INSTITUTE OF PHYSICS PUBLISHING REPORTS ON PROGRESS IN PHYSICS

Rep. Prog. Phys. **67** (2004) 755–811 PII: S0034-4885(04)36775-8

# Desperately seeking supersymmetry (SUSY)

**Stuart Raby**

Department of Physics, The Ohio State University, 174 W. 18th Ave., Columbus, OH 43210, USA

E-mail: raby@pacific.mps.ohio-state.edu

Received 13 January 2004
Published 27 April 2004
Online at stacks.iop.org/RoPP/67/755
DOI: 10.1088/0034-4885/67/5/R04

## Abstract

The discovery of x-rays and radioactivity in the waning years of the 19th century led to one of the most awe-inspiring scientific eras in human history. The 20th century witnessed a level of scientific discovery never before seen or imagined. At the dawn of the 20th century only two forces of nature were known—gravity and electromagnetism. The atom was believed by chemists to be the elemental, indestructible unit of matter, coming in many unexplainably different forms. Yet J J Thomson, soon after the discovery of x-rays, had measured the charge to mass ratio of the electron, demonstrating that this carrier of electric current was ubiquitous and fundamental. All electrons could be identified by their unique charge to mass ratio.

In the 20th century the mystery of the atom was unravelled, the atomic nucleus was smashed, and two new forces of nature were revealed—the weak force (responsible for radioactive $\beta$ decay and the nuclear fusion reaction powering the stars) and the nuclear force binding the nucleus. Quantum mechanics enabled the understanding of the inner structure of the atom, its nucleus and further inward to quarks and gluons (the building blocks of the nucleus) and thence outward to an understanding of large biological molecules and the unity of chemistry and microbiology.

Finally the myriad of new fundamental particles, including electrons, quarks, photons, neutrinos, etc and the three fundamental forces—electromagnetism and the weak and the strong nuclear forces—found a unity of description in terms of relativistic quantum field theory. These three forces of nature can be shown to be a consequence of symmetry rotations in internal spaces, and the particular interactions of each particle are solely determined by their symmetry charge. This unifying structure, describing all the present experimental observations, is known as the standard model (SM). Moreover, Einstein's theory of gravity can be shown to be a consequence of the symmetry of local translations and Lorentz transformations.

As early as the 1970s, it became apparent that two new symmetries, a grand unified theory of the strong, weak and electromagnetic interactions in conjunction with supersymmetry (SUSY), might unify all the known forces and particles into one unique structure.

Now 30 years later, at the dawn of a new century, experiments are on the verge of discovering (or ruling out) these possible new symmetries of nature. In this paper we try to clarify why SUSY and supersymmetric grand unified theories are the new SM of particle physics, i.e. the standard against which all other theories and experiments are measured.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

Supersymmetry (SUSY) is a space–time symmetry, an extension of the group of transformations known as the Poincaré group including space–time translations, spatial rotations and pure Lorentz transformations. The Poincaré transformations act on the three space coordinates, $\vec{x}$, and one time coordinate, $t$. The supersymmetric extension adds two anti-commuting complex coordinates $\theta_\alpha$, $\alpha = 1, 2$, satisfying $\theta_\alpha\theta_\beta + \theta_\beta\theta_\alpha = 0$. Together they make superspace, $z = \{t, \vec{x}, \theta_\alpha\}$, and supersymmetry transformations describe translations/rotations in superspace. Local SUSY implies a supersymmetrized version of Einstein's gravity known as supergravity.

In the standard model (SM), ordinary matter is made of quarks and electrons. All these particles are Fermions with spin $s = \frac{1}{2}\hbar$, satisfying the Pauli exclusion principle. Hence no two identical matter particles can occupy the same space at the same time. In field theory, they are represented by anti-commuting space–time fields we generically denote by $\psi_\alpha(\vec{x}, t)$. On the other hand, all the force particles, such as photons, gluons, $W^\pm$, $Z^0$, are so-called gauge Bosons with spin $s = 1\hbar$, satisfying Bose–Einstein statistics and represented by commuting fields $\phi(\vec{x}, t)$. As a result Bosons prefer to sit one on top of the other, thus enabling them to form macroscopic classical fields. A Boson–Fermion pair forms a supermultiplet that can be represented by a superfield $\Phi(z) = \phi(\vec{x}, t) + \theta\psi(\vec{x}, t)$. Hence a rotation in superspace rotates Bosons (force particles) into Fermions (matter particles) and vice versa.

