PROBLEM SET ONE: SETS

Background

In practical terms, sets can be considered the most basic mathematical entities, in the sense that the structures and systems used as idealized representations of observed phenomena (for us, linguistic ones) are defined in terms of them. We can begin to develop the kinds of skills used in careful linguistic argumention by proving assertions about sets, based on the assumptions we made in Chapter 1 about how set membership works. In this setting, to "prove" means to give a careful, persuasive, valid argument in English. You can use upper or lower case italic letters as metavariables (e.g. 'for any set A'); and you can introduce names (such as '$\emptyset$') for specific sets. But for the time being, in your arguments, please do **not** use logic-symbol abbreviations for 'and', 'or', 'implies', 'if ... then', 'iff', 'it is not the case that', 'for all x' 'there exists x such that', 'there exists unique x such that', etc. (This prohibition will be removed in Chapter 2, where we make clear exactly how these symbols are to be used.)

Some of the assertions you are asked to prove were already proved, albeit sketchily, in class, so if you took good notes, you may just be able to reconstruct the arguments given there. But it is not necessary to do so; you can just as well give an original argument, as long as it's valid.

For now, don't be too worried if you're not sure what kind of argumentation counts as 'valid': until we develop some of the logical tools for making this notion precise, you can take it to mean something like 'knockdown', 'irrefutable', or 'totally persuasive to any sane and reasonably intelligent person who knows English'.

For these problems, use only the *first five* of the assumptions in Chapter One (i.e. do not use the Assumption of Separation).

You can either email your completed assignment to Scott by midnight of Thur. Oct. 7, or turn in hard copy in class that day. Also, before writing up your work please review the paragraphs about written work and study groups on the Course Information page on the course website. And finally, start early, so that if you run into trouble you'll know what questions to ask in the Recitation this coming Friday!

Problem 1

Prove that for any sets a, b, and c, there is a set whose only members are a, b, and c. (Note: this way of wording the problem is *not* intended to imply that a, b, and c are necessarily distinct from each other.)

Problem 2

Prove the assertions (made without proof in class) that $\{0\}$ (aka 1) is the successor of $\emptyset$ (aka 0), and that $\{0, 1\}$ (aka 2) is the successor of 1.

Problem 3

Prove that 0, 1, and 2 are all distinct (i.e. that no two of them are equal). Caution: it won't work to try to argue that no number is equal to its successor, because there is no valid proof of that assertion from the assumptions we have made so far, and in fact there are set theories in which it is false!

Problem 4

What is the powerset of 4? Your answer should use the curly-rackets notation, with the names of the members separated by commas, in any order, but without any repetitions (that is, there should one fewer commas than there are members). (Note: you are not required to prove anything here.)

Problem 5

How many members does $\bigcup\langle 2, 3\rangle$ have? What are they? (Note: You will have to use the definition of ordered pair. You are not required to prove anything here.)

Problem 6

Prove that for any sets a, b, c, and d, if $\langle a, b\rangle = \langle c, d\rangle$, then $a = c$ and $b = d$. (Hint: notice that either $a = b$ or not, so you can split the proof into two cases.)