

Agnostic hyperintensional semantics

Carl Pollard

Received: 31 May 2013 / Accepted: 13 November 2013 / Published online: 19 December 2013
© Springer Science+Business Media Dordrecht 2013

Abstract A hyperintensional semantics for natural language is proposed which is agnostic about the question of whether propositions are sets of worlds or worlds are (maximal consistent) sets of propositions. Montague’s theory of intensional senses is replaced by a weaker theory, written in standard classical higher-order logic, of fine-grained senses which are in a many-to-one correspondence with intensions; Montague’s theory can then be recovered from the proposed theory by identifying the type of propositions with the type of sets of worlds and adding an axiom to the effect that each world is the set of propositions which are true there. Senses are compositionally assigned to linguistic expressions by a categorial grammar with only two rule schemas, based on the implicative fragment of intuitionistic linear propositional logic, and a fully explicit grammar fragment is provided that illustrates the compositional assignment of sense to a variety of constructions, including dummy-subject constructions, infinitive complements, predicative adjectives and nominals, raising to subject, ‘*tough*-movement’, and quantifier scope ambiguities. Notably, the grammar and the derivations that it licenses never make reference to either worlds or to the extensions of senses.

Keywords Hyperintension · Proposition · World · Higher order logic · Linear logic · Categorial grammar

1 Introduction

The first to realize that Carnapian intensions—functions from worlds to extensions—were too coarsely grained to successfully model senses of natural language expressions

C. Pollard (✉)
The Ohio State University, Columbus, OH, USA
e-mail: pollard@ling.ohio-state.edu; pollard.4@osu.edu

was of course Carnap (1947) himself, who ended up making recourse to *structured meanings* of a certain kind, essentially grammatical analyses (in somewhat more contemporary parlance, phrase structure trees) with intensions assigned to the grammatical constituents (or nodes). Nevertheless, Montague (1974a, b) wholeheartedly embraced the identification of senses with intensions, though he followed Kripke (1963) in treating worlds as unanalyzed primitives, not as Carnap's complete state descriptions.¹ Though aware that the closure of belief under logical equivalence "might under certain circumstances appear unreasonable", Montague (1974a, p. 139) seems not to have regarded this as problematic, suggesting that "its counterintuitive character can perhaps be traced to the existence of another notion of belief, of which the objects are sentences or, in some cases, complexes consisting in part of open formulas."

Among formally inclined linguistic semanticists from the 1970s to the 1990s, Montague's view prevailed, endorsed and elaborated by Kaplan (1978); Stalnaker (1976); Thomason (1976), and Dowty (1979), inter multa alia. Nowadays, though, it seems that advocates of Carnap/Montague-style intensions *qua* models of linguistic senses are thin on the ground. Stalnaker (1984) mounted a vigorous defense, but I don't know whether he would today. Rather, it is now generally acknowledged that senses must be more finely grained than intensions. If we agree to use Cresswell's (1975) term *hyperintension* for notions of sense finer-grained than intensions, then it seems fair to say that hyperintensionality is entering, or is on the verge of entering, the semantic mainstream, at least in principle if not in practice.

But what *are* hyperintensions? As yet, no answer to this question has gained widespread acceptance. A number of philosophers (such as Lewis 1970; Cresswell 1985; Soames 1987, and King 1996, 2007) and a few linguists (such as von Stechow (1989)), have followed Carnap in embracing some notion or other of structured meanings. Others, including Tichý (2004), Moschovakis (2006), and Muskens (2005), have proposed that senses are algorithms or procedures.² In the practice of linguistic semanticists, who are mostly concerned with empirical questions, not philosophical or mathematical ones, there has been little sense of urgency about the resolution of this issue. As one highly influential textbook put it:

The good news is, however, that the uncertainty in the area of propositional attitudes does not seem to have a lot of repercussions on the way linguists do semantics every day. A slight change [in the theory presented in the text—CP] led us from an extensional system to an intensional one. The switch to a hyperintensional system should not be much more eventful. What we have learned about particular extensional or intensional phenomena should be adaptable to a new foundation without too much ado. [Heim and Kratzer 1998, p. 311]

This is in the context of a discussion of the inadequacy of Carnap/Montague propositions for the semantic analysis of propositional attitude verbs, on the very last page of the book.

¹ More precisely, Montague's intensions assigned extensions not to worlds but rather to world-time pairs; but for the sake of expository simplicity, we ignore temporality altogether.

² Being a procedure need not preclude being a structured meaning; Tichý's *constructions* have been characterized as 'structured procedures' by Duži et al. (2010, p. 3).

In the 15 years since the publication of Heim and Kratzer's textbook, the proliferation of hyperintensional alternatives has been roundly ignored in linguistic circles. To the extent that semantic analyses of natural language phenomena are formalized (or more often, semi-formalized or pseudo-formalized), they are for the most part couched in extensional, or in cases where reference to propositions is unavoidable (e.g. interrogatives, dynamic semantics, presupposition/conventional implicature, modality, etc.) intensional terms, in spite of the general recognition that neither is quite right. Why is this? I think the most important reason is that none of the alternatives proposed so far possess the combination of features—rigor, simplicity, and accessibility—that enabled Montague semantics to prevail for as long as it did. Without naming names, it is fair to say that no version of hyperintensions presently on offer is worked out with the technical precision of Montague's (1974b) PTQ grammar, has such straightforward formal prerequisites (the simple theory of types, basic categorial grammar), and at the same time has been presented to the linguistic community in such a leisurely and digestible form as, say, the exemplary textbook of Dowty et al. (1981).

The remainder of this paper is organized as follows. In Sect. 2, I present an axiomatic theory of hyperintensional senses and their extensions at worlds which is *agnostic* with respect to the fundamental nature of the objects (propositions, entities, truth values, and worlds) that the theory is about, and which I believe possesses the requisite degrees of rigor and simplicity. In Sect. 3, I provide a small categorial grammar fragment—couched in terms of a syntactic formalism much simpler than the one Montague employed—which shows that the theory in question is not just about senses and references, but also works in tandem with a proof-theoretic system—the grammar—that relates linguistic form to sense in a compositional way. That is, the theory of senses and extensions together with the grammar provides an *agnostic hyperintensional semantics* (hereafter, AHS) for a fragment of English, much in the same way that Montague's theory of senses and extensions, together with a roughly Ajdukiewicz-style categorial grammar, provided an intensional semantics for a somewhat similar fragment. Finally, Sect. 4 uses the resulting AHS framework as a vehicle for exploring and, hopefully, illuminating some of the basic issues about hyperintensionality posed by the issue editors, especially as they bear on natural language.

2 AHS basics

2.1 The underlying logic

As Gallin (1975) showed, Montague might as well have written his theory of intensions and extensions in Henkin's (1950) classical higher order logic (hereafter HOL) with one additional basic type for worlds, rather than Montague's idiosyncratic intensional logic (IL). For expository simplicity, I will ignore IL altogether, so by 'Montague semantics' (hereafter MS) I will mean the now-familiar translation of the PTQ semantic theory into HOL. The reader is reminded that HOL crucially differs from Church's (1940) simple theory of types by the addition of the following axiom of truth-value extensionality explicitly rejected by Church:

(1) Axiom of truth-value extensionality

$$\vdash \forall_{s,t}.(s \leftrightarrow t) \rightarrow (s = t)$$

which identifies biimplication with truth value equality.

I adhere to the following notational conventions. Lambda-bound variables are sub-scripted, e.g. $\lambda_x.a$ rather than $\lambda x.a$. Successive abstractions are written with a single lambda, e.g. $\lambda_{xy}.a$ rather than $\lambda_x.\lambda_y.a$. Combinations are written $(a b)$ (or often just $a b$ when outermost) rather than $a(b)$. Combination is left-associative, so $a b c$ abbreviates $(a b) c$. The familiar truth-value connectives (as distinct from the propositional connectives introduced below) are written as follows: T, F, \neg , \wedge , \vee , \rightarrow , \leftrightarrow , \forall , and \exists ; $\exists \lambda_x.\phi$ is abbreviated $\exists_x.\phi$, and likewise with \forall .

Functional types are written $A \rightarrow B$, not $\langle A, B \rangle$, so the implication of the (intuitionistic) type logic is not distinguished notationally from the implication of the (classical) logic of truth-value terms. Type implication associates to the right, so $A \rightarrow B \rightarrow C$ abbreviates $A \rightarrow (B \rightarrow C)$. For the sake of familiarity, I resist the urge to add the type constructor \wedge (cartesian product), and so multivariate functional terms are always curried. I do, however, make use of the unit type T and the associated logical constant $*$, for senses of semantically vacuous linguistic expressions (e.g. the dummy pronoun *it* in weather constructions). Note that the unit type is not distinguished notationally from the truth constant of type t.

2.2 Semantic types

Following Montague and Gallin, AHS employs the basic types e (Henkin's ι , entities, also called individuals), t (Henkin's o, truth values) and w (Montague's s, worlds). What about the type for propositions, the things that potentially serve as senses of declarative sentential utterances? For Montague, of course, this was $w \rightarrow t$, since he followed Carnap in identifying propositions with (characteristic functions of) sets of worlds. But that identification, of course, leads straightaway to the logical omniscience problem (that whoever believes or knows some necessary truth believes or knows all necessary truths) discussed below on Sect. 4.7.1), since there is only one necessary truth (the set of all worlds). Instead, I follow Thomason (1980) in introducing an additional basic type p for propositions, an idea which, as Thomason pointed out, had earlier been suggested by Ramsey, Prior, and Cresswell. Although not couched in type-theoretic terms, other important antecedents of our notion of proposition *qua* theoretical primitive include Bolzano's (1837) *Satz an sich*, Wittgenstein's (1921) *Tatsache*, and especially Lewis' (1923) *fact*; for discussion of these notions, see Plummer and Pollard (2012).

