PROBLEM SET FIVE: INFINITIES

Note that these problems follow the same order in which they appear in the main text of Chapter 5. Note also that in each problem, any results from earlier in the chapter, or from earlier chapters, can be assumed and used, even ones from problems you didn't manage to do.

Be sure not to overlook hints given in Chapter 5 about some of the proofs.

Problem 1

Prove the theorem that no natural number is Dedekind infinite.

Problem 2

Prove the corollary that no finite set is Dedekind infinite.

Problem 3

Prove the corollary that any Dedekind infinite set is infinite.

Problem 4

Prove the corollary that no two distinct natural numbers are equinumerous.

Problem 5

Prove the corollary that for any finite set A, there is a unique natural number equinumerous with A.

Problem 6

Prove the lemma that if $C \subsetneq n \in \omega$, then $C \approx m$ for some $m < n$.

Problem 7

Prove that for any sets A, B, and C:

a. $A \preceq A$;
b. if $A \preceq B$ and $B \preceq C$ then $A \preceq C$; and
c. $A \preceq \wp(A)$.

Problem 8

Prove the corollary that any countably infinite set is equinumerous with ω.

Problem 9

Prove the corollary that any infinite subset of ω is equinumerous with ω.

Problem 10

Prove that $\wp(\omega)$ is nondenumerable.

Problem 12

Show how the Recursion Theorem justifies the definition of the function h used in the proof of the theorem (in the Appendix) that every infinite set dominates ω.