Electricity & Magnetism – Problem Set # 2

Rest mass of the electron $m_e = 9.11 \times 10^{-31}$ kilogram $= 9.11 \times 10^{-28}$ gram

Magnitude of the electron charge $e = 1.60 \times 10^{-19}$ coulomb $= 4.80 \times 10^{-10}$ statcoulomb (esu)

Avogadro’s number $N_0 = 6.02 \times 10^{23}$ per mole

Universal gas constant $R = 8.32$ joules/(mole · K)

Boltzmann’s constant $k = 1.38 \times 10^{-23}$ joule/K $= 1.38 \times 10^{-16}$ erg/K

Speed of light $c = 3.00 \times 10^8$ m/s $= 3.00 \times 10^{10}$ cm/s

Planck’s constant $h = 6.63 \times 10^{-34}$ joule · second $= 4.14 \times 10^{-15}$ eV · second
$\hbar = h/2\pi$

Vacuum permittivity $\varepsilon_0 = 8.85 \times 10^{-12}$ coulomb$^2$/newton · meter$^2$

Vacuum permeability $\mu_0 = 4\pi \times 10^{-7}$ weber/(ampere · meter)

Universal gravitational constant $G = 6.67 \times 10^{-11}$ meter$^3$/kilogram · second$^2$

Acceleration due to gravity $g = 9.80$ m/s$^2 = 980$ cm/s$^2$

1 atmosphere pressure $1$ atm $= 1.0 \times 10^5$ newton/meter$^2 = 1.0 \times 10^5$ pascals (Pa)

1 angstrom $1$ Å $= 1 \times 10^{-10}$ meter

1 weber/m$^2 = 1$ tesla $= 10^4$ gauss
55. What is the magnitude of the net force on the loop when the induced current is \( i \)?

(A) \( \frac{\mu_0 IL}{2\pi} \ln\left(\frac{r + a}{r}\right) \)

(B) \( \frac{\mu_0 IL}{2\pi} \ln\left(\frac{r}{r + a}\right) \)

(C) \( \frac{\mu_0 IL}{2\pi} \frac{b}{a} \)

(D) \( \frac{\mu_0 IL}{2\pi} \frac{ab}{r(r + a)} \)

(E) \( \frac{\mu_0 IL}{2\pi} \frac{r(r + a)}{ab} \)
57. A uniform and constant magnetic field $\mathbf{B}$ is directed perpendicularly into the plane of the page everywhere within a rectangular region as shown above. A wire circuit in the shape of a semicircle is uniformly rotated counterclockwise in the plane of the page about an axis $A$. The axis $A$ is perpendicular to the page at the edge of the field and directed through the center of the straight-line portion of the circuit. Which of the following graphs best approximates the emf $\mathbf{E}$ induced in the circuit as a function of time $t$?

(A)  

(B)  

(C)  

(D)  

(E)  

\[ \text{Graphs (A), (B), (C), (D), (E)} \]
64. If an electric field is given in a certain region by $E_x = 0, E_y = 0, E_z = kz$, where $k$ is a nonzero constant, which of the following is true?

(A) There is a time-varying magnetic field.
(B) There is charge density in the region.
(C) The electric field cannot be constant in time.
(D) The electric field is impossible under any circumstances.
(E) None of the above

65. Two point charges with the same charge $+Q$ are fixed along the $x$-axis and are a distance $2R$ apart as shown. A small particle with mass $m$ and charge $-q$ is placed at the midpoint between them. What is the angular frequency $\omega$ of small oscillations of this particle along the $y$-direction?

(A) $\frac{Qq}{2\pi \varepsilon_0 m R^2}$
(B) $\frac{Qq}{4\pi \varepsilon_0 m R^3}$
(C) $\frac{Qq}{2\pi \varepsilon_0 m R}$
(D) $\left(\frac{Qq}{4\pi \varepsilon_0 m R^2}\right)^{\frac{1}{2}}$
(E) $\left(\frac{Qq}{2\pi \varepsilon_0 m R^3}\right)^{\frac{1}{2}}$
79. The dispersion curve shown above relates the angular frequency $\omega$ to the wave number $k$. For waves with wave numbers lying in the range $k_1 < k < k_2$, which of the following is true of the phase velocity and the group velocity?