Thus the basic types of our theory of hyperintensions and their extensions are e, t, w, p, and the unit type T; the remaining types are formed from these by using the single type constructor \rightarrow in the familiar way. But not everything contemplated by this theory is a sense. For example, truth values cannot be senses, nor can worlds. Those types which are constructed only from the other basic types e, p, and T I call *sense types*.

Like Montague, I assume that for each type of sense, there is an ‘extensional’ type for the corresponding reference. For Montague, of course, a sense always has a type of the form $w \rightarrow A$, and so the type of the corresponding reference is A . In AHS, though, the correspondence is given by the function Ext from sense types to types recursively defined as follows (here A and B range over sense types):

- (2) Types for extensions of senses
 - a. $\text{Ext}(e) = e$
 - b. $\text{Ext}(p) = t$
 - c. $\text{Ext}(T) = T$
 - d. $\text{Ext}(A \rightarrow B) = A \rightarrow \text{Ext}(B)$

Here clause (a) embodies the [Kripke \(1980\)](#) doctrine that the meaning of a name coincides with its reference. An alternative treatment more consonant with the Frege/Carnap/Montague tradition would posit a distinct basic sense type i for *individual concepts* and define $\text{Ext}(i)$ to be e ; see Sect. 4.7.1 below for details. The interesting clause is (d): note that the domain of a reference which is a function is the same as the domain of the corresponding sense. In particular, for any sense type A , the extensional type corresponding to A -properties (type $A \rightarrow p$) is A -sets (type $A \rightarrow t$).

2.3 Truth, facts, extensions, and equivalence

For Montague, as for Carnap, to say that proposition p is true at world w is to say that w is a set-theoretic member of p ; that is, the relation of being-true-at between propositions and worlds of is (a restriction of) the relation inverse to set membership. But as we have seen, propositions don’t have to be conceptualized as sets of worlds, not even in possible worlds semantics. Both Wittgenstein (1921) and [Lewis \(1923\)](#) conceptualized worlds as (maximal consistent) sets of propositions, so that for p to be true at w is for p to be a set-theoretic member of w ; that is, being true at is (a restriction of) set membership.

The position that AHS takes on this question—and this is the sense in which it is *agnostic*—is that being-true-at is *some relation or other* between propositions and worlds, and we can characterize its essential properties without committing to the Montague-Carnap line, the Wittgenstein–Lewis one, or something else altogether. To this end, I introduce the constant $@$ of type $p \rightarrow w \rightarrow t$, written infix-style, so that $p@w$ can be read as ‘ p is true at w ’, ‘the truth value of p at w ’, or ‘the extension of p at w ’. More generally, for each sense type A , I introduce the constant $@_A$ of type $A \rightarrow w \rightarrow \text{Ext}(A)$, and read $a@_A w$ as ‘the extension of a at w ’. In practice, the type of the sense term a can usually be inferred from the context, in which case the subscript A is omitted. Then the first axioms of AHS are as follows:

- (3) Axioms for extensions
 - a. $\vdash \forall_{uw}. u@_T w = u$
 - b. $\vdash \forall_{xw}. x@_e w = x$
 - c. $\vdash \forall_{fw}. f@_{A \rightarrow B} w = \lambda_x.(f x)@_B w$

Thus, at any world, for a sense which is vacuous or an entity, the extension coincides with the sense; but for a functional sense f , the extension at w is the function mapping each x in the domain to the extension at w of the sense obtained by applying f to x .

For example, the extension at w of the property of being a donkey is the set of all entities x such that the proposition that x is a donkey is true at w . This is just an instance of the familiar notion of a property as a propositional function. Formally, we introduce a constant **donkey** of type $e \rightarrow p$, and then the theory proves that

$$\vdash \forall_w.(\text{donkey}@w) = \lambda_x.(\text{donkey } x)@w$$

So if **w** is a constant of type w (intuitively, the actual world) then

$$\vdash \text{donkey}@w = \lambda_x.(\text{donkey } x)@w$$

That is: the actual donkeys are those entities x such that the proposition (**donkey** x) is one of the facts of the actual world.

This simple example bears some reflection, since it captures a fundamental philosophical difference between AHS and MS. To frame it in terms of a thought experiment, a Montagovian figures out what the sense of the noun *donkey* is by visiting a world, making a record of all the donkeys there, repeating this action for every other world, and then stapling all the reports together. S/he has to know which things are donkeys in order to figure out what *donkey* means. But in AHS, it goes the other way around: you use the sense of *donkey* to *find* the donkeys! That is, whichever world you find yourself in (transworld travel being what it is these days, one's options are rather limited), you ask, of each entity x that you find, whether the proposition that x is a donkey is true. But consider: wouldn't the Montagovian investigator have had to do just that at each world in order to know which entities to report for that world? The point is that you use the sense to determine the extension (and not the other way around). No matter which world this is that we are in, we figure out whether one of its entities is a donkey by knowing the sense of *donkey* (and not the other way around).

It should be noted, by the way, that for any sense a of type A , $a@$ is of type $w \rightarrow \text{Ext}(A)$. Thus, AHS has analogs of Carnap-Montague intensions, whether we want them or not. For this reason, I call $a@$ the *intension* of a . For example, for any proposition p , $p@$ is precisely the set of worlds at which p is true. It is crucially important that, in general, the intension of a sense is not *the same thing* as the sense; in particular, a proposition is not the same thing as the set of worlds at which it is true. For that to be so, $@_p$ would have to be the identity function on propositions; but nothing in AHS warrants this identification. More generally, for a given sense type A other than T or e , nothing in AHS even warrants the conclusion that $@_A$ is injective. That is, the mapping from senses to intensions is many-to-one. And that is why AHS senses are hyperintensional: they are more fine-grained than intensions.

I adopt the following abbreviations for certain much-employed terms (again A ranges over sense types):

(4) Abbreviations for terms

$$\begin{aligned} \text{facts} &=_{\text{def}} \lambda_{wp}.p@w : w \rightarrow p \rightarrow t \\ \text{entails} &=_{\text{def}} \lambda_{pq}.\forall_w.p@w \rightarrow q@w : p \rightarrow p \rightarrow t \\ \equiv_A &=_{\text{def}} \lambda_{xy}.\forall_w.x@w = y@w : A \rightarrow A \rightarrow t \end{aligned}$$

Thus, the facts of a world are the propositions which are true there; proposition q is entailed by proposition p just in case it is true at every world where p is true; and two senses are equivalent just in case, at each world, their extensions there coincide. A simple consequence of these definitions is that entailment is a preorder (reflexive and transitive). A second is that two propositions are equivalent just in case they are mutually entailing:

$$\vdash \forall_{pq}.(p \equiv q) \leftrightarrow ((p \text{ entails } q) \wedge (q \text{ entails } p))$$

and from this it follows that (so-called) equivalence is in fact an equivalence relation. Crucially—and this AHS’s solution to the logical omniscience problem—nothing in the theory warrants the conclusion that equivalent propositions are equal; there is no propositional-level analog of the axiom (1) of truth-value extensionality:

(5) A non-axiom

$$\not\vdash \forall_{p,q}.(p \equiv q) \rightarrow (p = q)$$

In MS, by contrast, entailment is just the subset inclusion relation on sets of worlds, and so, by the extensionality of sets, mutual entailment is equality of propositions. That is, in MS entailment is antisymmetric, and therefore partially orders the propositions. Indeed, MS is nothing more than the extension of AHS obtained by identifying type p with $w \rightarrow t$ and adding the following:

(6) Montagovian axiom

$$@_p = \lambda_p.p$$

Obviously, I mention this not because I wish to advocate the addition of this axiom! Rather, the point is that MS is an *extension* of AHS, and so there cannot be anything about AHS for Montagovians to take issue with.

What about Wittgenstein–Lewis-style possible worlds theory? There, worlds are ultrafilters—maximal consistent sets of propositions—which in turn presupposes that the set of propositions is endowed with boolean structure, with entailment as the induced preorder. But is it antisymmetric? The tractarian Wittgenstein thought so:

If p follows from q and q from p then they are one and the same proposition.
[Wittgenstein 1921:5.4.1]

That is, tractarian propositions form a boolean algebra ordered by entailment. However, the conceptualization of Lewis (1923) is entirely consistent with the view that propositions (Lewis’s *facts*) are only *preordered* by entailment. Such a view emerges if we gloss Lewis’s terminology algebraically as follows: *requires* = entails, *joint fact* = conjunctive proposition; *contradictory of a fact* = negation of a proposition; *incompatible* = inconsistent; *system* = proper filter; *world* = ultrafilter. That is, we

could implement Lewis's conceptualization algebraically by modelling worlds as *pre-boolean algebras*, which are to boolean algebras as preorders are to orders: all the expected boolean identities are satisfied, but only up to equivalence.³

A more conservative position, the AHS position described in the following subsection, is that (1) the propositions form a preboolean algebra preordered by entailment, and that (2) for each world w , the set of facts of w form an ultrafilter. Something close to the algebraicized Lewis-like view described above can then be obtained as AHS extended by an axiom to the effect that the function mapping each world to its set of facts is a bijection onto the set of ultrafilters.⁴ Thus, AHS is consistent with both MS and with Wittgenstein–Lewis-style possible worlds theory, even though the latter two are mutually inconsistent.⁵

2.4 The propositional connectives and quantifiers

Like Thomason (1980), AHS is equipped with a set of proposition-level connectives in addition to the truth-value connectives supplied by the underlying HOL:

(7) AHS propositional connectives

- ⊢ **truth** : p (a necessary truth)
- ⊢ **falsity** : p (a necessary falsehood)
- ⊢ **not** : $p \rightarrow p$ (propositional negation, the sense of the expression *it is not the case that*)
- ⊢ **and** : $p \rightarrow p \rightarrow p$ (propositional conjunction, the sense of the sentence coordinator *and*, written infix)
- ⊢ **or** : $p \rightarrow p \rightarrow p$ (propositional disjunction, the sense of the sentence coordinator *or*, written infix)
- ⊢ **implies** : $p \rightarrow p \rightarrow p$ (propositional implication, the sense of the subordinator *if (... then)*, written infix)

Unlike Thomason's propositional connectives, some of these are explicitly intended to serve as senses of English expressions. The propositional connectives are subject to the following axioms, which relate them to the truth-value connectives:

(8) Axioms for the AHS propositional connectives

- ⊢ $\forall w. \text{truth}@w$
- ⊢ $\forall w. \neg(\text{falsity}@w)$
- ⊢ $\forall pw. (\text{not } p)@w \leftrightarrow \neg(p@w)$

³ For details of one such implementation, see Pollard (2008).

