Minimalist Approach to Kahler Moduli Stabilization in Type IIB orientifolds and constraints on the String Axiverse

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Outline

• Introduction and motivation
• Moduli fixing in Type IIB CY orientifolds
• Axiverse and strong CP
• Conclusions
Introduction and motivation

• By compactifying to 4D, we obtain a multitude of scalar fields – moduli, parameterizing internal metric deformations, brane positions, etc.

• In String/M-theory all masses and couplings are functions of the moduli vevs

• Once the moduli are stabilized, all couplings are completely fixed and are computable in principle

• Can we stabilize all moduli so that we can make predictions?
• Moduli masses must be large enough to be compatible with observations ($\text{BBN} \Rightarrow M_{\text{mod}} > O(10)\text{TeV}$)

• Addressing the strong CP problem

\[ f_{QCD} = \frac{4\pi}{g_3^2} + i \frac{\theta_{QCD}}{2\pi} \]

Very heavy, the vev should be fixed at some high scale

Very light, the vev should be fixed by the QCD instanton effects

In SUSY Minkowski vacua $\theta_{QCD}$ is too heavy!

Conlon hep-th/0602233

• CC should be tiny but positive (discrete fine tuning)

• SUSY must be broken such that new CP violating phases are compatible with current exp. limits

• FCNCs are suppressed, no rapid proton decay, etc.
Moduli fixing in Type IIB Calabi-Yau orientifolds

- Complex structure moduli and the axio-dilaton can be stabilized at tree-level by turning on fluxes Dasgupta, Rajesh, Sethi; Giddings, Kachru, Polchinski; Larsen, O’Connell, Robbins

- Kahler moduli can be stabilized by including non-perturbative effects (instantons, gaugino condensation) Kachru, Kallosh, Linde, Trivedi (KKLT)

- Can obtain de Sitter vacua once supersymmetry breaking effects are included: alpha prime corrections (Balasubramanian, Berglund, Conlon, Quevedo), matter F-terms plus D-terms (Lebedev, Nilles, Ratz), anti D3-branes (KKLT)
Here we concentrate on Kahler moduli

\[ T_i = \tau_i + i\chi_i ; \quad i = 1, \ldots, h^{1,1}_+ \]

Volumes of 4-cycles (fluctuations of the metric)

Axions (periods of the 4-form, transform under a shift symmetry)

\[ K_{\text{Kahler}} = -2\ln V_{\text{CY}}(\tau_i) \quad W = W_{\text{flux}} + \sum_k A_k e^{-\frac{2\pi}{c_k} \tilde{T} \cdot \tilde{n}^{(k)}} \]

\[ \tau^{(k)} = \tilde{\tau} \cdot \tilde{n}^{(k)} = \text{Re} \tilde{T} \cdot \tilde{n}^{(k)} = \sum_{i=1}^{h^{1,1}_+} \tau_i n_i^{(k)} \]

Volume of the k-th divisor (supersymmetric 4-cycle)

Integers, specifying the homology of the k-th divisor
• Common practice: assume as many divisors contributing to the superpotential as there are Kahler moduli and go to a basis where

\[ W = W_{\text{flux}} + \sum_{k=1}^{h^{1,1}} A_k e^{- \frac{2\pi}{c_k} T_k} \]

• For a k-th divisor \( D \) to contribute the instanton must possess precisely two fermionic zero modes:

Witten hep-th/9604030

Necessary condition: \( \chi_+(D, \mathcal{O}_D) - \chi_-(D, \mathcal{O}_D) = 1 \)

Sufficient: \( h^{02}(D) = 0, \ h^{01}(D) = 0 \)
• Naively, one needs at least as many rigid divisors to contribute to the superpotential as there are Kahler moduli. If $h^{1,1}_+(X)=O(100)$ this is a rather daunting task.

• Furthermore, visible sector divisor $D_{\text{visible}}$ supports chiral matter $\Rightarrow$ extra zero modes. Blumenhagen et. al. hep-th/0711:3389

Visible sector charged chiral matter

$$\delta W_{\text{visible}} = \prod_i \Phi_i e^{-2\pi T_{\text{visible}}}$$

• We already know that $\langle \Phi_i \rangle = 0$ in the visible sector.

• Hence, despite the success of the KKLT approach the problem seems to comes back.
• Is it possible to stabilize all Kahler moduli with fewer than $h^{1,1}_+(X)$?