This simple extension of ordinary space into two infinitesimal directions has almost miraculous consequences, making it one of the most studied possible extensions of the SM of particle physics. It provides a 'technical' solution to the so-called gauge hierarchy problem, i.e. why is $M_Z/M_{\rm pl} \ll 1$. In the SM, all matter derives its mass from the vacuum expectation value (VEV), $v$, of the Higgs field. The $W^\pm$ and $Z^0$ mass are of order $gv$, where $g$ is the coupling constant of the weak force, while quarks and leptons (the collective name for electrons, electron neutrinos and similar particles having no strong interactions) obtain mass of order $\lambda v$, where $\lambda$ is called a Yukawa coupling, a measure of the strength of the interaction between the Fermion and Higgs fields. The Higgs VEV is fixed by the Higgs potential and in particular by its mass, $m_H$, with $v \sim m_H$. The problem is that in quantum field theory, the Lagrangian (or bare) mass of a particle is subject to quantum corrections. Moreover, for Bosons, these corrections are typically large. This was already pointed out in the formative years of quantum field theory by Weisskopf (1939). In particular, for the Higgs we have $m_H^2 = m_0^2 + \alpha\Lambda^2$, where $m_0$ is the bare mass of the Higgs, $\alpha$ represents some small coupling constant and $\Lambda$ is typically the largest mass in the theory. In electrodynamics, $\alpha$ is the fine-structure constant and $\Lambda$ is the physical cutoff scale, i.e. the mass scale where new particles and their new interactions become relevant. For example, it is known that gravitational interactions become strong at the Planck scale, $M_{\rm pl} \sim 10^{19}$ GeV; hence we take $\Lambda \sim M_{\rm pl}$. In order to have $M_Z \ll M_{\rm pl}$, the bare mass must be fine-tuned to one part in $10^{17}$, order by order in perturbation theory, against the radiative corrections in order to preserve this hierarchy. This appears to be a particularly 'unnatural' accident or, as most theorists believe, an indication that the SM is incomplete. Note that neither Fermions nor gauge Bosons have this problem. This is because their mass corrections are controlled by symmetries. For Fermions these chiral symmetries become exact only when the Fermion mass vanishes. Moreover, with an exact chiral symmetry the radiative corrections to the Fermion's mass vanish to all orders in perturbation theory. As a consequence, when chiral symmetry is broken the Fermion mass corrections are necessarily proportional to the bare mass. Hence $m_F = m_0 + \alpha m_0 \log(\Lambda/m_0)$ and a light Fermion mass does not require any 'unnatural' fine-tuning. Similarly, for gauge Bosons, the local gauge symmetry prevents any non-zero corrections to the gauge Boson mass. As a consequence, massless gauge Bosons remain massless to all orders in perturbation theory. What can we expect in a supersymmetric theory? Since SUSY unifies Bosons and Fermions, the radiative mass corrections of the Bosons are controlled by the chiral symmetries of their Fermionic superpartners. Moreover, for every known Fermion with spin $\frac{1}{2}\hbar$, we have necessarily a spin 0 Boson (or Lorentz scalar) and for every spin $1\hbar$ gauge Boson, we have a spin $\frac{1}{2}\hbar$ gauge Fermion (or gaugino). Exact SUSY then requires Boson–Fermion superpartners to have identical mass. Thus in SUSY an electron necessarily has a spin 0 superpartner, a scalar electron, with the same mass. Is this a problem? The answer is Yes since the interaction of the scalar electron with all SM particles is determined by SUSY. In fact, the scalar electron necessarily has the same charge as the electron under all SM local gauge symmetries. Thus it has the same electric charge, and it would have been observed long ago. We thus realize that SUSY can only be an approximate symmetry of nature. Moreover, it must be broken in such a way as to raise the mass of the scalar partners of all SM Fermions and the gaugino partners of all the gauge Bosons. This may seem like a tall order. But what would we expect to occur once SUSY is softly broken at a scale $\Lambda_{\mathrm{SUSY}}$? Then scalars are no longer protected by the chiral symmetries of their Fermionic partners. As a consequence, they receive radiative corrections to their mass of order $\delta m^2 \propto \alpha \Lambda^2_{\mathrm{SUSY}} \log(\Lambda/m_0)$. As long as $\Lambda_{\mathrm{SUSY}} \leqslant 100\,\mathrm{TeV}$, the Higgs Boson can remain naturally light. In addition, the gauge Boson masses are still protected by gauge symmetries. The gauginos are special since; however, since even if SUSY is broken, gaugino masses may still be protected by a chiral symmetry known as $R$ symmetry (Farrar and Fayet 1979). Thus gaugino masses are controlled by both the SUSY and $R$ symmetry breaking scales.

Before we discuss SUSY theories further, let us first review the SM in some more detail. The SM of particle physics is defined almost completely in terms of its symmetry and the charges (or transformation properties) of the particles under this symmetry. In particular, the symmetry of the SM is $SU(3) \times SU(2) \times U(1)_Y$. It is a local, internal symmetry, by which we mean it acts on internal properties of states as a rotation by an amount that depends on the particular space–time point. Local symmetries demand the existence of gauge Bosons (or spin 1 force particles) such as the gluons of the strong $SU(3)$ interactions or the $W^{\pm}$, $Z^0$ or photon ($\gamma$) of the electroweak interactions $SU(2) \times U(1)_Y$. The strength of the interactions are determined by parameters called coupling constants. The values of these coupling constants however are not determined by the theory but must be fixed by experiment.