⁴ Note that we cannot simply *identify* worlds with ultrafilters, because ultrafilters have type $p \rightarrow t$ and so, in any Henkin model of the theory, the interpretation of w would be required to be a subset of the interpretation of $p \rightarrow t$. Pollard (2008) got around this problem by working in a higher order logic—the boolean version of Lambek and Scott's (1986) categorical logic—with a topos interpretation, so that subtypes corresponding to the kernel of a definable predicate (e.g. being an ultrafilter) are always available. In that version of the theory, the morphism that interprets $@_p$ is (the categorical generalization of) the Stone-dual mapping on a (pre-)boolean algebra (Pollard 2011).

⁵ They are mutually inconsistent because in MS not every ultrafilter corresponds to a world, only the principal ones do; see Sect. 4.7.2.

$$\begin{aligned} \vdash \forall_{pqw}.(p \text{ and } q)@w &\leftrightarrow (p@w \wedge q@w) \\ \vdash \forall_{pqw}.(p \text{ or } q)@w &\leftrightarrow (p@w \vee q@w) \\ \vdash \forall_{pqw}.(p \text{ implies } q)@w &\leftrightarrow (p@w \rightarrow q@w) \end{aligned}$$

Note—and this is another difference from Thomason—that the connection between the two sets of connectives is explicated in terms of worlds.⁶ Collectively, these axioms have as a consequence that, in any interpretation I , the set of propositions form a preboolean algebra with respect to the entailment preorder. That is: $I(\text{truth})$ is a top; $I(\text{falsity})$ is a bottom; $I(\text{not})$ is a complement operation up to equivalence; $I(\text{and})$ is a greatest lower bound operation; $I(\text{or})$ is a least upper bound operation; and $I(\text{implies})$ is a relative complement operation up to equivalence. Informally speaking, all the usual boolean identities are satisfied, but only up to equivalence (mutual entailment).

From these axioms, we can now prove (in the object language) that for each world w , the set of facts of w forms an ultrafilter, that is:

(9) The facts of a world form an ultrafilter

$$\vdash \forall_w.\text{ultra}(\text{facts } w)$$

where ($\text{ultra } s$) says of a set of propositions s that it is closed under conjunction, closed under entailment, does not have **falsity** as a member, and for every proposition p , has either p or (**not** p) as a member. For details, see [Plummer and Pollard \(2012\)](#).

Additionally, I introduce, for each sense type A , constants forall_A and exists_A of type $(A \rightarrow p) \rightarrow p$ for the proposition-level quantifiers, which are subject to the following (here x is of type A , P and Q of type $A \rightarrow p$, and type subscripts are suppressed):

(10) Axioms for propositional quantifiers

$$\begin{aligned} \vdash \forall_{Pw}.(\text{forall } P)@w &\leftrightarrow \forall_x.(P x)@w \\ \vdash \forall_{Pw}.(\text{exists } P)@w &\leftrightarrow \exists_x.(P x)@w \end{aligned}$$

Then senses for the determiners *some* and *every*, which have (schematic) type $(A \rightarrow p) \rightarrow (A \rightarrow p) \rightarrow p$, are defined as follows:

(11) Some useful abbreviations

$$\begin{aligned} \text{that} &= \text{def } \lambda_{PQx}.(P x) \text{ and } (Q x) \text{ (property conjunction, sense of the relativizer } \textit{that}, \text{ written infix)} \\ \text{some} &= \text{def } \lambda_{PQ}.\text{exists } (\lambda_x.(P x) \text{ and } (Q x)) = \lambda_{PQ}.\text{exists}(P \text{ that } Q) \\ &\text{(sense of the determiner } \textit{some}) \\ \text{every} &= \text{def } \lambda_{PQ}.\text{forall } (\lambda_x.(P x) \text{ implies } (Q x)) \text{ (sense of the determiner } \textit{every}) \end{aligned}$$

It follows from these definitions that the truth conditions for the determiners are as one might expect:

⁶ In fact, worlds are absent from Thomason’s system, though he tentatively introduces them toward the end of his exposition.

(12) Truth conditions for some and every

$$\begin{aligned} &\vdash \forall_{PQ}.(\text{some } P \ Q)@w \leftrightarrow \exists_x.(P \ x)@w \wedge (Q \ x)@w \\ &\vdash \forall_{PQ}.(\text{every } P \ Q)@w \leftrightarrow \forall_x.(P \ x)@w \rightarrow (Q \ x)@w \end{aligned}$$

2.5 Word senses

Finally, in order to construct a Montague-style grammar fragment, we will need a generous inventory of constants whose interpretations provide senses for nonlogical English words, such as the following (here p_1 abbreviates $e \rightarrow p$ and p_{n+1} abbreviates $e \rightarrow p_n$ for all $n > 0$):

(13) Some constants for word senses

$$\begin{aligned} &\vdash j : e \text{ (John)} \\ &\vdash c : e \text{ (Chiquita)} \\ &\vdash m : e \text{ (Mary)} \\ &\vdash \text{donkey} : p_1 \\ &\vdash \text{farmer} : p_1 \\ &\vdash \text{rain} : p \\ &\vdash \text{yell} : p_1 \\ &\vdash \text{please} : p_2 \\ &\vdash \text{give} : p_3 \\ &\vdash \text{believe} : p \rightarrow p_1 \\ &\vdash \text{persuade} : e \rightarrow p \rightarrow p_1 \\ &\vdash \text{easy} : e \rightarrow p_1 \rightarrow p \end{aligned}$$

Note that, by convention, for senses of verbs or predicate adjectives that take two or more arguments, the arguments corresponding to grammatical complements are ‘curried out’ first, in left-to-right order, and the argument corresponding to the grammatical subject last. This convention will simplify the notation in the grammatical derivations, because the lexical entries usually combine with the complements one at a time from left to right, and with the subject last. For example, the term denoting the sense of *Mary gives John Chiquita* is written (give j c m); and the sense term for *to please John is easy for mary* is (easy m (please j)), i.e. being easy is a two-place relation between an entity, encoded by the prepositional phrase complement, and a property, encoded by the infinitive verb phrase subject.⁷

Once the grammar is in place, we will have a means of relating strings of words (and, as we will see, also functions over such strings) to ‘nonbasic senses’, that is, to the interpretations of semantic terms constructed from constants such as these. For example, the grammar will see to it that the string *Pedro yells* is related in the appropriate way to the interpretation of the term (yell p) of type p. Notwithstanding the obvious fact that this term is structured by the syntactic rules of HOL, there is nothing about HOL, or about AHS as it stands, that compels us to regard its interpretation (the model-theoretic counterpart of the proposition that Pedro yells) as a ‘structured meaning’ with Pedro and the property of yelling as ‘parts’, no more than one’s favorite theory

⁷ Predicate adjectives like *easy* also permit dummy-subject constructions (*it is easy for Mary to please John*), and ‘tough-movement’ constructions (*John is easy for Mary to please*), as discussed below.

of arithmetic compels one to regard the interpretation of the term $7 - 2$ as a structure whose parts are the number seven, the number two, and the subtraction operation.

3 A grammar fragment

3.1 Linear grammar basics

I turn now to the task of providing a grammar that characterizes the relation between expressions of English and their senses. Indirectly, the grammar will also connect expressions to their references, since once the sense of a given expression is known to be (the interpretation of) a , its reference is then known to be $a@w$.⁸ (Well, at least it *would* be known, if only we knew exactly which world w was; but this is not a grammar question.)

Like Montague, I use a categorial grammar (CG) to give this characterization. However, CG has come along way since Montague's day, and there are better choices on offer nowadays than Montague's PTQ grammar. The kind of grammar I will use, here called *linear categorial grammar* (LCG), based largely on Oehrle (1994), has the outstanding virtue that it makes the composition of sense completely straightforward, even for grammatical constructions whose analysis is awkward for PTQ (such as Montague's quantifying-in, or Karttunen's (1977) analysis of *wh*-questions). The straightforwardness arises from the fact that the syntactic combinatorics, which drive the semantic composition, are based on the implicative fragment of linear logic (in a sequent-style natural-deduction presentation), which has only two rules and one logical axiom scheme (hypothesize).⁹ The two rules, implication elimination (modus ponens) and implication introduction (hypothetical proof), correspond to combination and abstraction of sense terms.

A subsidiary virtue of LCG is that it is easy to explain to people who have never been exposed to CG, but only to some version or other of Chomskyan generative grammar. For example, the natural-deduction proof trees correspond more or less directly to phrase structure trees; modus ponens to Merge, hypothetical proof to Move; and the logical axioms to traces. Since this is not a syntax paper, the exposition of LCG here is necessarily a bit on the telegraphic side. For more detailed presentations, see Smith (2010); Mihaliček (2012) or Martin (2013).

LCG draws on five main ideas. First, as originally proposed by Lambek (1958), syntactic analyses of linguistic expressions are *logical proofs*. Second, as suggested by Curry (1961), *phenogrammatical structure* (*pheno* for short, also called concrete syntax), which has to do with surface form, is systematically distinguished from *tectogrammatical structure* (*tecto* for short, also called abstract syntax), which has to do with semantically relevant combinatorics. Third, following Montague (1974b), the meaning of a complex linguistic expression is *recursively composed* from the meanings

⁸ Here I depart from both Montague and Frege in taking the determination of sense, but not of reference, to be compositional. This obviates the need to stipulate that, in certain contexts, the reference of an expression is its customary sense, or to apply a Montague-style $\hat{\cdot}$ -operator to the arguments of a predicate.