• Consider an extreme case with a single divisor $D$

$$W = W_{\text{flux}} + Ae^{\frac{2\pi i c}{c} T_D}$$
where $\tau_D = \text{Re} T_D = \sum_{i=1}^{h^{1,1}_+(X)} n_i \tau_i$

• A divisor $D$ corresponds to a zero locus of a section of a holomorphic line bundle $\Theta(D)$. If $D$ is ample, its Poincare dual $[\omega]$ is inside the Kahler cone.

$$\{[\omega] | \omega = g_{i\bar{j}} dz^i \wedge d\bar{z}^j \} = Kc(X) \subset H^{1,1}(X)$$

$$c_1(\Theta(D)) = \lambda[\omega], \text{where } 0 < \lambda \in \mathbb{R}$$
• The Kahler cone is given by a set of linear inequalities in the two-cycle volumes $t_i$. After a linear change of coordinates

$$Kc(X) = \{ \vec{t} \mid t_i > 0 \} \subset H^{1,1}(X)$$

• An ample divisor is transverse (has a positive intersection with) to all holomorphic curves

$$D = \sum_{i=1}^{h_+^{1,1}} n_i D_i \quad \text{is ample} \implies n_i > 0, \; i = 1,\ldots,h_+^{1,1}$$

• Hence, if a single ample divisor contributes to the superpotential, the vevs of all moduli are automatically inside the Kahler cone
• Recall: $K = -2 \ln V_{CY}$, where $V_{CY} = \sum d_{ijk} t_i t_j t_k = \frac{1}{3} \sum \tau_i t_i$

• SUSY extremum (pick for definiteness $W_{flux} < 0, \ A > 0$)

$$D_i W = 0 \Rightarrow \hat{n} \cdot \hat{\chi} = 0 \mod \frac{N}{2\pi}$$

$$t_i = \frac{n_i \tau_D^{1/2}}{\sqrt{3 \sum d_{ijk} n_i n_j n_k}}, \ \tau_i = \frac{\tau_D}{3} \frac{\partial}{\partial n_i} \ln \sum d_{ijk} n_i n_j n_k, \ V_{CY} = \frac{\tau_D^{3/2}}{3 \sqrt{3 \sum d_{ijk} n_i n_j n_k}}$$

where $\tau_D = \frac{N}{2\pi} \left( \ln \left| \frac{2A}{3W_{flux}} \right| + \ln \left( \frac{3}{2} + \ln \left| \frac{2A}{3W_{flux}} \right| \right) + ... \right)$

• All two- and four-cycle volumes as well as the volume of the Calabi-Yau are parameterized by $\tau_D$!
- Introduce anti D3-branes to break SUSY

\[ V_{total} = e^K \left( G^{ij} D_i W D_j \bar{W} - 3|W|^2 \right) + \frac{D}{V_{CY}^{4/3}} \]

\[ t_i = \frac{n_i \tau_D^{1/2}}{\sqrt{3} \sum d_{ijk} n_i n_j n_k} \]
\[ \tau_i = \frac{\tau_D}{3} \frac{\partial}{\partial n_i} \ln \sum d_{ijk} n_i n_j n_k \]
\[ V_{CY} = \frac{\tau_D^{3/2}}{3 \sqrt{3} \sum d_{ijk} n_i n_j n_k} \]

\[ \tau_D = \tau_D^{SUSY} + \frac{N^2}{48 \pi^2} \left( \frac{4 \pi^2}{N^2} \tau_D^{SUSY} + 3 \right)^2 \left( \frac{\pi}{N} \tau_D^{SUSY} + 1 \right) \left( \frac{4 \pi}{N} \tau_D^{SUSY} + 1 \right) \tau_D^{SUSY} W_{flux}^2 V_{CY}^{-3} \]
• A three-parameter example CICY:

Define the orientifold action:

\[ \Omega([x_0 : x_1 \ | \ y_0 : y_1 : y_2 \ | \ z_0 : z_1 : z_2 : z_3]) = [x_0 : -x_1 \ | \ y_0 : -y_1 : -y_2 \ | \ z_0 : -z_1 : -z_2 : -z_3] \]

Demand that the polynomials transform as

\[ p_{(0,0,3)} \circ \Omega = -p_{(0,0,3)}, \quad p_{(1,0,1)} \circ \Omega = p_{(1,0,1)}, \quad p_{(1,3,0)} \circ \Omega = p_{(1,3,0)} \]

