charges $+q$ and $-q$ and a surface that encloses the $+q$ but not the $-q$ charge.

How does this flux compare to the flux in Case #1? (Explain)

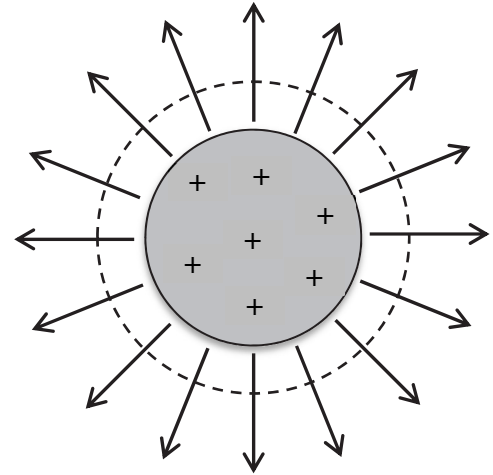
Based on your conclusions in the seven examples that you have just studied, what is a plausible (but not proven) relationship between the flux and the charged enclosed by the surface?

Activity 3 Electric Field Outside a Charged Sphere: A 10mm-radius solid sphere has $+10 \times 10^{-10}$ C of charge distributed uniformly throughout the sphere. We will go through the explicit steps necessary to use Gauss' Law to determine the electric field caused by this charge at a 15mm distance from the center of the sphere.

The electric field can be determined from the electric flux by use of the following equation:

$$\Phi = \int \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

(Gauss' Law)



1. First, we must see if this equation can be simplified:
 - a. What is the direction of $d\vec{A}$, a small part of the Gaussian sphere's surface area?
 - b. Explain why the dot product $\vec{E} \cdot d\vec{A}$ simplifies to $E dA$ for every element on the surface of the sphere shown in the Figure above.

- c. Is it possible to simplify the integral by pulling the electric field E outside, as shown in the equation below? Explain:

$$\int \vec{E} \cdot d\vec{A} = \int E \, dA = E \int dA$$

- d. What is the integral of dA ?

By means of these two steps we have simplified the expression for flux as follows:

$$\text{flux} = \Phi = \int \vec{E} \cdot d\vec{A} = \int E \, dA = E \int dA = E(\text{Area})$$

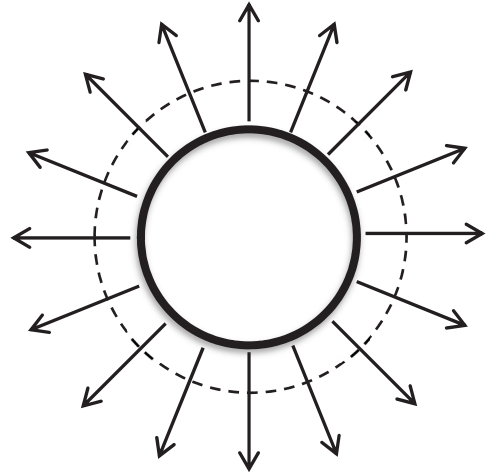
What two conditions must hold in general in order to be able to simplify the expression for flux in this way?

What is the area of our chosen Gaussian surface?

2. Next we will work on the right side of the equation: determine the amount of charge enclosed in the Gaussian surface Q_{enc} .
3. Insert the electric flux and Q_{enc} into Gauss' Law and find the electric field

Activity 4 **Electric Field outside a charged spherical shell:** A $+10 \times 10^{-10} \text{ C}$ charge is distributed uniformly on a 10 mm radius spherical shell, similar to a basketball. Use Gauss' Law to determine the electric field produced by this charge at a distance of 15 mm from the center of the sphere.

$$\Phi = \int \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$



Activity 5 **Electric Field inside the charged spherical shell:** For the charged spherical shell in Activity 4, use Gauss' Law to predict the value of the electric field *inside* the shell, specifically at $R = 5 \text{ mm}$ from the center of the 10 mm radius spherical shell.

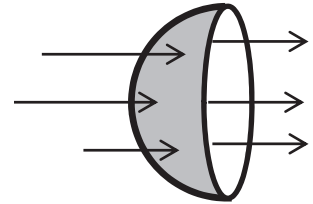
Activity 6 **Electric Field outside an asymmetric distribution of charge:** When a charge distribution has an irregular shape (e.g. a shape like that of your chair or a stapler), is Gauss' Law still mathematically valid? In this case, can we use Gauss' Law as an effective tool to determine the electric field outside the object? If not, what part of the procedure that we followed above for a symmetric charge distribution breaks down?

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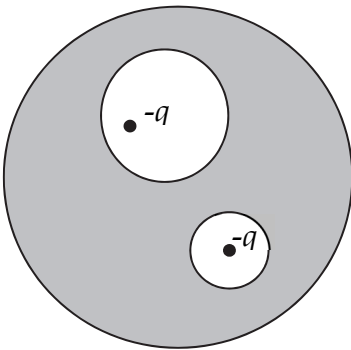
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Gauss's Law

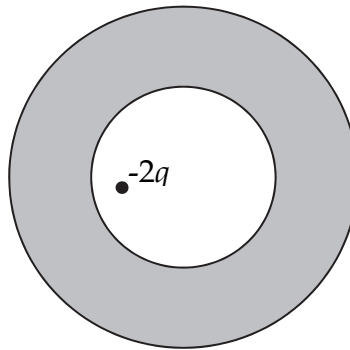
- (1) In the figure, a hollow hemisphere is in a uniform electric field of magnitude $E = 3.0 \text{ kN/C}$. The circular rim has a circle of radius $R = 12 \text{ cm}$ and is aligned perpendicular to the field. Find the electric flux through the curved hemispherical surface (not the complete closed surface). You can either do a double integral over the curved surface (ugh) or find a clever way to find the flux through that side using Gauss's Law.



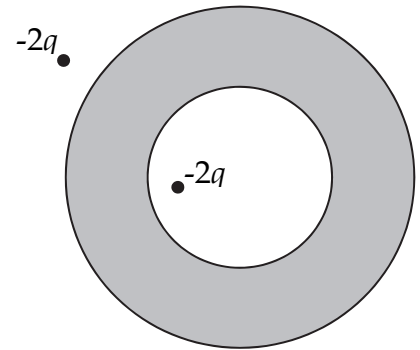
- (2) In each case, a conductor with cavities is shown *in cross-section* along with some point charges in the hollow spaces inside. The net charge on the conductor itself, q_{cond} , is given below each figure. Determine the amount of charge on all surfaces, inside and out, and sketch the charge distribution on all surfaces.



$$q_{\text{cond}} = -q$$



$$q_{\text{cond}} = +5q$$



$$q_{\text{cond}} = -4q$$

- (3) Find the electric field in all space due to a uniform, solid sphere of charge of radius R and volume charge density $+\rho$. Give your answer for the magnitude in the form of an equation " $E(r) = \dots$ " where r is the distance from the center of the sphere. The easiest way to specify the direction is in words. Note that the charge divides space into regions. *You will need to supply a separate answer for each region.*

(4) Use Gauss's Law to find the electric field in all space due to a thick, infinitely-long cylindrical shell of charge with volume charge density $-\rho$. The shell has inner and outer radii, a and b .

(5) Use Gauss's Law to find the electric field in all space due to a thick, infinitely-wide slab of charge of thickness T and volume charge density $-\rho$.