⁹ There are also nonlogical axioms, corresponding to lexical entries.

of its immediate syntactic constituents, with the recursion grounded in the meanings of lexical items specified by the grammar. Fourth, as proposed by Oehrle (1994), the phenogrammatical component of an expression is not a string, as per Montague, but rather a *term which denotes a string, or a (possibly higher-order) function over strings*, and is recursively composed from the phenos of the immediate constituents, in parallel with the meaning composition.¹⁰ And fifth, rather than composing *references* of expressions, we instead compose *senses*, so that the grammar defines a relation between phenos (roughly, surface forms), tectos (syntactic types), and meanings (hyperintensional senses) without ever involving reference at all.

In formal terms, an LCG is a sequent-style natural-deduction system each of whose proof trees can be thought of as ordered triple of ‘parallel’ proof trees, one for each of the three components pheno, tecto, and sense. Thus, the statement of each sequent (the part to the right of the turnstile), which we think of as modelling the form, syntactic category, and meaning of a linguistic expression, is a triple of the form

$$a : A; B; c : C$$

where $a : A$ is a typed pheno-term (described below) which characterizes the expression’s (contribution to) surface form, B is a logical formula which specifies the expression’s syntactic combinatoric potential, and $c : C$ characterizes the expression’s sense. Such triples are called *signs*.

In a sign, the sense term $c : C$ is a typed AHS term, where C is a sense type. The tecto component B is a formula in an (intuitionistic propositional implicative) linear logic where the atomic formulas are basic syntactic types such as NP (noun phrase) and S_f (finite sentence), and the only connective is linear implication (\multimap). Readers familiar with Lambek’s (1958) syntactic calculus or with Steedman’s (2000) combinatory categorial grammar can think of \multimap as replacing the two directional implications $/$ and \backslash ; this is possible because the directionality of combination is dealt with in the pheno component. Finally, the pheno term $a : A$ is a typed term in the higher-order theory of monoids. That is, besides the AHS theory of senses and extensions, there is a separate higher order theory with basic types t (truth values) and s (strings); with constants $\mathbf{e} : s$ (null string) and $\cdot : s \rightarrow s \rightarrow s$ (concatenation, written infix), and the *monoid axioms* (the variables here are of type s):

$$\begin{aligned} &\vdash \forall_{stu} . (s \cdot t) \cdot u = s \cdot (t \cdot u) \\ &\vdash \forall_s . (\mathbf{e} \cdot s) = s \\ &\vdash \forall_s . (s \cdot \mathbf{e}) = s \end{aligned}$$

Additionally, we have string constants corresponding to lexical phonologies, e.g. *mary*, *yells*, etc.

The LCG itself is a sequent-style natural deduction system whose sequents are of the form

¹⁰ The idea that phenos are string terms rather than strings is prefigured by Montague’s use of the ‘syntactic variables’ \mathbf{he}_n , which are replaced by ‘real’ strings when the rules of quantification T14–T16 are applied. In Oehrle’s, and LCG’s, pheno-terms, the ‘syntactic variables’ are eliminated in favor of authentic variables of the higher-order pheno theory.

$$\Gamma \vdash a : A; B; c : C$$

where the statement of the sequent is a sign as described above and the context Γ is a set of *hypothetical* signs, i.e. signs whose pheno and sense terms are variables. There are two kinds of axioms: logical, corresponding to hypotheses (analogous to traces in generative grammar), and nonlogical, corresponding to lexical entries; and there are two inference rule schemas: modus ponens (MP) and hypothetical proof (HP). In the tecto component, MP and HP are, respectively, \rightarrow -elimination and \rightarrow -introduction (there are no tecto terms); in the pheno and sense components, they are, respectively, \rightarrow -elimination and \rightarrow -introduction at the level of types, and combination and abstraction, respectively, at the level of terms. Then a sign $a : A; B; c : C$ is said to be *generated* by the grammar provided the (null-context) sequent $\vdash a : A; B; c : C$ is derivable. Intuitively, this means there is an English expression that sounds like a , means c , and has syntactic category B . Usually, we will be most interested in signs corresponding to declarative sentences, where $A = s$, $B = S_f$, and $C = p$.

3.2 A formal LCG fragment

3.2.1 Pheno and Tecto types

(14) Pheno types

- a. s (string) is a basic pheno type.
- b. If A and B are pheno types, so is $A \rightarrow B$.
- c. Nothing else is a pheno type.

(15) Tecto types

- a. The following are basic tecto types:
 - S_f (finite clause)
 - S_i (infinitive clause)
 - S_b (base-form clause)
 - PrdA (predicative adjectival clause)
 - NP (noun phrase)
 - N (common noun phrase)
 - It (dummy *it*)
 - PP_{for} (*for*-PP)
- b. If A and B are tecto types, so is $A \rightarrow B$.
- c. Nothing else is a tecto type.

3.2.2 Axioms

(16) Logical axioms (traces)

$$s : A; B; x : C \vdash s : A; B; x : C$$

Logical Axioms correspond roughly to traces in mainstream generative grammar. In the pheno and sense components, they introduce variables which remain free in

derivations until bound by an application of hypothetical proof (see below). A can be instantiated as any pheno type (always s in our examples), B as any tecto type, and C as any sense type. For example, a noun phrase trace has the form:

$$s : s; \text{NP}; x : e \vdash s : s; \text{NP}; x : e$$

(17) Nonlogical axioms (lexical entries)

mary = \vdash mary; NP; m

john = \vdash john; NP; j

it = \vdash it; It; *

rains = $\vdash \lambda_{s.t}.s \cdot \text{rains}; \text{It} \multimap \text{S}_f; \lambda_{u.t}.\text{rain}$

yells = $\vdash \lambda_{s.t}.s \cdot \text{yells}; \text{NP} \multimap \text{S}_f; \text{yell}$

pleases = $\vdash \lambda_{s.t}.t \cdot \text{pleases} \cdot s; \text{NP} \multimap \text{NP} \multimap \text{S}_f; \text{please}$

please = $\vdash \lambda_{s.t}.\text{please} \cdot s; \text{NP} \multimap \text{NP} \multimap \text{S}_b; \text{please}$

is = $\vdash \lambda_{s.t}.t \cdot \text{is} \cdot s; (B \multimap \text{PrdA}) \multimap A \multimap \text{S}_f; \lambda_{p.P} : (C \rightarrow p) \rightarrow C \rightarrow p$

to = $\vdash \lambda_{s.t}.\text{to} \cdot s; (B \multimap \text{S}_b) \multimap (B \multimap \text{S}_i); \lambda_{p.P} : (C \rightarrow p) \rightarrow C \rightarrow p$

for = $\vdash \lambda_{s.t}.\text{for} \cdot s; \text{NP} \multimap \text{PP}_{\text{for}}; \lambda_{x.x} : e \rightarrow e$

easy₁ = $\vdash \lambda_{s.t}.\text{easy} \cdot s \cdot t; \text{PP}_{\text{for}} \multimap \text{VP}_i \multimap \text{It} \multimap \text{PrdA}; \lambda_{x.Pu}.\text{easy } x \text{ } P : e \rightarrow p_1 \rightarrow \text{T} \rightarrow p$

easy₂ = $\vdash \lambda_{s.f}.\text{easy} \cdot s \cdot (f \text{ } e); \text{PP}_{\text{for}} \multimap (\text{NP} \multimap \text{VP}_i) \multimap \text{AP}; \lambda_{x.Ry}.\text{easy } x \text{ } (R \text{ } y) : e \rightarrow p_2 \rightarrow p_1$

donkey = \vdash donkey; N; donkey

farmer = \vdash farmer; N; farmer

every = $\vdash \lambda_{s.f}.f \text{ } (\text{every} \cdot s); \text{N} \multimap \text{QP}; \text{every}$

some = $\vdash \lambda_{s.f}.f \text{ } (\text{some} \cdot s); \text{N} \multimap \text{QP}; \text{some}$

Each lexical entry is given a mnemonic nickname, to save space in derivations below. Here and subsequently, types of pheno and sense terms are omitted when they can be inferred from the types of the constants. A and B are metavariables ranging over tecto types and sense types respectively.¹¹

In the two lexical entries for *easy*, VP_i (infinitive verb phrase) abbreviates $\text{NP} \multimap \text{S}_i$ and AP abbreviates $\text{NP} \multimap \text{PrdA}$. Thus the second tecto argument of **easy₂**, namely $\text{NP} \multimap \text{VP}_i$, corresponds to an infinitive verb phrase complement with a noun phrase gap, as in *John is easy for Mary to please*. The pheno component of this lexical entry, $\lambda_{s.f}.\text{easy} \cdot (f \text{ } e)$, also bears careful attention. Here the two lambda-bound variables s (of type s) and f (of type $s \rightarrow s$) correspond to the phenos of the PP_{for} and VP_i complements respectively. The body of the abstract says, in essence, that the word *easy* is to be followed first by the PP_{for} -complement (here, *for Mary*), and then by the string that results from applying the pheno of the gappy infinitive complement, namely $\lambda_{s.t}.\text{please} \cdot s$, to the null string e . Informally speaking, the null string is ‘lowered’ into the gap position, corresponding to what would be described in traditional generative-

¹¹ The double occurrences of A and B in the lexical entries **is** and **to** correspond to the fact that auxiliary verbs are ‘raising-to-subject’ predicates, i.e. the unrealized subject of the complement is identified with the subject of the auxiliary.

grammar terms as the ‘*tough*-movement’ of the grammatical object of *please* out of the infinitive verb phrase.¹²

In the tecto components of the lexical entries for *every* and *some*, QP (quantification noun phrase) abbreviates $(NP \multimap S_f) \multimap S_f$. Of special note in these entries are the pheno components, which illustrate Oehrle’s (1994) upgrade of Montague’s quantifying-in, which uses ordinary function application in place of Montague’s special-purpose machinery for replacing the ‘syntactic variables’ he_n with strings corresponding to QPs.

