BOLZANO-LEWIS POSSIBLE WORLDS SEMANTICS: AN IMPROVEMENT OVER ITS SUCCESSORS

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COMPONENTS OF THIS PRESENTATION

RECENT HISTORY OF MAINSTREAM LINGUISTIC SEMANTICS

- Montague’s (1974) style of possible worlds semantics (PWS) and its modeling of declarative utterance meaning.
- The ‘granularity problem’ – distinct utterances with the same truth conditions mean the same thing – and its impact on Montague’s PWS, and the field, generally.

EARLY MODERN HISTORY OF SEMANTICS

- 19th century antecedents of the modern mainstream approach with attention to the foundational metaphysics.
- An early 20th century precursor to Montague’s PWS, stemming from the work of Bernard Bolzano and C.I. Lewis, that does not suffer from the granularity problem.
Montague’s Contributions to Semantics

Proper Treatment

- Richard Montague was a mathematical logician, trained in logic and set theory under Alfred Tarski at UC, Berkeley in the 1950s.
- During the 1960s, Montague pioneered the systematic application of mathematical logic to the analysis of natural language meaning.
- In particular, Montague employed a certain kind of modal logic that made use of ‘possible worlds’ to model the meanings of natural language expressions.

Mainstream Linguistic Semantics

Montague’s (1974) style of PWS became, and remains, the mainstream framework for theorizing about natural language meaning in the linguistic semantics community.
The four main concepts within the mainstream approach – sense, reference, intension, extension – relate to one another as follows:

**Organizing the Concepts**

A. The meanings of natural language expressions are things called *senses*.

B. Senses of declarative utterances are called *propositions*.

C. A sense has an *extension*, and what that extension is in general depends on contingent facts (how things are).

D. The extension of an expression’s sense is called the expression’s *reference*. That is, extension and reference are identified.

E. The *intension* of an expression is simply its sense.
Montague’s PWS formulation borrowed two key ideas from contemporary philosophical logic and philosophy of language, one from Saul Kripke, and another from Rudolph Carnap.

**Possible Worlds**

From Kripke (1963) came the assumption of “an arbitrary set \( K \) of ‘possible worlds’,...and a function \( \Phi(P, H) \) assigning to each proposition \( [= \text{atomic formula}] P \) a truth-value in the world \( H \)” (pp. 69-70).

Montague took the arbitrary set of possible worlds to consist of unanalyzed primitives.
Carnapian intensions

From Carnap (1947) came the idea of a linguistic meaning as a Carnapian ‘intension’ – a function whose domain is the set of possible worlds.

In the case of a declarative utterance, its sense – the proposition expressed by the utterance – is nothing more or less than (the characteristic function of) a set of these possible worlds.

This is due to Carnap following Frege (1892) in assuming that the reference of a declarative utterance is the truth value (an element of the set \{true, false\}) of the proposition that it expresses.
It has long been recognized – at least as early as C.I. Lewis (1943) and Carnap (1947) – that treating utterance meanings as sets of worlds also has the troubling consequence that distinct utterances with the same truth conditions mean the same thing.

This is the best-known aspect of what is more generally known as the ‘granularity problem’ – that distinct linguistic expressions which have the same extensions at each world (and thus identical intensions) have the same meaning.
Montague seems to have simply ignored these known problems.

Mainstream responses

- **It’s a problem, but not our problem**: the position of mainstream semantics, which mostly sticks with intensional semantics and ignores its shortcomings.
- **It’s not a bug, it’s a feature**: the apparent bad consequences of intensional PWS are actually just what we want. Evidently only Stalnaker (1984) still takes this position.
RESPONSES TO THE PROBLEM

MEANINGS ARE ‘STRUCTURED’ OBJECTS

- Meanings are not intensions (or extensions), but ‘structured’ syntactic objects (such as nested tuples, trees, or LF s) with intensions (or extensions) at their ‘leaves’.

