The unique $\mathbb{Z}_4^R$ of the MSSM

and

its string theory realization using

the C++ orbifolder

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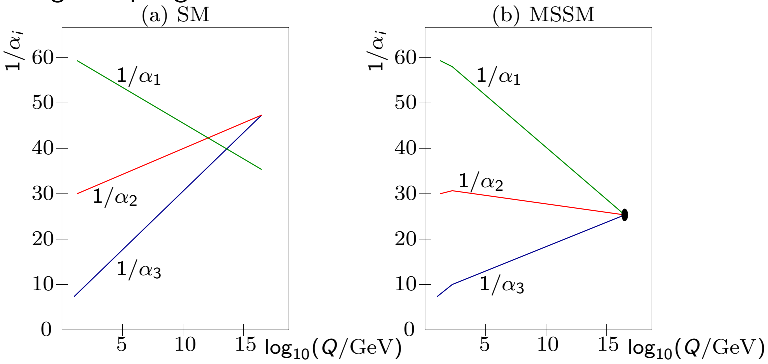
November 06, 2010

Based on:
H. M. Lee, S. Raby, M. Ratz, G. G. Ross, R. Schieren, K. Schmidt-Hoberg and P. V.:
arXiv:1009.0905 [hep-ph], to appear in PLB

H. P. Nilles, S. Ramos-Sánchez, P. V., A. Wingerter: to appear 2010/11

From the Standard Model to the MSSM

- Supersymmetry: squarks and sleptons
- Introduced as a solution to hierarchy problem
- Gauge coupling unification



- Dark matter candidate (LSP) ...

Problems of the MSSM

- μ-problem:

$$\mathcal{W} \supset \mu \bar{H} H$$

  $\mu \sim m_{3/2}$ needed, but $\mu \sim M_P$ expected
- Proton decay via operators, like $\bar{U}\bar{D}\bar{D}$ and $QQQL$
- CP and flavor problems

$\Rightarrow$ new (discrete) symmetries needed

Matter-parity ($= R$ parity)

Dimopoulos, Georgi/ Dimopoulos, Raby, Wilczek

- $\mathbb{Z}_2$ with charges:

matter	1
Higgs	0

- Allowed couplings: $\mathcal{W} \supset Q\bar{U}H + Q\bar{D}\bar{H} + L\bar{E}\bar{H} + \mu\bar{H}H$

- Forbidden couplings: $LH + LL\bar{E} + Q\bar{D}L + \bar{U}\bar{D}\bar{D}$

- LSP is stable $\Rightarrow$ dark matter candidate

- Still problematic: $\mathcal{W} \supset \mu\bar{H}H + QQQL + \bar{U}\bar{U}\bar{D}\bar{E}$

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Proton-Hexality

- Anomaly-free $\mathbb{Z}_6$ with charges: Dreiner, Luhn, Thormeier 2005

Q	$\bar{U}$	$\bar{D}$	L	$\bar{E}$	$\bar{H}$	H
0	1	-1	-2	1	1	-1

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Förste, Nilles, Ramos-Sánchez, P.V. 2010

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Förste, Nilles, Ramos-Sánchez, P.V. 2010

Discrete Symmetries of the MSSM

- Extend the MSSM by a discrete symmetry that
  - Allows the Yukawa couplings
  - Suppresses proton decay
  - Solves μ-problem ($\mu = 0$ at the perturbative level)
  - Symmetry should be anomaly free!
    (Allowing for the Green-Schwarz mechanism)
- Discrete anomalies without being embedded in gauge symmetry!
  Example: heterotic orbifolds

  Araki, Kobayashi, Kubo, Ramos-Sánchez, Ratz and P.V. 2008

$\mathbb{Z}_N$ (R) symmetry

- Matter superfields with charges $q^{(f)}$
  $\Rightarrow$ fermion components with charges $q^{(f)} - R$,
  with $R = 0$ or 1 for non-R or R symmetry
- Superpotential $\mathcal{W}$ has charge $2R$
- Anomaly coefficients for gauge group G

$$A_{G-G-\mathbb{Z}_N} = \sum_{r^{(f)}} \ell\left(r^{(f)}\right)\left(q^{(f)} - R\right) + \ell\,(\mathrm{adj})\, R$$

- Anomaly cancellation

$$A_{G-G-\mathbb{Z}_N} = \rho \bmod \eta \quad \text{where} \quad \eta = \begin{cases} N & \text{for } N \text{ odd,} \\ N/2 & \text{for } N \text{ even.} \end{cases}$$

$\rho \neq 0$ Green-Schwarz anomaly cancellation

SU(5) universal charges

Consider simple situation:

- SU(5) universal charges for matter $q_{\mathbf{10}_i}$ and $q_{\overline{\mathbf{5}}_i}$
- $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$ anomaly coefficients

$$
\begin{aligned}
A_{\mathrm{SU}(3)_C-\mathrm{SU}(3)_C-\mathbb{Z}_N} &= \frac{1}{2}\sum_i \left[3q_{\mathbf{10}_i} + q_{\overline{\mathbf{5}}_i} - 4R\right] + 3R \\
A_{\mathrm{SU}(2)_L-\mathrm{SU}(2)_L-\mathbb{Z}_N} &= \frac{1}{2}\sum_i \left[3q_{\mathbf{10}_i} + q_{\overline{\mathbf{5}}_i} - 4R\right] + 2R \\
&\quad + \frac{1}{2}\left(q_H + q_{\bar{H}} - 2R\right)
\end{aligned}
$$

SU(5) universal charges

- Anomaly cancellation
(Allowing for the Green-Schwarz mechanism)

$$\begin{aligned} A_{\mathrm{SU}(2)_L-\mathrm{SU}(2)_L-\mathbb{Z}_N} - A_{\mathrm{SU}(3)_C-\mathrm{SU}(3)_C-\mathbb{Z}_N} &= 0 \bmod \eta \\ \Rightarrow \qquad q_H + q_{\bar{H}} &= 4R \bmod 2\eta \end{aligned}$$

- In contrast, μ-term allowed if

$$q_H + q_{\bar{H}} = 2R \bmod N \,.$$

- For non-R-symmetry ($R=0$): In the SU(5) case, anomaly cancellation generically implies the $\mu-$problem

SU(5) universal charges for $\mathbb{Z}_N^R$

Demand:

- $\mathbb{Z}_N^R$-charges for matter are SU(5) universal
- $G - G - \mathbb{Z}_N^R$ anomalies are universal for $G \in \mathrm{SM}$
- Yukawa couplings and Weinberg operator are allowed

Result: N divides 24 and:

N	$q_{\mathbf{10}}$	$q_{\overline{\mathbf{5}}}$	q_H	$q_{\bar{H}}$	ρ
4	1	1	0	0	1
6	5	3	4	0	0
8	1	5	0	4	1
12	5	9	4	0	3
24	5	9	16	12	9
3	2	0	1	0	0
6	2	0	4	0	0

(last two cases allow $\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}} \Rightarrow$ Proton decay)

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SO(10) universal charges for $\mathbb{Z}_N^R$

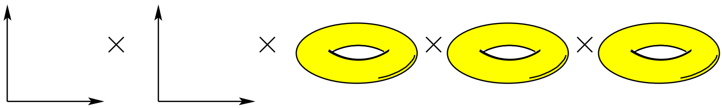
good $\mathbb{Z}_4^R$ of the MSSM is unique! good

- $\mathbb{Z}_4^R$ with SO(10) universal R-charges for matter
  Babu, Gogoladze, Wang
- It is: unique, forbidds Proton decay, allows Yukawas and neutrino masses and forbidds the μ-term
- $\rho = 1 \Rightarrow$ Green-Schwarz mechanism to cancel the anomaly
- Non-perturbative effects break $\mathbb{Z}_4^R$ to $\mathbb{Z}_2^R$
- Hence: dim. 5 Proton decay and μ-term at the non-pert. level
- Dangerous? See at explicit (string) model with $\mathbb{Z}_4^R$

The C++ orbifolder

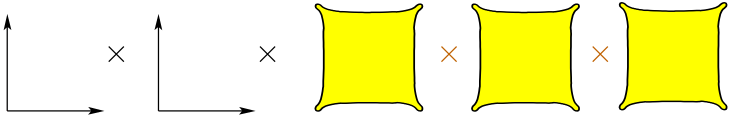
Part 2: The C++ orbifolder

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification



- 10d heterotic string on torus $T^6 = T^2 \times T^2 \times T^2$
- divided by discrete group $\mathbb{Z}_2 \times \mathbb{Z}_2$
- Closed strings:
  - Untwisted: gravity, gauge group and some matter
  - Twisted: matter
- Construct models using the C++ orbifolder

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification



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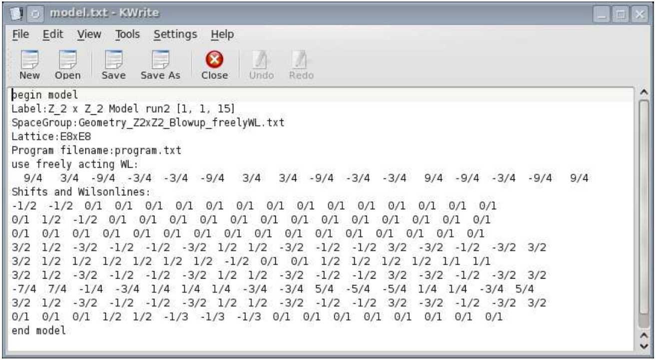
The C++ orbifolder

The C++ orbifolder

- automatically constructs inequivalent orbifold models,
- checks anomaly freedom (including Green-Schwarz mechanism) and
- offers linux-style command system to analyze the model.

