A Systematic, Statistical Search of Gauge Content in Free-Fermionic Heterotic String Models

Douglas Moore

05 Nov 2010

Outline

- Classification of Calabi-Yau Geometries
- String Model-Building
- Moduli Stabilization and Supersymmetry Breaking
- Low-Energy String Theory and Warped Compactifications
- String Cosmology
- Tabulation of String Vacua Within Classes of String Theories
- Surveys of String Theory Landscapes
- The Nature of Neutrinos in String Models
- Exotic Particles and Extensions of the SM Gauge Group
- Geometrical Analysis of Supersymmetric Field Theories

- Classification of Calabi-Yau Geometries
- String Model-Building
- Moduli Stabilization and Supersymmetry Breaking
- Low-Energy String Theory and Warped Compactifications
- String Cosmology
- Tabulation of String Vacua Within Classes of String Theories
- Surveys of String Theory Landscapes
- The Nature of Neutrinos in String Models
- Exotic Particles and Extensions of the SM Gauge Group
- Geometrical Analysis of Supersymmetric Field Theories

- Classification of Calabi-Yau Geometries
- String Model-Building
- Moduli Stabilization and Supersymmetry Breaking
- Low-Energy String Theory and Warped Compactifications
- String Cosmology
- Tabulation of String Vacua Within Classes of String Theories
- Surveys of String Theory Landscapes
- The Nature of Neutrinos in String Models
- Exotic Particles and Extensions of the SM Gauge Group
- Geometrical Analysis of Supersymmetric Field Theories

- Classification of Calabi-Yau Geometries
- String Model-Building
- Moduli Stabilization and Supersymmetry Breaking
- Low-Energy String Theory and Warped Compactifications
- String Cosmology
- Tabulation of String Vacua Within Classes of String Theories
- Surveys of String Theory Landscapes
- The Nature of Neutrinos in String Models
- Exotic Particles and Extensions of the SM Gauge Group
- Geometrical Analysis of Supersymmetric Field Theories

- Classification of Calabi-Yau Geometries
- String Model-Building
- Moduli Stabilization and Supersymmetry Breaking
- Low-Energy String Theory and Warped Compactifications
- String Cosmology
- Tabulation of String Vacua Within Classes of String Theories
- Surveys of String Theory Landscapes
- The Nature of Neutrinos in String Models
- Exotic Particles and Extensions of the SM Gauge Group
- Geometrical Analysis of Supersymmetric Field Theories

- Classification of Calabi-Yau Geometries
- String Model-Building
- Moduli Stabilization and Supersymmetry Breaking
- Low-Energy String Theory and Warped Compactifications
- String Cosmology
- Tabulation of String Vacua Within Classes of String Theories
- Surveys of String Theory Landscapes
- The Nature of Neutrinos in String Models
- Exotic Particles and Extensions of the SM Gauge Group
- Geometrical Analysis of Supersymmetric Field Theories

Objectives of These Studies

- Systematic landscape surveys.
  - Fixed GSO-coefficients.
  - Fixed Basis Vectors.
  - Varied GSO-coefficients and Basis Vectors.
- Flexibility
  - A software framework specifically designed for gauge content searches.
  - The software should scale and be adaptable.
  - Longevity.
- Speed
  - Show that systematic searches, especially of low Layer-Order combinations are viable.
  - Pique interest in such approaches.

Free-Fermionic Heterotic String Models

- Heterotic Strings
  - Left- and Right- moving modes of a closed string are independent.
  - Left-movers are supersymmetric.
  - Right-movers are bosonic.
- What is needed to build a model?
  - A set of L basis vectors.
  - An $L \times L$ GSO coefficient matrix.

FF Basis Vectors

Free-Fermionic basis vectors are really boundary conditions. Working in light-cone gauge...

- Left-Movers
  - 8 Real Fermions in $D = 10$
  - Add 2 Real Fermions for each compactified dimension.
  - Total of 20 Real Fermions in $D = 4$
  - Complexifying gives us 10 Complex Fermions.
- Right-Movers
  - 32 Real Fermions in $D = 10$
  - Add 2 Real Fermions for each compactified dimension.
  - Total of 44 Real Fermions in $D = 4$
  - Complexifying to 22 Complex Fermions

Sectors and Charges

Within the Free-Fermionic formalism, we consider transport of fermions around non-contractable loops on the world-sheet.
Consistency requires:

$$\psi_j \to -e^{i\pi V_j^i} \psi_j$$

with $V_j^i \in (-1, 1] \cap \mathbb{Q}$. With 6 compactified dimensions, this is a 32 dimensional vector in the complex basis. We can write these sectors as linear combinations of basis vectors,

$$\vec{\boldsymbol{V}}^i = \sum_{i=1}^{L} m_j^i \vec{\boldsymbol{\alpha}}^j$$

with m_j^i are integers from 0 to $N_j - 1$. N_i is the smallest integer such that $N_i \vec{\boldsymbol{\alpha}}^i = 0 \pmod 2$.
We can then find the charges to be

$$\vec{\boldsymbol{Q}}^i = \frac{1}{2} \vec{\boldsymbol{V}}^i + \vec{\boldsymbol{F}}^i$$

Modular Invariance

We need the world-sheet to be modular invariant. That is, transformations under the group $SL(2\mathbb{Z})/\mathbb{Z}_2$ leave the physics invariant.

Within the context of FFHS models, this results in the following conditions on the basis vectors:

$$N_i \; \vec{\boldsymbol{\alpha}}^i \cdot \vec{\boldsymbol{\alpha}}^i = 0 \pmod{\left\{ \begin{array}{c} 8 \\ 4 \end{array} \right\}}$$
$$N_{ij} \; \vec{\boldsymbol{\alpha}}^i \cdot \vec{\boldsymbol{\alpha}}^j = 0 \pmod{4}$$

where $N_{ij} = \mathsf{LCM}(N_i, N_j)$.

Note that if you have a 32 dimensional vector, each of these constraints requires 64+ operations. If you knew a set of basis vectors were modular invariant you wouldn't have to apply these equations to the set.

The GSO Projection and k_{ij}-Matrix

As mentioned, the bosonic ground state is problematic: it is tachyonic. We need to get rid of this state if we want the model to be viable. Hence, the GSO projection:

$$\vec{\boldsymbol{\alpha}}_i \cdot \vec{\boldsymbol{Q}}^j = \sum_{n=1}^{L} m_n^j k_{in} + s_i \pmod 2$$

k_{ij} is the GSO coefficient matrix. Sadly, it also must satisfy modular invariance constraints.

$$k_{ij} + k_{ji} = \frac{1}{2}\vec{\boldsymbol{\alpha}}_i \cdot \vec{\boldsymbol{\alpha}}_j \pmod 2$$
$$k_{ii} + k_{i1} = \frac{1}{4}\vec{\boldsymbol{\alpha}}_i \cdot \vec{\boldsymbol{\alpha}}_i + s_i \pmod 2$$
$$N_j k_{ij} = 0 \pmod 2$$

Gauge Searches

From the charges you can delve into the gauge and matter content.

Since we will be dealing only with gauge content:

- We will work in the complex basis $\rightarrow$ 10 left-movers, 22 right-movers.
- Our left-moving boundary conditions are 0.
- We need both the left- and right-movers to be massless

$$\begin{aligned} \alpha' m^2_{\text{left}} &= \frac{1}{2}(\vec{Q}^i_{\text{left}})^2 - \frac{1}{2} \\ \alpha' m^2_{\text{right}} &= \frac{1}{2}(\vec{Q}^i_{\text{right}})^2 - 1 \end{aligned} \implies \begin{aligned} (\vec{Q}^i_{\text{left}})^2 &= 1 \\ (\vec{Q}^i_{\text{right}})^2 &= 2 \end{aligned}$$

- And the GSO Projection becomes

$$\vec{\alpha}_i \cdot \vec{Q}^j = \sum_{n=1}^{L} m^j_n k_{in} \pmod 2$$

$$N_j k_{ij} = 0 \pmod 2$$

More About Basis Vectors

A set of basis vectors can be described in terms of the number of layers, L, and the order N_i associated with the i^{th} basis vector, $\vec{\boldsymbol{\alpha}}^i$.

The structure of a basis vector is such that each element it a rational number, with an even numerator and all of the elements have the same denominator:

$$\alpha^i_j \in \frac{2\mathbb{Z}}{N_i} \cap (-1, 1]$$

Examples:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & \vec{\mathbf{0}}^{18} \end{pmatrix}, \ (N = 2)$$

$$\begin{pmatrix} 1 & 1 & \vec{\mathbf{0}}^{18} & 1 & 1 \end{pmatrix}, \ (N = 2)$$

$$\begin{pmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{2}{3} & \vec{\mathbf{0}}^{19} \end{pmatrix}, \ (N = 3)$$

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \vec{\mathbf{0}}^{17} \end{pmatrix}, \ (N = 4)$$

Brute Force Approach

Now, we want to generate ALL of the layer, L, order N_i models.

Consider just the simplest possible set of models: $L1O2$.

- If we created every possible two order 2 basis vector, there are 2^{22} vectors.
- Modular invariance check will be ~ 45 calculations per basis vector.

Just considering those two number, around 10^{24} calculations to determine what basis vectors work. This does not reduce the permutation redundancies as noted previously.

- Around 1.2×10^6 order 2 basis vectors that are modular invariant.
- This takes about 3 s.

Higher order brute force approaches fail because of hardware constraints.

We will do the following to ensure that only modular invariant basis vectors are created:

- Choose a preferred ordering.
- Use the modular invariance constraints to determine the number of nonzero elements.

Our ordering convention:

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}^{n_0} \begin{pmatrix} 1 \\ -1 \end{pmatrix}^{n_1} \begin{pmatrix} 1 \\ 0 \end{pmatrix}^{n_2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}^{n_3} \begin{pmatrix} 0 \\ -1 \end{pmatrix}^{n_4}$$

Modular Invariance Constraints:

$$n_k = -\sum_{i=0}^{k-1} a_i^2 n_i + \left\{ \begin{array}{c} 2 \\ 1 \end{array} \right\} N_i \mathbb{Z}$$

$$n_l = -\sum_{i=0}^{l-1} a_i b_i n_i + \frac{N_i N_j}{N_{ij}} \mathbb{Z}$$

Improvements?

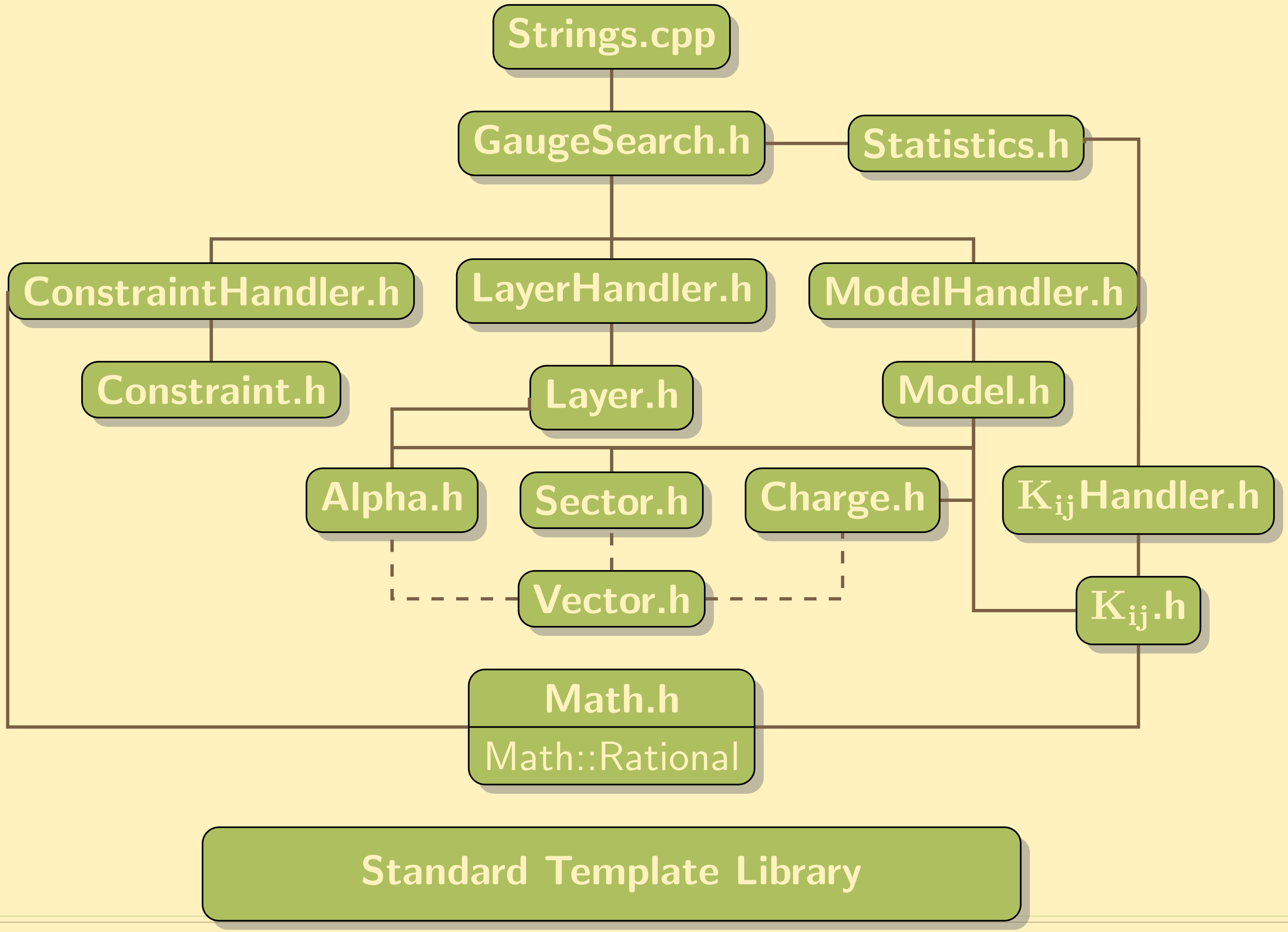
Table 1: Comparison of Basis Vector Generation Speeds

N_i	$\mathcal{N}_{BF}$	t_{BF}	$\mathcal{N}_{MI}$	t_{MI}
2	2^{22}	3	5	2×10^{-3}
3	3^{22}	N/A	47	2×10^{-3}
4	4^{22}	N/A	149	4×10^{-3}
5	5^{22}	N/A	873	10^{-2}
6	6^{22}	N/A	2021	3.2×10^{-2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
13	13^{22}	N/A	2×10^{6}	15

These are the time to build all modular invariant basis vectors in the two approaches, brute force and modular invariant.

Build time for the charges and determination of the gauge groups has not yet been determined.

Basic Structure



Current Status

Table 2: Layer 1 Runtimes

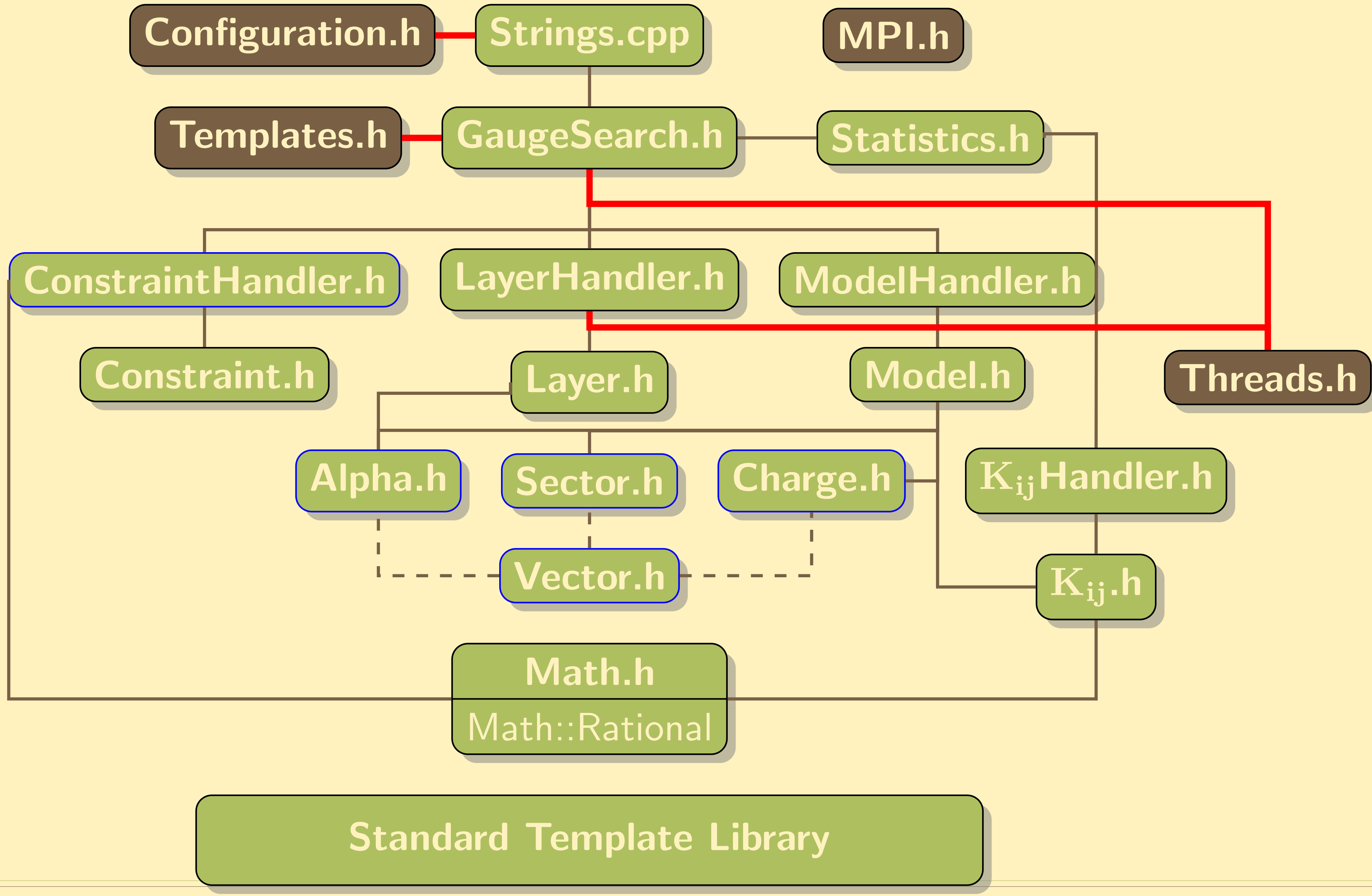
Orders	$\mathcal{N}_{\text{models}}$	t_{total}	f	f_{exp}
2	5	$0.024s$	208	3120
3	47	$0.183s$	252	3780
4	149	$0.734s$	202	3030
5	873	$3.637s$	240	3600
6	2021	$9.176s$	220	3300
7	9352	$41.34s$	226	3390
8	17604	$82.65s$	213	3195
9	70426	$358.9s$	196	2940
10	115109	$533.2s$	215	3225

Current Status - Con't.

Table 3: Layer 2 Runtimes

Orders	$\mathcal{N}_{\text{models}}$	t_{total}	f	f_{exp}
$2, 2$	70	$0.452s$	154	2323
$2, 3$	3626	$16.705s$	217	3255
$2, 4$	15923	$89.13s$	178	2670
$3, 3$	16092	94.425	170	2550

Future Development



Conclusion

- Systematic searches of regions of the string landscape, with regard to gauge content, is within the current technological limits.

- By generating only modular invariant basis vectors we have significantly lowered the computational requirements of FFHS model building.

References

M. Robinson, G. Cleaver, and M. Hunziker. Free fermionic heterotic model building and root systems. *MOD.PHYS.LETT.A*, 24:2703, 2009

Keith R. Dienes. Statistics on the heterotic landscape: Gauge groups and cosmological constants of four-dimensional heterotic strings. *Phys. Rev.*, D73:106010, 2006