(A) They are in opposite directions.
(B) They are in the same direction and the phase velocity is larger.
(C) They are in the same direction and the group velocity is larger.
(D) The phase velocity is infinite and the group velocity is finite.
(E) They are the same in direction and magnitude.

81. In the $RLC$ circuit shown above, the applied voltage is

$$E(t) = E_m \cos \omega t.$$ 

For a constant $E_m$, at what angular frequency $\omega$ does the current have its maximum steady-state amplitude after the transients have died out?

(A) $\frac{1}{RC}$

(B) $\frac{2L}{R}$

(C) $\frac{1}{\sqrt{LC}}$

(D) $\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

(E) $\sqrt{\left(\frac{1}{RC}\right)^2 - \left(\frac{L}{R}\right)^2}$
83. Two pith balls of equal mass $M$ and equal charge $q$ are suspended from the same point on long massless threads of length $L$ as shown in the figure above. If $k$ is the Coulomb's law constant, then for small values of $\theta$, the distance $d$ between the charged pith balls at equilibrium is

(A) $\left(\frac{2kq^2L}{Mg}\right)\frac{1}{3}$

(B) $\left(\frac{kq^2L}{Mg}\right)\frac{1}{3}$

(C) $\left(\frac{2kq^2L}{Mg}\right)\frac{1}{3}$

(D) $\left(\frac{kq^2L}{Mg}\right)\frac{1}{2}$

(E) $\frac{L}{4}$

84. An electron oscillates back and forth along the + and $-x$-axes, consequently emitting electromagnetic radiation. Which of the following statements concerning the radiation is NOT true?

(A) The total rate of radiation of energy into all directions is proportional to the square of the electron’s acceleration.

(B) The total rate of radiation of energy into all directions is proportional to the square of the electron’s charge.

(C) Far from the electron, the rate at which radiated energy crosses a perpendicular unit area decreases as the inverse square of the distance from the electron.

(D) Far from the electron, the rate at which radiated energy crosses a perpendicular unit area is a maximum when the unit area is located on the + or $-x$-axes.

(E) Far from the electron, the radiated energy is carried equally by the transverse electric and the transverse magnetic fields.
86. The circuit shown above is used to measure the size of the capacitance $C$. The $y$-coordinate of the spot on the oscilloscope screen is proportional to the potential difference across $R$, and the $x$-coordinate of the spot is swept at a constant speed $s$. The switch is closed and then opened. One can then calculate $C$ from the shape and the size of the curve on the screen plus a knowledge of which of the following?

(A) $V_0$ and $R$
(B) $s$ and $R$
(C) $s$ and $V_0$
(D) $R$ and $R'$
(E) The sensitivity of the oscilloscope

88. A parallel-plate capacitor is connected to a battery. $V_0$ is the potential difference between the plates, $Q_0$ the charge on the positive plate, $E_0$ the magnitude of the electric field, and $D_0$ the magnitude of the displacement vector. The original vacuum between the plates is filled with a dielectric and then the battery is disconnected. If the corresponding electrical parameters for the final state of the capacitor are denoted by a subscript $f$, which of the following is true?

(A) $V_f > V_0$
(B) $V_f < V_0$
(C) $Q_f = Q_0$
(D) $E_f > E_0$
(E) $D_f > D_0$
92. A flat coil of wire is rotated at a frequency of 10 hertz in the magnetic field produced by three pairs of magnets as shown above. The axis of rotation of the coil lies in the plane of the coil and is perpendicular to the field lines. What is the frequency of the alternating voltage in the coil?

(A) \( \frac{10}{6} \) Hz

(B) \( \frac{10}{3} \) Hz

(C) 10 Hz

(D) 30 Hz

(E) 60 Hz