The C++ orbifolder

The model defined in model.txt:

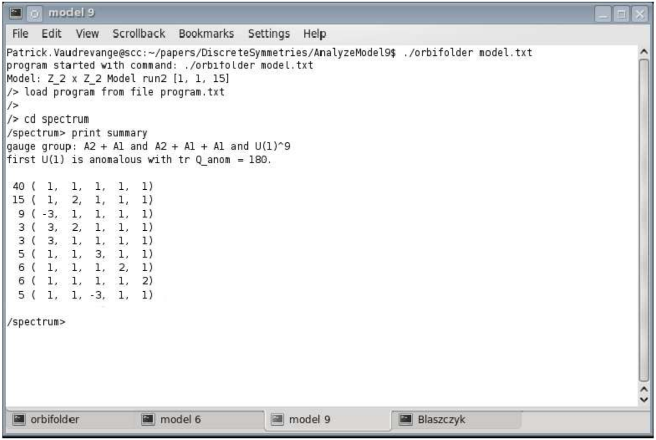


model.txt - KWrite
File Edit View Tools Settings Help
New Open Save Save As Close Undo Redo

```
begin model
Label:Z_2 x Z_2 Model run2 [1, 1, 15]
SpaceGroup:Geometry_Z2xZ2_Blowup_freelyWL.txt
Lattice:E8xE8
Program filename:program.txt
use freely acting WL:
  9/4   3/4  -9/4  -3/4  -3/4  -9/4   3/4   3/4  -9/4  -3/4  -3/4   9/4  -9/4  -3/4  -9/4   9/4
Shifts and Wilsonlines:
-1/2  -1/2  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1
0/1  1/2  -1/2  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1
0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1
3/2  1/2  -3/2  -1/2  -1/2  -3/2  1/2  1/2  -3/2  -1/2  -1/2  3/2  -3/2  -1/2  -3/2  3/2
3/2  1/2  1/2  1/2  1/2  1/2  1/2  -1/2  0/1  0/1  1/2  1/2  1/2  1/2  1/1  1/1
3/2  1/2  -3/2  -1/2  -1/2  -3/2  1/2  1/2  -3/2  -1/2  -1/2  3/2  -3/2  -1/2  -3/2  3/2
-7/4  7/4  -1/4  -3/4  1/4  1/4  1/4  -3/4  -3/4  5/4  -5/4  -5/4  1/4  1/4  -3/4  5/4
3/2  1/2  -3/2  -1/2  -1/2  -3/2  1/2  1/2  -3/2  -1/2  -1/2  3/2  -3/2  -1/2  -3/2  3/2
0/1  0/1  0/1  1/2  1/2  -1/3  -1/3  -1/3  0/1  0/1  0/1  0/1  0/1  0/1  0/1  0/1
end model
```

The C++ orbifolder

Load the model from file model.txt:



The C++ orbifolder

Show some data:

```
model 9
File  Edit  View  Scrollback  Bookmarks  Settings  Help
first U(1) is anomalous with tr Q_anom = 180.

 40 (  1,  1,  1,  1,  1)
 15 (  1,  2,  1,  1,  1)
  9 ( -3,  1,  1,  1,  1)
  3 (  3,  2,  1,  1,  1)
  3 (  3,  1,  1,  1,  1)
  5 (  1,  1,  3,  1,  1)
  6 (  1,  1,  1,  2,  1)
  6 (  1,  1,  1,  1,  2)
  5 (  1,  1, -3,  1,  1)

/spectrum> cd ..
/> cd model
/model> print shifts
Shift V_1:
(-0.50 -0.50  0.00  0.00  0.00  0.00  0.00  0.00 ) ( 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00 )
Shift V_2:
( 0.00  0.50 -0.50  0.00  0.00  0.00  0.00  0.00 ) ( 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00 )
/model> print Wilson lines
Wilson line A_2: Order 2
( 1.50  0.50 -1.50 -0.50 -0.50 -1.50  0.50  0.50 ) (-1.50 -0.50 -0.50  1.50 -1.50 -0.50 -1.50  1.50 )
Wilson line A_3: Order 2
( 1.50  0.50  0.50  0.50  0.50  0.50  0.50 -0.50 ) ( 0.00  0.00  0.50  0.50  0.50  0.50  1.00  1.00 )
Wilson line A_4: Order 2
( 1.50  0.50 -1.50 -0.50 -0.50 -1.50  0.50  0.50 ) (-1.50 -0.50 -0.50  1.50 -1.50 -0.50 -1.50  1.50 )
Wilson line A_5: Order 2
(-1.75  1.75 -0.25 -0.75  0.25  0.25  0.25 -0.75 ) (-0.75  1.25 -1.25 -1.25  0.25  0.25 -0.75  1.25 )
Wilson line A_6: Order 2
( 1.50  0.50 -1.50 -0.50 -0.50 -1.50  0.50  0.50 ) (-1.50 -0.50 -0.50  1.50 -1.50 -0.50 -1.50  1.50 )

/model> 
orbifolder   model 6   model 9   Blaszczyk
```

