MECHANICS – Problem Set #1

Questions 4-5

The magnitude of the Earth's gravitational force on a point mass is \( F(r) \), where \( r \) is the distance from the Earth's center to the point mass. Assume the Earth is a homogeneous sphere of radius \( R \).

4. What is \( \frac{F(R)}{F(2R)} \)?
   (A) 32
   (B) 8
   (C) 4
   (D) 2
   (E) 1

5. Suppose there is a very small shaft in the Earth such that the point mass can be placed at a radius of \( R/2 \). What is \( \frac{F(R)}{F\left(\frac{R}{2}\right)} \)?
   (A) 8
   (B) 4
   (C) 2
   (D) \( \frac{1}{2} \)
   (E) \( \frac{1}{4} \)
6. Two wedges, each of mass \( m \), are placed next to each other on a flat floor. A cube of mass \( M \) is balanced on the wedges as shown above. Assume no friction between the cube and the wedges, but a coefficient of static friction \( \mu < 1 \) between the wedges and the floor. What is the largest \( M \) that can be balanced as shown without motion of the wedges?

(A) \( \frac{m}{\sqrt{2}} \)

(B) \( \frac{\mu m}{\sqrt{2}} \)

(C) \( \frac{\mu m}{1 - \mu} \)

(D) \( \frac{2\mu m}{1 - \mu} \)

(E) All \( M \) will balance.

7. A cylindrical tube of mass \( M \) can slide on a horizontal wire. Two identical pendulums, each of mass \( m \) and length \( L \), hang from the ends of the tube, as shown above. For small oscillations of the pendulums in the plane of the paper, the eigenfrequencies of the normal modes of oscillation of this system are 0, \( \sqrt{\frac{g(M + 2m)}{kM}} \), and

(A) \( \sqrt{\frac{g}{k}} \)

(B) \( \sqrt{\frac{g}{k} \frac{M + m}{M}} \)

(C) \( \sqrt{\frac{g}{k} \frac{m}{M}} \)

(D) \( \sqrt{\frac{g}{k} \frac{m}{M + m}} \)

(E) \( \sqrt{\frac{g}{k} \frac{m}{M + 2m}} \)
8. A solid cone hangs from a frictionless pivot at origin $O$, as shown above. If $\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors, and $a$, $b$, and $c$ are positive constants, which of the following forces $\mathbf{F}$ applied to the cone at a point $P$ results in a torque $\tau$ or cone with a negative component $\tau_z$?

(A) $\mathbf{F} = a\hat{k}$, $P$ is $(0, b, -c)$
(B) $\mathbf{F} = -a\hat{k}$, $P$ is $(0, -b, -c)$
(C) $\mathbf{F} = a\hat{j}$, $P$ is $(-b, 0, -c)$
(D) $\mathbf{F} = a\hat{j}$, $P$ is $(b, 0, -c)$
(E) $\mathbf{F} = -a\hat{k}$, $P$ is $(-b, 0, -c)$

19. Which of the following is most nearly the mass of the Earth? (The radius of the Earth is about $6.4 \times 10^6$ meters.)

(A) $6 \times 10^{24}$ kg
(B) $6 \times 10^{27}$ kg
(C) $6 \times 10^{30}$ kg
(D) $6 \times 10^{33}$ kg
(E) $6 \times 10^{36}$ kg
Questions 41-42

A cylinder with moment of inertia 4 kg \cdot m^2 about a fixed axis initially rotates at 80 radians per second about this axis. A constant torque is applied to slow it down to 40 radians per second.

41. The kinetic energy lost by the cylinder is
   (A) 80 J
   (B) 800 J
   (C) 4000 J
   (D) 9600 J
   (E) 19,200 J

42. If the cylinder takes 10 seconds to reach 40 radians per second, the magnitude of the applied torque is
   (A) 80 N \cdot m
   (B) 40 N \cdot m
   (C) 32 N \cdot m
   (D) 16 N \cdot m
   (E) 8 N \cdot m

43. If \( \frac{\partial L}{\partial q_n} = 0 \), where \( L \) is the Lagrangian for a conservative system without constraints and \( q_n \) is a generalized coordinate, then the generalized momentum \( p_n \) is
   (A) an ignorable coordinate
   (B) constant
   (C) undefined
   (D) equal to \( \frac{d}{dt} \left( \frac{\partial L}{\partial q_n} \right) \)
   (E) equal to the Hamiltonian for the system