Kahler potential:

\[ K = -2 \ln \left( \frac{3}{2} t_2 t_3 (6t_1 + t_2 + t_3) \right) = -2 \ln \left( \frac{1}{6\sqrt{3}} \sqrt{\tau_1 (6\tau_2 - \tau_1)(2\tau_3 - \tau_1)} \right) \]

Ample divisor: \( D = \{ p_{(1,1,1)} = 0 \} \subset X \quad \tau_D = \tau_1 + \tau_2 + \tau_3 \)
• Recall the general expressions for the moduli vevs

\[ t_i = \frac{n_i \tau_D^{1/2}}{\sqrt{3} \sum d_{ijk} n_i n_j n_k}, \quad \tau_i = \frac{\tau_D}{3} \frac{\partial}{\partial n_i} \ln \sum d_{ijk} n_l n_j n_k, \quad V_{CY} = \frac{\tau_D^{3/2}}{3 \sqrt{3} \sum d_{ijk} n_l n_j n_k} \]

• For the specific example we get:

\[ \tau_1 = \frac{6}{30} \tau_D, \quad \tau_2 = \frac{11}{30} \tau_D, \quad \tau_3 = \frac{13}{30} \tau_D, \quad t_1 = t_2 = t_3 = \frac{\tau_D^{1/2}}{3 \sqrt{5}}, \quad V_{CY} = \frac{\tau_D^{3/2}}{9 \sqrt{5}} \]

• All Kahler moduli are parameterized by a SINGLE parameter! This approach to Kahler moduli fixing is highly constraining and therefore potentially predictive

\[ \alpha_G^{-1} = \sum_{i=1}^{h^{1,1}} n_i \tau_i \quad \Rightarrow \quad \tau_D = 3 \left( \sum_{i=1}^{h^{1,1}} n_i \frac{\partial}{\partial n_i} \ln \sum d_{ijk} n_l n_j n_k \right)^{-1} \times \alpha_G^{-1} \]

\[ \alpha_G^{-1} \approx 25 \]
• In fluxless $G_2$ compactifications of M-theory we find a qualitatively similar result when the non-perturbative superpotential receives a contribution from a supersymmetric three-cycle $Q$, which is Poincare dual to the co-associative four-form $*\Phi$

Integers specifying the homology of $Q$

$$\tau_i = N_i \frac{7V_7}{3V_Q} > 0, \quad \Leftrightarrow \quad PD_X(Q) = \alpha * \Phi, \quad 0 < \alpha \in R$$

$$\tau_i \equiv \frac{\partial V_7}{\partial s_i} = \frac{1}{3} \int_X \phi_i \wedge *\Phi = \frac{1}{3} \int *\Phi, \text{where } \Phi = \sum_{i=1}^{b_3(X)} s_i \phi_i; \quad \phi_i \in H^2(X, \mathbb{Z})$$

The four-form is dynamically fixed by the homology of $Q$

$$G_2: \quad s_i = \frac{4V_Q}{7} \frac{\partial}{\partial \tau_i} \ln V_7 \bigg|_{\tau_k = N_k}; \quad \text{TypeIIB: } \tau_i = \frac{\tau_D}{3} \frac{\partial}{\partial n_i} \ln \sum d_{ijk} n_i n_j n_k$$
• There may exist additional non-perturbative contributions. How robust are the vacua we found?

\[ W_{total} = W_{flux} + Ae \frac{2\pi}{N} \tau_D + Be \frac{2\pi}{M} \tilde{r} \cdot \tilde{m} \approx W_{flux} - \frac{3}{2} W_{flux} + B \left( \frac{3W_{flux}}{2A} \right) + O(1) \]

\[ \tau_D \approx \frac{N}{2\pi} \ln \left| \frac{2A}{W_{flux}} \right|, \quad \tau_i = \frac{\tau_D}{3} \frac{\partial}{\partial n_i} \ln \sum d_{ijk} n_l n_j n_k \]

• Clearly, when \( N >> M \), the extra contributions are exponentially suppressed.

• This is especially true if the leading term is due to a gaugino condensate with \( N \sim O(10) \), while the remaining terms come from instantons, i.e. \( M=1 \).
Moduli masses:

one is heavy \( M_1 \approx O(60) \times m_{3/2} \)

N-1 are light \( m_i \approx O(1) \times m_{3/2} < 2 \times m_{3/2} \)

• \( m_{3/2} \approx O(10) \text{TeV} \Rightarrow \) moduli are probably heavy enough to decay before BBN

• For the explicit toy example we numerically obtain \( m_i < 1.3 \times m_{3/2} \Rightarrow \) the light moduli cannot decay into the gravitinos. Large entropy production at late times, but before BBN, helps to avoid the gravitino problem

Acharya, et.al, hep-ph/0804.0863
String Axiverse and Strong CP

• Until now we only discussed fixing the volumes $\tau_i$ of four-cycles (and one linear combination of the axions). What about the remaining axions? Problem or virtue?

• Recall:

$$\chi_i = \int C_4; \quad [\tau_i] \in H_4(X, Z); \quad i = 1, \ldots, h^{1,1}_+$$

• Axions (arising from PQ symmetry) – the most elegant dynamical solution to the strong CP problem (Peccei, Quinn; Weinberg, Wilczek)

• May provide a significant fraction of Dark Matter

• Typically very light, so could have important consequences for cosmology and astrophysics
• Solving Strong CP via axions requires that the QCD axion acquires most of its mass from QCD instantons

\[ m_{QCD} \sim \frac{\Lambda_{QCD}^2}{f_a} \sim 10^{-9}\, eV, \quad \text{for } f_a \approx 10^{16}\, GeV \]

• Typical moduli stabilization mechanisms give large masses to the axions, same as their saxion partners

• In Type IIB flux compactifications the dilaton and its axionic partner – the RR scalar are stabilized by fluxes near the string scale

• In a generic KKLT scenario as well as the LARGE volume scenario, the axionic partners of Kahler moduli get masses \( m_\chi \sim O(m_{3/2}) \gg m_{QCD} \) (Conlon hep-th/0602233)
• In the new scenario, a single non-perturbative term freezes the volumes $\tau_i$ and a single linear combination of axions

$$\vec{n} \cdot \vec{\chi} = 0 \mod \frac{N}{2\pi}$$

• To fix all axions we must include the truncated non-perturbative contributions B. Acharya, KB, P. Kumar: hep-th/1004.5138

$$W = W_{\text{flux}} + A_1 e^{ib_1 f} + \sum_{k>1} A_k e^{ib_k f_k}$$

• Effective scalar potential after including the remaining non-perturbative terms and freezing all $\tau_i$

$$V \approx V_0 - m_{3/2} m_{pl} e^{K/2} \sum_{j>2} D_k e^{-b_j \tau^{(j)}} \cos \chi_j ; \ \chi_j = b_j \vec{n}_j \cdot \vec{\chi}$$
• The axion mass spectrum before the QCD effects

\[ m_1 \approx O(60) \times m_{3/2}, \quad m_i^2 \approx O(1) \frac{m_{pl}^3 m_{3/2}^{3/2}}{f_i^2} e^{K/2} e^{-b_i \tau^{(i)}}, \quad i = 2, ..., h_+^{1,1} \]

where

\[ \tau^{(i)} = \vec{n}^{(i)} \cdot \vec{\tau} \sim O(1) \times \tau_D \]

• Consider \( b_i = 2\pi, \quad m_{3/2} \sim 10 \text{ TeV}, \quad V_{CY} = 1000, \quad f_i = 10^{16} \text{GeV} \)

\[ 15 < \tau^{(j)} < 40 \quad \Rightarrow \quad 10^{-33} \text{eV} < m_i < 1 \text{eV} \]

• In generic compactifications \( h_+^{1,1}(X) \sim O(100-1000) \)

\( \Rightarrow \) String Axiverse with a multitude of light axions

Arvanitaki et. al hep-th/0905.4720, hep-th/1004.3558
String Axiverse

Arvanitaki et. al hep-th/0905.4720, hep-th/1004.3558

• For masses below $10^{-33}$ eV, the axion mass is below the current Hubble scale, so they are essentially irrelevant although may contribute to dark energy

• Between $10^{-33}$ eV and $10^{-28}$ eV axions start oscillating between recombination and today. If they couple to E.B they can induce the rotation in the CMB polarization

• For $m > 10^{-28}$ eV , axions can be a significant fraction of DM and can suppress power in small scale density perturbations

• Between $10^{-22}$ eV and $10^{-10}$ eV axions affect dynamics of rotating black holes. Superradiance.
What about the QCD axion?

- From QCD instantons \( \mathcal{V}_{QCD} = \Lambda_{QCD}^4 (1 - \cos \theta_{QCD}) \)

\[
\theta_{QCD} = 2\pi \sum_{i=1}^{h_{+1,1}} n_{vis}^i \chi_i + \text{arg det } m_q = \theta_0 + \sum_{k=1}^{h_{+1,1}} \psi_k \frac{1}{f_k} ; \quad \tilde{f}_l \equiv \sum_{i=1}^{h_{+1,1}} \sum_{k=1}^{h_{+1,1}} n_{vis}^i U_{ik} \frac{2\pi}{f_k} \tilde{U}_{kl}
\]

Fixed by the vevs of the C.S. moduli

- The QCD instanton mass matrix is rank one \( \Rightarrow \)

\[
(m_k^2)_{QCD} = 0, \forall k = 1,..., h_{+1,1} - 1; \quad (m_{h_{+1,1}}^2)_{QCD} \approx \Lambda_{QCD}^4 \sum_{l=1}^{h_{+1,1}} \frac{1}{\tilde{f}_l^2}
\]

- QCD effects give mass to the lightest mass eigenstate \( \psi_k \) inside the linear combination representing \( \theta_{QCD} \)

- Can easily achieve \( |\theta_{QCD}| < 10^{-10} \), as long as at least one of the mass eigenstates inside \( \theta_{QCD} \) is lighter than

\[
m^2_{\text{exp}} \approx 10^{-10} \left( m^2_{h_{+1,1}} \right)_{QCD} \approx \left( 10^{-14} \text{eV} \right)^2
\]
• Pre BBN cosmological history is non-thermal so the standard estimates on the axion relic density do not apply. The mass of the QCD axion is such that it starts coherent oscillations during moduli dominated era. For all axions whose masses are greater then
\[ \Gamma_{X_0} \approx O(1) \frac{m_{X_0}^3}{m_{pl}^2} \sim 10^{-14} \text{eV}; \quad m_{X_0} \approx 50 \text{TeV} \]
the relic abundance is independent of their mass!

\[ \Omega_k h^2 = O(10) \left( \frac{f_k}{2 \times 10^{16} \text{GeV}} \right)^2 \left( \frac{T_{RH}^{X_0}}{1 \text{MeV}} \right) \langle \theta_{I_k}^2 \rangle \chi \]

Fox, Pierce, Thomas hep-th:0409059, Visinelli, Gondolo astro-ph:0912.0015

\[ \Omega_{DM} h^2 \leq 0.11 \Rightarrow \langle \theta_{I_k}^2 \rangle \leq 10^{-2} \]

• Modest fine tuning of the misalignment angle. The entropy dilution due to late time moduli decays allows decay constants \( f_k \) much closer to the GUT scale

a) Non-thermal cosmology \( (H_{\text{inf}} > M_{\text{moduli}}) \)  

b) Standard cosmology \( (H_{\text{inf}} < M_{\text{moduli}}) \)

- Consider one axion/e-folding between \( 10^{-33} \text{eV} \) and \( 1 \text{ eV} \)
- Constraints from the relic abundance require a much smaller fine tuning of \( \langle \theta_I \rangle \) for case a) compared to case b)
- Isocurvature fluctuations can easily distinguish between the two scenarios
Falsifiable predictions

• Discovery of tensor modes in the near future would rule out the entire Axiverse (isocurvature fluctuations produced during inflation would be too large)
• Discovery of isocurvature component of in the CMB would strongly favor the non-thermal scenario
• Expect O(1) fraction of DM to come from axions
• Couplings of axions with masses \( m_i \ll m_{\text{QCD}} \sim \Lambda_{\text{QCD}}^2 / f \) to E.B are highly suppressed by a factor \( (m_i / m_{\text{QCD}})^2 \) essentially due to grand unification \( \Rightarrow \) axion decays to photons and the rotation of the CMB polarization are suppressed for light non-QCD axions
Conclusions

• Constructed a new class of compactifications in Type IIB on CY orientifolds, completely analogous to the M-theory models found earlier.

• All Kahler moduli can be stabilized inside the Kahler cone by a single non-perturbative contribution. Divisor supporting the visible sector is then fixed automatically.

• SUSY CP violation is suppressed since the superpotential as well as the F-terms have the same overall phase.

• Strong CP problem can be solved with all the moduli stabilized. Moreover, obtain a multitude of light axions!