3.2.3 Rules

(18) Modus Ponens (MP)

$$\begin{array}{c} \Gamma, \Delta \vdash (ba) : B; D; (dc) : F \\ \diagdown \quad \diagup \\ \Gamma \vdash b : A \rightarrow B; C \multimap D; d : E \rightarrow F \quad \Delta \vdash a : A; C; c : E \end{array}$$

Instances of MP, which corresponds roughly to the Merge rule of mainstream generative grammar, combine an argument-seeking sign with its argument. In the tecto component, MP is \multimap -elimination; in the pheno and sense components, it is \rightarrow -elimination at the level of types, and combination at the level of terms.

(19) Hypothetical proof (HP)

$$\begin{array}{c} \Gamma \vdash \lambda_s a : A \rightarrow D; B \multimap E; \lambda_x b : C \rightarrow F \\ | \\ \Gamma, s : A; B; x : C \vdash a : D; E; b : F \end{array}$$

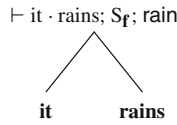
Instances of HP, which corresponds roughly to the Move rule of mainstream generative grammar (or, more specifically, to the trace-binding aspect of Move), discharge a hypothetical sign (trace). In the tecto component, HP is \multimap -introduction; in the pheno and sense components, it is \rightarrow -introduction at the level of types, and abstraction at the level of terms.

3.2.4 Example derivations

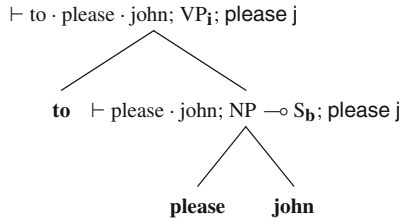
In these derivations, the following conventions are observed. Lexical entries (nonlogical axioms) are replaced by their abbreviations. All other leaf nodes are traces (logical axioms). Binary-branching nodes all correspond to instances of modus ponens. Unary-branching nodes all correspond to instances of hypothetical proof. In a derivation, if a subtree was derived earlier, it is replaced by the example number of the earlier derivation.

¹² This technique of simulating transformational movement by applying a string function to the null string is due to Muskens (2007).

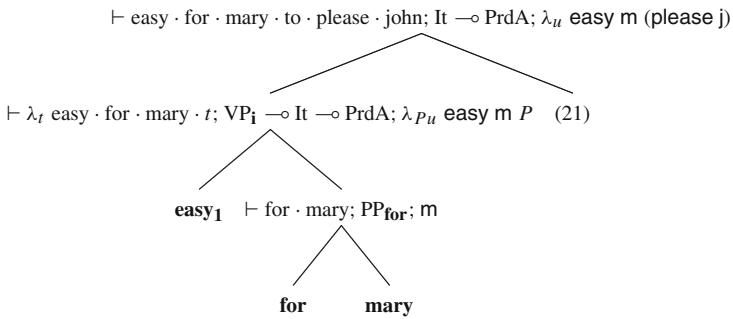
(20) A simple sentence



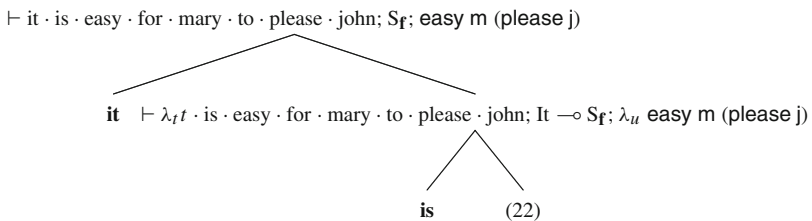
(21) An infinitive verb phrase



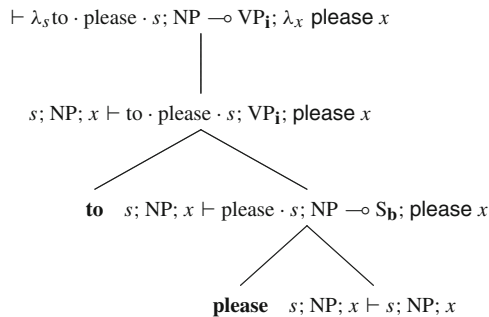
(22) A predicative adjective phrase



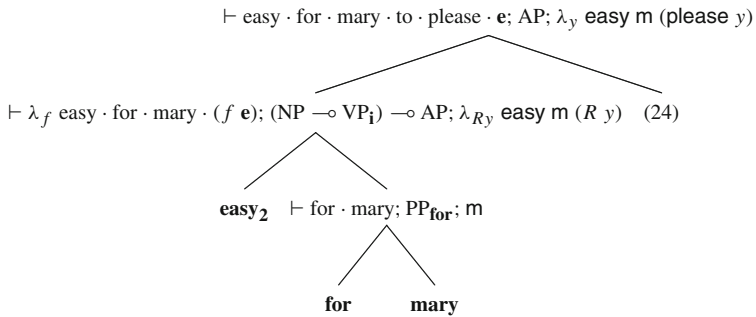
(23) A dummy-subject sentence



(24) An infinitive verb phrase with a noun phrase gap

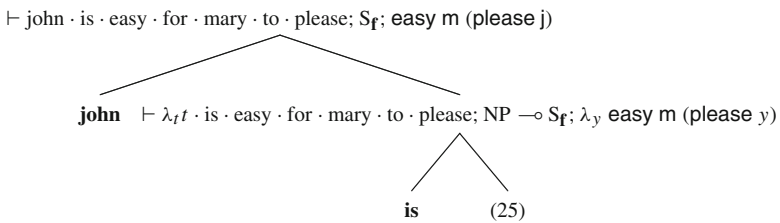


(25) Another predicative adjective phrase



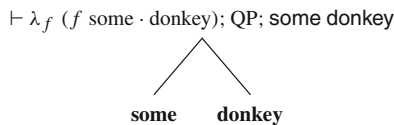
Here AP abbreviates NP \multimap PrdA. Note that when *easy* combines with the gappy infinitive verb phrase, its trace has already been bound.

(26) A ‘*Tough*-movement’ sentence

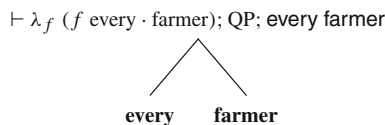


Note that this sentence has the same sense as the dummy-subject sentence (23). In traditional (1960s) generative grammar, it would have been derived from (23) by deleting the dummy subject *it* and moving the object of the infinitive, *John*, into the deletion site.¹³ On the categorial analysis, there is no syntactic connection between the two sentences; they contain different inventories of lexical entries and have different derivations. In particular, the *tough*-movement sentence, whose derivation is given in (24–26), involves a trace and an instance of HP to bind it, whereas the dummy-subject sentence, whose derivation is given in (21–23), involves only lexical entries and instances of MP.

(27) A quantified noun phrase



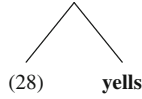
(28) Another quantified noun phrase



¹³ The movement operation in question was called ‘*tough*-movement’ because it was occasioned only in sentences containing certain predicates, such as *tough*, *easy*, *a bitch*, etc.

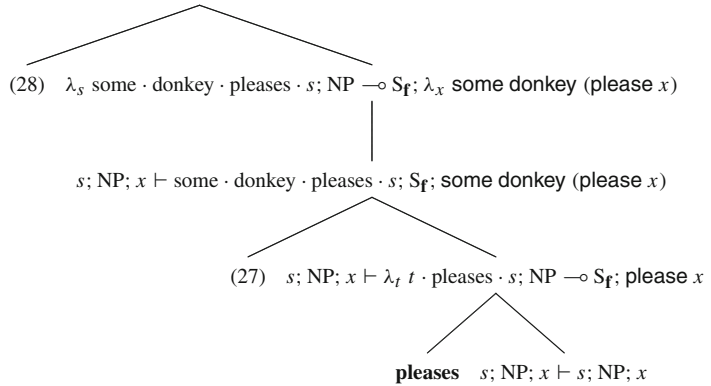
(29) A quantificational sentence

\vdash every · farmer · yells; S_F ; every farmer yell



(30) A quantifier scope ambiguity (1/2)

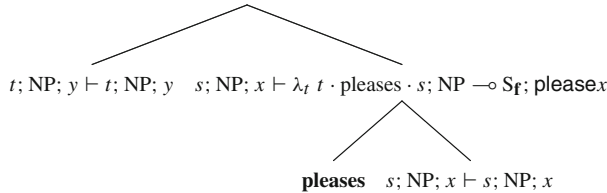
\vdash some · donkey · pleases · every · farmer; S_F ; every farmer (λ_x some donkey (please x))



To get the other scope reading, we have to introduce *two* traces and then successively quantify in:

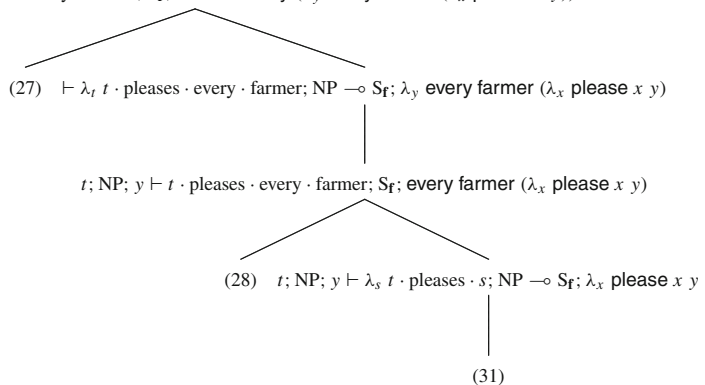
(31) A sentence with two gaps

t ; NP; y , s ; NP; $x \vdash t \cdot$ pleases · s ; S_F ; please x y



(32) A quantifier scope ambiguity (2/2)

\vdash some · donkey · pleases · every · farmer; S_F ; some donkey (λ_y every farmer (λ_x please x y))



To summarize: in LCG, linguistic expressions are modelled as pheno/tecto/sense triples. A sign $a : A; B; c : C$ is in the language in question just in case it is a theorem of the grammar, i.e. it is the statement of a hypothesis-free sequent that labels the root node of an LCG proof tree. In particular, the root sequents of the derivations (20), (23), (26), (29), (30), and (32), namely

- ⊢ it · rains; S_f ; rain
- ⊢ it · is · easy · for · mary · to · please · john; S_f ; easy m (please j)
- ⊢ john · is · easy · for · mary · to · please; S_f ; easy m (please j)
- ⊢ every · farmer · yells; S_f ; every farmer yell
- ⊢ some · donkey · pleases · every · farmer; S_f ; every farmer (λ_x some donkey (please x))
- ⊢ some · donkey · pleases · every · farmer; S_f ; some donkey (λ_y every farmer (λ_x please x y))

are all English signs of syntactic category (tectogrammatical type) S_f .