The C++ orbifolder

Create all trilinear couplings and the mass matrix for some exotics:

model 9

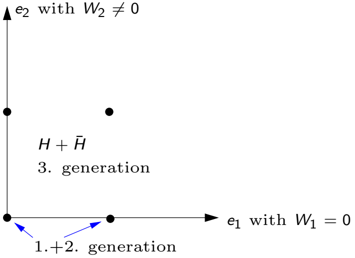
File Edit View Scrollback Bookmarks Settings Help

```
Shift V_1:
(-0.50 -0.50  0.00  0.00  0.00  0.00  0.00  0.00 ) ( 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00 )
Shift V_2:
( 0.00  0.50 -0.50  0.00  0.00  0.00  0.00  0.00 ) ( 0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00 )
/model> print Wilson lines
Wilson line A_2: Order 2
( 1.50  0.50 -1.50 -0.50 -0.50 -1.50  0.50  0.50 ) (-1.50 -0.50 -0.50  1.50 -1.50 -0.50 -1.50  1.50 )
Wilson line A_3: Order 2
( 1.50  0.50  0.50  0.50  0.50  0.50  0.50 -0.50 ) ( 0.00  0.00  0.50  0.50  0.50  0.50  1.00  1.00 )
Wilson line A_4: Order 2
( 1.50  0.50 -1.50 -0.50 -0.50 -1.50  0.50  0.50 ) (-1.50 -0.50 -0.50  1.50 -1.50 -0.50 -1.50  1.50 )
Wilson line A_5: Order 2
(-1.75  1.75 -0.25 -0.75  0.25  0.25  0.25 -0.75 ) (-0.75  1.25 -1.25 -1.25  0.25  0.25 -0.75  1.25 )
Wilson line A_6: Order 2
( 1.50  0.50 -1.50 -0.50 -0.50 -1.50  0.50  0.50 ) (-1.50 -0.50 -0.50  1.50 -1.50 -0.50 -1.50  1.50 )

/model> cd ..
/> cd yukawa
/yukawa> create coupling(* * *)
new child process: PID 13352 from command: create coupling(* * *)
/yukawa>
PID 13352 done: create coupling(* * *) TIME: 00:00:01
86 Yukawa couplings created.
/yukawa> mass matrix(D bD)

mass matrix: D_i M_ij bD_j      (3 x 6 matrix)
0       0       0       0       s^1     s^1
0       s^1     0       0       s^1     0
0       s^1     0       0       0       s^1

mass matrix saved as number 0.
/yukawa>
```

orbifolder | model 6 | model 9 | Blaszczyk

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification and $\mathbb{Z}_4^R$

6d intermediate SU(6) GUT:



- 1. + 2. generation form D_4 doublet
- 3. generation and Higgs-pair in the 6d bulk

$$\Rightarrow \quad \mu \sim \langle \mathcal{W} \rangle \sim m_{3/2}$$

Kappl, Nilles, Ramos-Sánchez, Ratz, Schmidt-Hoberg and P.V.
Brümmer, Kappl, Ratz and Schmidt-Hoberg

- From 10d Lorentz symmetry $\Rightarrow \mathbb{Z}_4^R$

Conclusion

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- Anomaly-free $\mathbb{Z}_N$ (R) symmetries of the MSSM, allowing for Green-Schwarz mechanism
- First, SU(5) universal charges for matter
- Avoid Proton decay and μ-problem
  $\Rightarrow \mathbb{Z}_N^R$-symmetry with N divides 24
- SO(10) universal charges for matter $\Rightarrow$ unique $\mathbb{Z}_4^R$
- C++ orbifolder
- $\mathbb{Z}_4^R$ arises naturally in $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds