PROBLEM SET TWO: CARTESIAN (CO-)PRODUCT; MATHESE

Problem 1

Prove that for any set a, $a \times \emptyset = \emptyset$.

Problem 2

Review the definition of (cartesian) coproduct at the end of Chapter One (not covered in class). Then show that for any sets a and b, there exists a unique set which is the coproduct $a + b$. [Hint: the harder part is the existence.]

Problem 3

Translate the following Mathese sentences into Mathese symbols. Don't worry about whether the sentences are true or false, or provable or unprovable; in math it's necessary to be able to express things that are false, and things that are true but unprovable.

a. There exists a set x such that [(the empty set is in x) and (for every set y, if y is in x then the successor of y is in x)].

b. There does not exist a set x such that [(x is a member of x) and (for every set y, if y is in x then y is equal to x)].

c. For every set n, n belongs to the successor of n.

d. For every set n, either n is zero or there exists a set m such that n is the successor of m.

Problem 4

Translate the Mathese sentences in Problem 3 into sentences of ordinary English (If you are starting to have trouble distinguishing Mathese from ordinary English, inagine you have to translate these sentences for your parents, dentist, vet, bartender, etc.)

Problem 5

Translate the following sequences of symbols into clear, unambiguous English, either standard English or Mathese or a mixture of the two, whichever you prefer. Again, don't worry about whether the sentences are true, or whether they are provable. [*Note*: In Mathese, there is a standard way of avoiding repeating sequences of quantifiers of the same kind (i.e. all universal or all existential), e.g.:

instead of "For every x, for every y ...", say "For all x and y ..." or "For any two sets x and y, ..."

instead of "There exists x such that there exists y such that there exists z ...", say "There exist three sets x, y, and z such that ...'.]

a. $\forall x \exists y \forall z (z \in y \leftrightarrow \exists u \exists v [z \in u \wedge u \in v \wedge v \in x])$

b. $\exists ! x \forall y (y \in x \leftrightarrow y \notin y)$

c. $\forall x \forall y [x \neq y \rightarrow (s(x) \neq s(y))]$

d. $\forall x ([0 \in x \wedge \forall y (y \in x \rightarrow s(y) \in x)] \rightarrow z \subseteq x)$