MORE RADICAL MEASURES

- abandoning worlds (Thomason, 1980),
- switching to partial worlds (Barwise and Perry, 1983),
- switching to untyped $\lambda$-calculus (Turner, 1985), or
- treating meanings as extremely fine-grained algorithms (Muskens, 2005).
RESPONSES TO THE PROBLEM

LOOKING BACK

Another kind of response involves looking carefully at the choice Montague made in piecing together his PWS approach, i.e., combining

(1) the (primitive) worlds of Kripke (1963) with

(2) Carnap’s (1947) identification of propositions with sets of possible worlds.

It turns out that this choice is neither empirically nor formally motivated. Moreover, both (1) and (2) are in contradistinction to earlier conceptualizations of possible worlds and propositions.
Toward Earlier Conceptualizations

Looking back

Indeed, long before Carnap, Kripke, and Montague, there were well worked out conceptions of

- propositions as things in their own right independent of utterances that might express them or conditions under which they might be true; and of

- possible worlds, not as unanalyzed primitives, but rather as certain sets of propositions (the maximal consistent ones).

In the remainder of this presentation we focus on the development of these earlier conceptualizations.
Metaphysical Foundations

During the German Enlightenment, Kant argued for a clear conceptual separation of certain aspects of philosophy from the emerging field of human psychology.

Kant’s distinction

Kant’s basic distinction between “things as they are in themselves” and “things as they are as objects of our knowledge” bolstered a metaphysical stance wherein

- *reason*, in the abstract sense, is a “thing in itself,” independent from human reasoning (see Smith, 1997, pp. 200-14), and
- *logic* is the “science of reason” (see Kant, 1800, p. 18).
Kant’s logic was put to use by philosophers charged with the task of organizing a *Wissenschaft*, or “universal science,” that integrated the variegated approaches to scientific inquiry emerging in 19th century Germany universities.

**Bolzano’s theory of science**

- Bernard Bolzano’s (1837) *Theory of Science* (*Wissenschaftslehre*) concerned the nature of such a theory, and its elements, propositions.

- Bolzano’s notion of “proposition in itself” (*Satz an sich*) embodied most of the key properties that present-day semanticists attribute to propositions.
Metaphysical Foundations

Bolzano’s Proposition in Itself (*Satz an sich*)

A. They are expressed by declarative utterances.
B. They are the primary bearers of truth and falsity; an utterance is only secondarily, or derivatively, true or false, depending on what proposition it expresses.
C. They are the objects of the attitudes, i.e. they are the things that are known, believed, doubted, etc.
D. They are not linguistic. They are not mental. They are not located in space or time.
E. Utterances in different languages, or different utterances in the same language, can express the same proposition.
Frege’s Contributions

- Frege generalized the notion of “extension of a word’s meaning” by postulating that the meanings of utterances also have extensions, i.e., truth values, sharpening Bolzano’s propositions’ “bearing” a truth value.
- Frege also made explicit the notion of meaning-level compositionality that was prefigured into Bolzano’s approach.

Where is C. S. Peirce?

Across the pond, C. S. Peirce (1880) showed that the collection of propositions (together with the operations ‘and’, ‘or’, and ‘not’) formed a mathematical structure called a Boolean algebra.
Tractarian Semantics

The Tractatus (1921)

Ludwig Wittgenstein, a student of Bertrand Russell’s at Cambridge, seems to have been the first to explicitly identify possible worlds with (maximal consistent) sets of “facts.”

A. A fact (Tatsache), the closest counterpart in the Tractatus to Bolzano’s propositions, consists of the existence (or nonexistence) of a state of affairs (Sachverhalt);

B. The possible worlds of the Tractatus are maximal consistent assemblages of positive and negative facts;

C. The term ‘proposition’ (Satz) is reserved for the linguistic entities that express (potential) facts, or equivalently, describe states of affairs.
**TRACTARIAN SEMANTICS**

**POTENTIAL WAY FORWARD**

Importantly, within the general Tractarian approach possible worlds are sets of propositions. This flips the relationship within the mainstream approach where propositions are sets of possible worlds.