4 Hyperintensionality and natural language

4.1 How fine-grained must hyperintensions be?

Linguistics is an empirical discipline, and in linguistic semantics the gold standard for whether (the proposition expressed by) sentence S entails sentence S' is whether native speakers of the language in question judge that, under any conditions where S is true, so is S' . More generally, in light of the widespread agreement that the determination of sense is compositional with respect to syntactic combinatorics, together with the equally widespread agreement that the extension of a proposition (at a given world) is its truth value, substitutability *salva veritate* of one linguistic subexpression for another with the same sense in natural-language sentences must obtain. Conversely, lack of substitutability requires that the senses of linguistic expressions be distinguished. This is precisely the sense in which intensions are insufficiently fine-grained to model natural-language senses. For the purposes of this paper, I *defined* hyperintensions to be models of natural-language senses that are more fine-grained than intensions. But once we move to the realm of non-natural languages (e.g. logical or mathematical languages, type theories, programming languages, etc.), the term *hyperintension* no longer has a fixed meaning, and so the question of how fine-grained they should be cannot arise.

4.2 Intensions in hyperintensional semantics

As we have seen, in AHS, for any sense a , $a@$ is an intension; so we are stuck with them, whether we want them or not. Are they of any use? Well, given a declarative sentence S whose sense is the proposition p , we could ascertain the truth of S by applying $p@$ to w , if only we knew which world w was. But we don't, so we can't. From the point of view of AHS, it appears that the best that can be said of intensions is that they do no harm.

4.3 What is the expressive power of AHS?

AHS is not a logic. It is just an axiomatic theory expressed in simple type theory, or more specifically, the classical HOL of Henkin 1950 with some simple upgrades (unit type and basic types for worlds and propositions). So it seems that the question of expressive power does not arise.

4.4 Must a hyperintensional theory be an HOL?

AHS is not an HOL; it is a theory written in one.

4.5 Could a hyperintensional theory be extensional?

Is AHS extensional? Well, the HOL it is written in is, by certain measures: it validates Leibniz's Law, Church's rule (η), Henkin's axiom of truth-value estensionality, and substitutability of equals. But AHS does not validate substitutability of equivalents, if by equivalents we mean senses with identical intensions.

These facts suggest the following definitions. Suppose A and B are sense types and $f : A \rightarrow B$. Then I call f :

extensional provided

$$\vdash \forall_{wxy}.(x@w = y@w) \rightarrow ((f x)@w = (f y)@w)$$

that is, the extension (at a given world) of the application of f to an argument depends only on the extension of the argument;

intensional provided

$$\vdash \forall_{xy}.(x \equiv y) \rightarrow ((f x) \equiv (f y))$$

that is, the intension of the application of f to an argument depends only on the intension of the argument (and so, extensionality entails intensionality);

strictly intensional provided it is intensional but not extensional;

modal provided it is strictly intensional and $A = B = p$; and

hyperintensional if it is not intensional.

4.6 Hyperintensionality and truth conditions

In AHS truth conditions are too coarse-grained to determine sense, but the sense of a sentence uniquely determines its truth conditions, because, for a sentence S with sense p , the truth value of S at any world w is $p@w$. To put it another way, the intension associated with any sentence's sense uniquely determines its truth value at all worlds. One might say that the intension $p@$ is the same thing as the truth conditions of S .

4.7 What count as puzzles of hyperintensionality?

For linguistic semanticists raised on MS, there has always been a famous hyperintensionality puzzle: the *granularity problem*, namely the fact that intensions are too coarse-grained as models of linguistic senses to account for robust native speaker intuitions about synonymy. But there are a couple more less well-known hyperintensionality puzzles, the *nonprincipal ultrafilters* problem and the *total omniscience problem* that perhaps deserve to be better known. Let us consider all three in turn.

4.7.1 The granularity problem

What is at stake here are cases where two linguistic expressions E and E' are assigned identical Carnap/Montague-style intensions, but are not mutually substitutable *salva veritate*. Here we consider three representative instances.

The *logical omniscience* problem is a special case of the granularity problem in which E and E' are both necessarily truths and therefore are assigned the common intension $\lambda_w.T$. This is problematic because it predicts that anyone who knows one necessary truth, say that Justin Bieber is Justin Bieber, must know every necessary truth, for example the proposition *p* which is whichever of the following three propositions is true: (1) the Riemann hypothesis (RH) is a theorem of ZFC; (2) RH is inconsistent with ZFC; and (3) RH is independent of ZFC. And so, if Lindsay Lohan knows that Justin Bieber is Justin Bieber, it follows that she also knows that *p*, this notwithstanding the fact that RH is among the most celebrated of unresolved mathematical conjectures. Logical omniscience is a nonproblem for AHS, because it is not a consequence of AHS (or of AHS combined with LCG) that any of (1), (2), or (3) has the same sense as *Justin Bieber is Justin Bieber*, i.e. (b equals b).¹⁴

The *woodchuck problem* is the special case of the granularity problem in which E and E' are common nouns with equivalent senses, such as (presumably) *woodchuck* and *groundhog*: whatever is a woodchuck is necessarily a groundhog, and conversely. Consequently, in an intensional theory like MS, anyone who believes that Punxsutawney Phil is a groundhog is predicted to also believe that Punxsutawney Phil is a woodchuck. In AHS, this unwelcome consequence does not arise, because the identity of *groundhog@* and *woodchuck@* does not force the identity of *groundhog* and *woodchuck*.

¹⁴ Here *b* is the sense component in the lexical entry for the name *Justin Bieber*, and *equals* : $e \rightarrow e \rightarrow p$ is a constant axiomatized as follows:

$$\vdash \forall_{wxy}.(x \text{ equals } y)@w \leftrightarrow (x = y)$$

This constant is introduced in the lexical entry for the *predicativization* sign that combines with an NP to form a predicate:

$$\vdash \lambda_s.s; \text{ NP} \multimap \text{Prd}; \lambda_{xy}.y \text{ equals } x.$$

Finally, the *Hesperus-Phosphorus* problem has to do with coreferential names.¹⁵ In MS, the problem arises because name senses are treated as rigid individual concepts (constant functions from worlds to entities). Thus, since in at least one world (ours, where presumably both names refer to Venus), the two senses *hesperus* and *phosphorus* must be the same constant function, namely $\lambda_w.v$. where v is the constant of type e that denotes the (model-theoretic counterpart of) the planet Venus. But we can easily imagine a community in which these names are in use, some member of which believes that Hesperus is Hesperus, but not that Hesperus is Phosphorus.

There are (so far) a couple of ways to handle this problem within AHS. The first, described in detail in Pollard (2008), is to drop the Kripkean identification of name meaning with name reference, and instead introduce a hyperintensional analog of Carnap/Montague individual concepts. This amounts to adding a new type i for individual concepts, of which e is the corresponding extensional type, replacing the first clause of (2) with

$$\text{Ext}(i) = e$$

and eliminating the second clause of (3) that identifies an entity *qua* sense with its own extension. Correspondingly, all occurrences of e in the types of the sense components of lexical entries have to be replaced by i . A less obvious consequence is that AHS becomes inconsistent with the tractarian doctrine that a world is uniquely determined by its facts, because two worlds with the same facts can now differ with respect to, say, which entity is the extension of the individual concept which is the sense of the name *Justin Bieber*. (These are not necessarily undesirable consequences.) This is a fundamentally Fregean solution, in the respect that it implements the idea that ‘Hesperus’ and ‘Phosphorus’ agree in reference but not in sense.

A different, and more radical, solution to the Hesperus-Phosphorus problem is to abandon the naive account of names as constants denoting fixed entities (or fixed individual concepts), and replace it with a more pragmatically informed account wherein uses of names either introduce or continue *discourse referents* (Karttunen 1976; Kamp 1981; Heim 1982, inter multa alia). This involves extending AHS, which is fundamentally a *static* theory of *sentence meanings*, into a theory of *discourse* which takes into account the *dynamic* nature of the relationship between utterance context and utterance interpretation: not only does the context in which an utterance is made partially determine its interpretation (Kaplan 1978), but the utterance itself *changes* the context with respect to which subsequent utterances are interpreted. Dynamic accounts of discourse meaning differ radically from static theories of sentence meaning in the important respect that they are no longer about the relationship between the forms of expressions and extralinguistic reality but rather about the relationship between utterances and *how the discourse participants represent the world as being*.

