# (Pre-)Algebras for Linguistics

## 7. Modelling Meaning and Reference

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Linguistics 680:
Formal Foundations

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- A source of confusion: the terms Frege used were *Sinn* and *Bedeutung*, usually glossed by German-English dictionaries as 'sense' and 'meaning'.
- In general, the reference of an expression can be **contingent** (depend on how things are), while the meaning is independent of how things are (examples coming soon).

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- But that distinction can no longer be ignored when one examines the interdependence between the meaning of an expression and the **context** in which it is uttered.
- This interdependence is the topic of the Winter/Spring 2011 Interdisciplinary Seminar on the Syntax/Semantics/Pragmatics Interface (Linguistics 812).

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- Names are controversial! Vastly oversimplifying:
    - **Descriptivism** (Frege, Russell) the meaning of a name is a **description** associated with the name by speakers; the reference is what satisfies the description.
    - **Direct Reference Theory** (Mill, Kripke) the meaning of a name **is** its reference, so names are **rigid** (their reference is independent of how things are.)

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- Grammar says nothing about reference.
- Instead, a separate, nonlinguistic, theory tells how the **extension** of a meaning depends on how things are.
- An expression's reference is just its meaning's extension.
- So reference also depends on how things are.

# Review of Propositions

We have a set P of **propositions** with the **entailment** preorder $\sqsubseteq$ and the following operations:

- $\sqcap$ a glb operation, the meaning of *and*
- $\rightarrow$ a residual operation for $\sqcap$, the meaning of *implies*
- $\sqcup$ a lub operation, the meaning of *or*
- $\neg$ a complement operation, the meaning of *no way*
- $\top$ a top, a necessary truth
- $\bot$ a bottom, a necessary falsehood

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d. operations given by the usual truth tables: $\wedge$ (glb), $\vee$ (lub), $\supset$ (residual of $\wedge$), and $\sim$ (complement).

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- I The **individuals** (things that can be meanings of names).
- W The **worlds** (ultrafilters of propositions)
- 1 The **unit set** $\{0\}$.
  It's conventional to call the member of this set $*$, rather than 0, since the important thing about it is that it is a singleton and not what its member is.

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d. Nothing else is a semantic domain. (In particular, W and B are not involved in the definition of semantic domains.)

Later we will see that an expression meaning is always a member of a semantic domain (which one depending on the syntactic category of the expression).

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The meaning of the sentential adverb **obviously** will be a function $\mathbf{obvious}' \colon \mathrm{P} \to \mathrm{P}$. For each proposition $p$, we think of $\mathbf{obvious}'(p)$ as the proposition that $p$ is obvious.

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e. If $A$ and $B$ are semantic domains, then $\mathrm{Ext}(A \to B) = A \to \mathrm{Ext}(B)$.

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d. For all $a \in A$, $b \in B$, and $w \in W$, $\langle a, b\rangle @w = \langle a@w, b@w\rangle$.
e. For all $f \in A \to B$ and $w \in \mathrm{W}$, $f@w$ is the function from $A$ to $\mathrm{Ext}(B)$ that maps each $a \in A$ to $f(a)@w$.

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The assignment of meanings to expressions is done by the grammar (next lecture).

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**obviously** is the function from propositions to truth values that maps each proposition $p$ to **t** if the proposition **obvious**′($p$) is in $w$, and to **f** otherwise. (Informally speaking, this is (the characteristic function of) the set of all propositions which are obvious at $w$.)

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Example: the extension of an individual property (e.g. $\mathbf{donkey}'$) is (the characteristic function of) a set of individuals.

Example: the extension of a property of propositions (e.g.$\mathbf{obvious}'$) is (the characteristic function of) a set of propositions.