**DRAWBACKS TO THE APPROACH**

- It is *atomistic*. That is, there is a requirement that there be a collection of ‘basic facts’, i.e. the ones expressed by elementary propositions (*Elementarsätze*).
- Logically equivalent facts are identical (a form of the granularity problem).
Around the same time, C.I. Lewis provided a more technically precise theory free of the defects found within Wittgenstein’s approach. In Lewis’ theory,

A. ‘facts’ correspond to Bolzano’s propositions, the ‘contradictory of a fact’ corresponds to the negation of a proposition, a ‘joint fact’ corresponds to a conjunctive proposition;

B. a ‘world’ is a set of facts called a ‘system,’ which is additionally maximal in the sense of containing each fact or its contradictory.

Importantly, nothing in Lewis’ theory forces facts with the same truth conditions to be equal, so the granularity problem does not arise.
Aside from the technical advantages...

- Despite C.I. Lewis’ prominence within the logic community, his approach to possible worlds and propositions was not widely recognized within it, and not at all within linguistics.

- Despite Wittgenstein’s immense influence within the logic and language communities, and his contributions to the philosophy of language, his approach to worlds and propositions seems to be unknown to the linguistics community.

- Descendants of the general Lewisian/Tractarian approach have been advocated by various philosophers, such as Adams (1974), Plantinga (1974) and Lycan (1979), but have also been generally ignored by linguists.
Generalizing Lewisian Semantics

Hyperintensional Semantics
Pollard’s (2008, 2011) “hyperintensional semantics” is the first attempt to rigorously formalize a theory that takes possible worlds to be sets of propositions, with wide-scale linguistic modeling in mind.

Agnostic Semantics
- Plummer and Pollard (2012) demonstrate that hyperintensional semantics is simply a re-invention of the old Lewisian semantics.
- Interestingly, it was shown that a core semantics can be “factored out” of both the mainstream and Lewisian/Tractarian semantics, yielding a practicable theory agnostic wrt the relationship between propositions and possible worlds.
The main concepts within the agnostic approach – sense, reference, intension, extension – relate to one another as follows:

**RE-ORGANIZING THE CONCEPTS**

A. The meanings of natural language expressions are things called *senses*.

B. Senses of declarative utterances are called *propositions*.

C. A sense has an *extension*, and what that extension is in general depends on contingent facts (how things are).

D. The extension of an expression’s sense is called the expression’s *reference*. That is, extension and reference are identified.

The *intension* of an expression is simply its *sense*.
We’ve covered...

- Montague’s (1974) style of PWS and its modeling of declarative utterance meaning;
- the ‘granularity problem’ – distinct utterances with the same truth conditions mean the same thing – and its impact on Montague’s PWS, and the field, generally;
- 19th century antecedents of the modern mainstream approach with attention to the foundational metaphysics;
- an early 20th century precursor to Montague’s PWS, stemming from the work of Bolzano and C.I. Lewis, that does not suffer from the granularity problem, in addition to having other advantages over the modern mainstream approach.
Improvement over its Successors

Tony Hoare (of Quicksort fame) made the following remarks on the programming language ALGOL 60.

ALGOL 60

“The more I ponder the principles of language design, and the techniques which put them into practice, the more is my amazement and admiration of ALGOL 60. Here is a language so far ahead of its time, that it was not only an improvement on its predecessors, but also on nearly all its successors. Of particular interest are its introduction of all the main program structuring concepts, the simplicity and clarity of its description, rarely equalled and never surpassed” (Hoare, 1973, p. 27).
References available at:
http://www.ling.ohio-state.edu/ plummer/presentations/andrew-
plummer-naahols2013-references.pdf

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