There is certainly ample motivation for shifting from static to dynamic semantics independent of considerations about Hesperus and Phosphorus! The well-known successes of dynamic semantics lie in such areas as distinguishing definiteness from

¹⁵ Frege’s (1892) problem is the analogous puzzle for definite descriptions (*the Evening Star* and *the Morning Star*).

indefiniteness, in accounting for puzzles about anaphora (including the notorious ‘donkey pronouns’) that seem beyond the reach of static accounts, and in classifying *conventional implicatures*, those aspects of conventionally signalled linguistic meaning (including, but not limited to, anaphora) that lie beyond compositionally determined sense.

In fact, a compositional, dynamic extension of AHS has already been developed (Martin 2013). On that account, sentences express not propositions, but rather functions from contexts to contexts, where a context in turn is a function that maps an n -tuple of entities to a proposition. In such contexts, the coordinates of the argument tuples (that is, the natural numbers less than n) play the role of *discourse referents* (DRs) in earlier versions of dynamic semantics (such as Kamp 1981 and Heim 1982); the context itself is analogous to the notion of *common ground* (Stalnaker 2002), roughly in the sense of what the discourse participants take to have been mutually agreed upon for the purposes of the conversation. According to Martin’s analysis, a name is a kind of *definite* expression. More specifically, an utterance of a name (such as *Hesperus*) expresses not an entity as such, but rather the uniquely most salient DR (provided there is one) which is entailed by the context to have the property of bearing the name ‘Hesperus’. Frege’s puzzle arises in discourses where two name utterances pick out different DRs but the real-world counterparts of those DRs are the same entity.

Another class of problems which I believe might lend itself to a dynamic solution has to do with so-called *strict/sloppy* ambiguities, such as *John claims he jogs, and so does Peter*. Does Peter claim that John jogs (the strict reading), or that Peter does (the sloppy reading)? Naively, *so* is anaphoric to the verb phrase *claims he jogs*, and so the ambiguity of the second sentence seemingly can be reduced to the ambiguity of *claims he jogs*, more specifically to the question of whether the pronoun *he* is *de re* (leading to the strict reading of the second clause) or *de dicto* (leading to the sloppy reading of the second clause). But *de re* vs. *de dicto* puzzles can be viewed as a special case of much more general issues about the *point-of-view anchoring* of conventional implicatures (which include but are by no means limited to the existential presuppositions associated with definite pronouns).

4.7.2 The nonprincipal ultrafilters problem

Let us assume, uncontroversially, that there is an infinite set of English sentences none of which entails any of the others, e.g. *Frege had exactly one cat*, *Frege had exactly two cats*, etc. Then the set of propositions, ordered by entailment, must be an infinite boolean algebra. Let us assume also, only slightly less controversially, Choice. Then that boolean algebra must have a nonprincipal ultrafilter, \mathbf{W} . Then \mathbf{W} is a maximal consistent set of propositions. Intuitively, it corresponds to a way that things could be (or perhaps, if we are lucky, actually are). But if we are good Montagovians, then there is a fixed inventory of worlds, the interpretation of the type w . For any one of these worlds w , the set of facts of w is also a maximal consistent set, namely the set of all propositions (sets of worlds) which are supersets of the singleton set $\{w\}$. But each of these maximal consistent sets is the *principal* ultrafilter whose generator is $\{w\}$, and so none of them is \mathbf{W} . And so, \mathbf{W} is a possible way things might be (maybe even the actual way), and yet it doesn’t correspond to any of the Montagovian worlds; it

doesn't get to participate in determining what senses are, since these are all functions whose domain contains only the official Montagovian worlds.

It seems to me that Montagovians, if there are any left, owe us an explanation of why it is that nonprincipal ultrafilters are, semantically speaking, second-class citizens. In AHS, by contrast, there is nothing about nonprincipal ultrafilters that confers a distinct status upon them. If a world w happens to have a nonprincipal ultrafilter U as its set of facts, its members are still facts of w . In particular, if $p \in U$, then the intension $p@$ has w in its domain.

4.7.3 The total omniscience problem

Suppose that MS is correct, let \mathbf{w} be the actual world, and let \mathbf{p} be the proposition which is actually true, but false at every nonactual world, namely $\{\mathbf{w}\}$. Does Justin Bieber know that \mathbf{p} ?

Well, at any nonactual world w , he clearly doesn't, since knowledge is factive. But what about at \mathbf{w} ? Either he does or he doesn't. Let's consider these options in turn. Suppose he doesn't know that \mathbf{p} . Then $(\text{know } \mathbf{p} \ \mathbf{b})$ is a necessary falsehood, hence its denial is a necessary truth. So \mathbf{p} is a contingent proposition, but Justin Bieber's knowledge of it is noncontingent, which, if not contradictory, certainly counts as puzzling.

The other option, of course, is that Justin Bieber *does* know that \mathbf{p} . But \mathbf{p} is the intersection of all the actual facts, the conjunction of all propositions true in the actual world. So under the uncontroversial assumption that anyone who knows a conjunction of facts knows each of the conjuncts, it follows that Justin Bieber is not merely *logically* omniscient (which, as good Montagovians, we have long since come to terms with), but *totally omniscient*: he knows *all* the facts, even the merely contingent ones.

We can choose either of these unappealing options. Or we can reject the Montagovian axiom, and not have to choose.

4.8 Hyperintensionality and model-theoretic semantics

AHS is a theory in HOL, and HOL has model theories (e.g. Henkin models, or topos models). This is no different than the situation with MS. In fact, as we have seen, MS *is* AHS with the identification of p with $w \rightarrow t$ and the addition of the Montagovian axiom

$$\vdash @_p = \lambda p. p$$

4.9 Is there an intuitionistic hyperintensional theory?

In MS, the underlying HOL must be classical, because the classicality of the proposition-level negation arises from the classicality of the truth-value-level negation:

$$\text{not} =_{\text{def}} \lambda_{pw}. \neg(p \ w)$$

In model-theoretic terms: the power set of the set of worlds is a boolean algebra. But in AHS, the situation is rather different because we directly axiomatize the set of propositions to be a preboolean algebra. This would still be possible if we had chosen to write the theory in an intuitionistic HOL (such as Lambek and Scott’s) instead, but then the axiom for **not** in (8) must be supplemented with the following:

$$\vdash \forall_{pw}. p@w \vee (\text{not } p)@w$$

and the axiom for **implies** there must be replaced by the following:

$$\vdash \forall_{pqw}. (p \text{ implies } q) \leftrightarrow ((\text{not } p)@w \vee q@w)$$

This is sufficient to ensure that, for each world w , the set of facts of w remains an ultrafilter.

On the model-theoretic side, reformulating the theory in Lambek and Scott’s categorical logic would necessitate relinquishing Henkin models in favor of non-set-theoretic topos models; then the ‘set’ of propositions is a preboolean algebra object, verb senses are morphisms that target this object, etc.

In practical terms, though, since the intent of AHS is to serve as a practical upgrade to MS for linguistic semanticists, there is little incentive to reformulate the theory on an intuitionistic basis.

4.10 Hyperintensionality and structured meanings

Although, as we have noted, there are rough correspondences between the technical devices of LCG and those of generative grammar, it is important to note that in LCG, it is the root sequents of the derivations that model linguistic expressions, not the derivations themselves. The LCG model of *John is easy for Mary to please* is the triple

$$\text{john} \cdot \text{is} \cdot \text{easy} \cdot \text{for} \cdot \text{mary} \cdot \text{to} \cdot \text{please}; S_f; \text{easy } m \text{ (please } j)$$

above, not a derivation which has this triple as the statement of its root sequent. And so, just as nothing about AHS entails that senses of signs are structured (beyond being triples of a pheno, a tecto, and a sense), likewise nothing about LCG entails that the signs themselves are. This is quite different from generative grammar, where the shapes of the phrase structure trees, and configurationally defined relations between nodes (such as domination, c-command, government, etc.) have always been taken to be linguistically significant. In particular, no matter how much the appearance of LCG derivations might suggest an affinity with Carnap’s or D. Lewis’s structured meanings, there is no such affinity. Senses are hyperintensions, and signs are associations between senses and surface forms (not arboreal structures).

That much said, nothing precludes the possibility that AHS has an interpretation in which senses are structured in some sense or other, and, if so, this might even be seen as

a positive development. Indeed, it seems intuitively clear that the propositional functions that serve as senses of natural-language predicates (such as verbs) should yield different propositions when applied to different arguments. For example, if Chiquita and Maria are different entities, then the proposition that Chiquita yells and the proposition that Maria yells should not be the same proposition! A straightforward way of preventing such accidental synonymies is by means of nonlogical axioms (meaning postulates) to the effect that the propositional functions in question are injective:

(33) A meaning postulate schema for ‘structured propositions’

$$\vdash \forall_{xy}.(\mathbf{P} x) = (\mathbf{P} y) \rightarrow (x = y)$$

Though a bit on the brutal side, such meaning postulates capture in a relatively conservative way something that is right about ‘structured meaning’ theories, without requiring that senses be modeled (*à la* Soames) as tuples or (*à la* Carnap or D. Lewis) as phrase structure trees with nodes labelled by intensions, and without abandoning Church’s rule (β).

In fact, meaning postulates of this kind do not go far enough. For example, there is nothing ruling out the possibility that (say) the proposition that Maria yells and the proposition that Chiquita brays are the same proposition! What we would like, it seems, is to preclude the possibility of two semantic objects being the same unless their identity is a consequence of AHS. To put it another way, we want our favored interpretation to be minimal, in the sense that it validates only those identities that are theorems of AHS. This perhaps provides an incentive for interpreting AHS not into (just) Henkin models, but rather into toposes, since then we are guaranteed a minimal model, namely the topos whose internal language is AHS.

Should hyperintensional senses be thought of as procedures or programs? AHS is agnostic about that question, unsurprisingly so since it does not say what senses are, no more so than axiomatic set theories say what sets are. On the other hand, a sense is a sense *of an expression*, and in order for the sense of a given expression to be determined compositionally, the expression has to be parsed. That is, a derivation (or something in our mental life corresponding to one) must be found. It is certainly natural to think of *the mental activity of parsing the expression that expresses the sense* in procedural terms. But it is much less clear to me that any advantage is gained from treating *senses themselves* in such terms. It might turn out that, as with MS and tractarian semantics, some form or other of procedural semantics can be implemented as an extension of AHS. But in advance of expending the effort of carrying out such an extension, one would first want to know what work it would be expected to do that AHS as it stands is not already doing.

Finally, although this is not one of the questions posed by the issue editors, it is fair to ask (as one of the anonymous referees did) what advantages AHS has over other forms of hyperintensional semantics that have already been proposed. The most obvious advantage is the very fact that it is *agnostic* about what senses (and worlds) are, and therefore is consistent with the two main types of possible worlds that have been advocated so far (the tractarian and the Montagovian). Second, since it is *weaker* than MS, which remains the standard framework for linguistic semantic analysis in spite

of its shortcomings, there is nothing in AHS for mainstream linguistic semanticists to take issue with; they could just as well use AHS instead of MS, and even add back in the Montagovian axiom if, for whatever reason, living with those shortcomings makes them feel more at home. Third, it does not require mastering any new formal technology, since *it is expressed entirely in completely standard classical higher-order predicate logic*. Fourth, it improves on the widely adopted Heim–Kratzer-style hybrid of (extensional) MS with a vaguely Chomskyan transformational syntax by restoring Montague-style *compositionality*, since it comes equipped with a fully explicit interface to LCG; at the same time, LCG, which was expressly designed to facilitate linguistic description and analysis, serves to deflect the widespread (and, unfortunately, valid) objections to Montague’s own, excessively complicated yet decidedly un-linguistic, categorial syntax. Fifth, the static semantics described herein is easily replaced by Martin’s explicit (and still fully compositional) *dynamic* semantics, with no attendant complication of the syntactic interface. Sixth, AHS is *simple*, in the crudely quantitative respect that it can be specified in its entirety in the space of seven printed pages (Sects. 2.2—2.5 above), with another five pages for the complete specification of the syntactic interface (Sects. 3.1—3.2.3 above). And seventh, the AHS/LCG package is *teachable* and *learnable*, in comparison with other semantic/syntactic frameworks which I have taught over the last three decades, as evidenced by the relative ease with which linguistics graduate students have mastered it and gone on to use as a practical framework for linguistic research.

Acknowledgments AHS is the result of joint work with Andy Plummer. The dynamic extension of the theory arises from a collaboration with Scott Martin. For corrections and helpful comments on an earlier version, thanks are due to Manjuan Duan, Murat Yasavul, and an anonymous referee.

References

- Bolzano, B. (1972). *Theory of science*. Translation of *Wissenschaftslehre*, 1837, edited and translated by R. George. Berkeley and Los Angeles, CA: University of California Press.
- Bolzano, B. (1973). *Theory of science*. Translation of *Wissenschaftslehre*, 1837, edited by J. Berg and translated by B. Terrell. Dordrecht: D. Reidel.
- Carnap, R. (1947). *Meaning and necessity*. Chicago: University of Chicago Press.
- Church, A. (1940). A formulation of the simple theory of types. *Journal of Symbolic Logic*, 5, 56–68.
- Cresswell, M. (1975). Hyperintensional logic. *Studia Logica*, 34, 25–38.
- Cresswell, M. (1985). *Structured meanings*. Cambridge, MA: MIT Press.
- Curry, H. (1961). Some logical aspects of grammatical structure. In R. Jakobson (Ed.), *Structure of language in its mathematical aspects* (pp. 56–68). Providence, RI: American Mathematical Society.
- Dowty, D. (1979). *Word meaning and Montague grammar*. Dordrecht: D. Reidel.
- Dowty, D., Wall, R., & Peters, S. (1981). *Introduction to Montague semantics*. Dordrecht: D. Reidel.
- Duži, M., Jespersen, B., & Materna, A. (2010). *Procedural semantics for hyperintensional logic*. Berlin: Springer.
- Frege, G. (1980). On sense and reference. Translation of *Über Sinn und Bedeutung*, 1892. In P. Geach & M. Black (Eds.), *Translations from the philosophical writings of Gottlob Frege* (3rd ed.). Oxford: Blackwell.
- Gallin, D. (1975). *Intensional and higher order modal logic*. Amsterdam: North-Holland.
- Heim, I. (1982). The semantics of definite and indefinite noun phrases. Ph.D. Dissertation, University of Massachusetts, Amherst, MA.
- Heim, I., & Kratzer, A. (1998). *Semantics in generative grammar*. Oxford: Blackwell.
- Henkin, L. (1950). Completeness in the theory of types. *Journal of Symbolic Logic*, 15, 81–91.

- Kamp, H. (1981). A theory of truth and semantic representation. In J. Groenendijk, T. Janssen, & M. Stokhof (Eds.), *Formal methods in the study of language* (Part 1, pp. 277–322). Amsterdam: Mathematical Centre.
- Kaplan, D. (1978). On the logic of demonstratives. *Journal of Philosophical Logic*, 8, 81–98.
- Karttunen, L. (1976). Discourse referents. In J. McCawley (Ed.), *Notes from the linguistic underground. Syntax and semantics* (Vol. 7, pp. 363–385). New York: Academic Press.
- Karttunen, L. (1977). Syntax and semantics of questions. *Linguistics and Philosophy*, 1, 3–44.
- King, J. (1996). Structured propositions and sentence structure. *Journal of Philosophical Logic*, 25, 495–521.
- King, J. (2007). *The nature and structure of content*. Oxford: Oxford University Press.
- Kripke, S. (1963). Semantic analysis of modal logic I: Normal modal propositional calculi. *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik*, 9, 67–96.
- Kripke, S. (1980). *Naming and necessity*. Oxford: Blackwell.
- Lambek, J. (1958). The mathematics of sentence structure. *American Mathematical Monthly*, 65, 154–170.
- Lambek, J., & Scott, P. (1986). *Introduction to higher-order categorical logic*. Cambridge: Cambridge University Press.
- Lewis, C. I. (1923). Facts, systems, and the unity of the world. *Journal of Philosophy*, 20, 141–151.
- Lewis, D. (1970). General semantics. *Synthese*, 22, 18–67.
- Martin, S. (2013). The dynamics of sense and implicature. Ph.D. Dissertation, Ohio State University, Columbus, OH.
- Mihaliček, V. (2012). Serbo-Croatian word order: A logical approach. Ph.D. Dissertation, Ohio State University, Columbus, OH.
- Montague, R. (1974). Pragmatics and intensional logic. In R. Thomason (Ed.), *Formal philosophy: Selected papers of Richard Montague* (pp. 119–146). New Haven, CT: Yale University Press.
- Montague, R. (1974). The proper treatment of quantification in ordinary English. In R. Thomason (Ed.), *Formal philosophy: Selected papers of Richard Montague* (pp. 247–270). New Haven, CT: Yale University Press.
- Moschovakis, Y. (2006). A logical calculus of meaning and synonymy. *Linguistics and Philosophy*, 29, 27–89.
- Muskens, R. (2005). Sense and the computation of reference. *Linguistics and Philosophy*, 28(4), 473–504.
- Muskens, R. (2007). Separating syntax and combinatorics in categorial grammar. *Research on Language and Computation*, 5, 267–285.
- Oehle, R. (1994). Term-labelled categorial type systems. *Linguistics and Philosophy*, 17, 633–678.
- Plummer, A., & Pollard, C. (2012). Agnostic possible worlds semantics. Logical aspects of computational linguistics. In *Seventh international conference (LACL 2012)*, Nantes, July 2012. Springer Lecture Notes in Computer Science (Vol. 7351, pp. 201–212). Berlin: Springer.
- Pollard, C. (2008). Hyperintensions. *Journal of Logic and Computation*, 18(2), 257–282.
- Pollard, C. (2011). *Are (linguists') propositions (topos) propositions?* Logical aspects of computational Linguistics 6. Berlin: Springer.
- Smith, E. (2010). Correlational comparison in English. Ph.D. Dissertation, Ohio State University, Columbus, OH.
- Soames, S. (1987). Direct reference, propositional attitudes, and semantic content. *Philosophical Topics*, 15, 47–87.
- Stalnaker, R. (1976). Propositions. In A. F. MacKay & D. D. Merrill (Eds.), *Issues in the philosophy of language* (pp. 79–91). New Haven, CT: Yale University Press.
- Stalnaker, R. (1984). *Inquiry*. Cambridge, MA: MIT Press/Bradford Books.
- Stalnaker, R. (2002). Common ground. *Linguistics and Philosophy*, 25, 701–721.
- Steedman, M. (2000). *The syntactic process*. Cambridge, MA: MIT Press/Bradford Books.
- Thomason, R. (1976). Some extensions of Montague grammar. In B. Partee (Ed.), *Montague grammar* (pp. 75–111). New York: Academic Press.
- Thomason, R. (1980). A model theory for propositional attitudes. *Linguistics and Philosophy*, 4, 47–70.
- Tichý, P. (2004). *Collected papers in logic and philosophy* (V. Svoboda, B. Jespersen, & C. Cheyne, Eds.). Filosofía/Dunedin: Czech Academy of Sciences/University of Otago Press.
- von Stechow, A. (1989). Focusing and backgrounding operators. Arbeitspapier No. 6, Fachgruppe Sprachwissenschaft, Universität Konstanz.
- Wittgenstein, L. (1961). *Tractatus Logico-Philosophicus*. Translation by D. F. Pears & B. F. McGuinness of Logisch-Philosophische Abhandlung in *Annalen der Naturphilosophie*, 1921. London and Henley: Routledge & Kegan Paul.

Copyright of Synthese is the property of Springer Science & Business Media B.V. and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.