There are three families of matter particles, spin $\frac{1}{2}$ quarks and leptons, each family carrying identical SM symmetry charges. The first and lightest family contains the up ($u$) and down ($d$) quarks, the electron ($e$) and the electron neutrino ($\nu_e$) (the latter two are leptons). Two up quarks and one down quark bind via gluon exchange forces to make a proton, while one up and two down quarks make a neutron. Together different numbers of protons and neutrons bind via residual gluon and quark exchange forces to make nuclei, and finally nuclei and electrons bind via electromagnetic forces (photon exchanges) to make atoms, molecules and us. The strong forces are responsible for nuclear interactions. The weak forces on the other hand are responsible for nuclear $\beta$ decay. In this process typically a neutron decays, thereby changing into a proton, electron and electron neutrino. This is the so-called $\beta^-$ decay since the electron (or $\beta$ particle) has negative charge, $-e$. $\beta^+$ decays also occur where a proton (bound in the nucleus of an atom) decays into a neutron, anti-electron and electron neutrino. The anti-electron (or positron) has positive charge, $+e$, but mass identical to the electron. If the particle and anti-particle meet, they annihilate, or disappear completely, converting their mass into pure energy in the form of two photons. The energy of the two photons is equal to the energy of the particle–anti-particle pair, which includes the rest mass of both. Nuclear fusion reactions where two protons combine to form deuterium (a p–n bound state), $e^+ + \nu_e$, is the energy source for stars like our sun and the energy source of the future on earth. Weak forces occur very rarely because they require exchange of $W^{\pm}$, $Z^0$, which are 100 times more massive than the proton or neutron.

The members of the third family, $\{t, b, \tau, \nu_\tau\}$, are heavier than the second family, $\{c, s, \mu, \nu_\mu\}$, which are heavier than the first family members $\{u, d, e, \nu_e\}$. Why there are three copies of families and why they have the apparent hierarchy of masses is a mystery of the SM. In addition, why each family has the following observed charges is also a mystery. A brief word about the notation. Quarks and leptons have four degrees of freedom each (except for the neutrinos, which in principle may only have two degrees of freedom) corresponding to a left- or right-handed particle or anti-particle. The field labelled $e$ contains a left-handed electron and a right-handed anti-electron, while $\bar{e}$ contains a left-handed anti-electron and a right-handed electron. Thus all four degrees of freedom are naturally (this is in accord with Lorentz invariance) contained in two independent fields, $e$, $\bar{e}$. This distinction is a property of nature since the charges of the SM particles depend on their handedness. In fact in each family we have five different charge multiplets, given by

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad \bar{u} \quad \bar{d} \quad L = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \bar{e}, \tag{1}$$

where $Q$ is a triplet under colour $SU(3)$ and a doublet under weak $SU(2)$ and carries $U(1)_Y$ weak hypercharge, $Y = \frac{1}{3}$. The colour anti-triplets $(\bar{u}, \bar{d})$ are singlets under $SU(2)$ with $Y = (-\frac{4}{3}, \frac{2}{3})$, and finally the leptons appear as a electroweak doublet ($L$) and singlet ($\bar{e}$) with $Y = -1, +2$, respectively. Note, by definition, leptons are colour singlets and thus do not feel the strong forces. The electric charge for all the quarks and leptons is given by the relation $Q_{\rm EM} = T_3 + Y/2$, where the (upper, lower) component of a weak doublet has $T_3 = (+\frac{1}{2}, -\frac{1}{2})$. Finally the Higgs boson multiplet,

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \tag{2}$$

with $Y = +1$, is necessary to give mass to the $W^{\pm}$, $Z^0$ and to all quarks and leptons. In the SM vacuum, the field $h^0$ obtains a non-zero VEV $\langle h^0 \rangle = v/\sqrt{2}$. Particle masses are then determined by the strength of the coupling to the Higgs. The peculiar values of the quark, lepton and Higgs charges is one of the central unsolved puzzles of the SM. The significance of this problem only becomes clear when one realizes that the interactions of all the particles (quarks, leptons, and Higgs bosons), via the strong and electroweak forces, are completely fixed by these charges.

Let us now summarize the list of fundamental parameters needed to define the SM. If we do not include gravity or neutrino masses, then the SM has 19 fundamental parameters. These include the $Z^0$ and Higgs masses ($M_Z$, $m_h$), setting the scale for electroweak physics, the three gauge couplings $\alpha_i(M_Z)$, $i = 1, 2, 3$, the nine charged Fermion masses and four quark mixing angles. Lastly, there is the QCD theta parameter which violates CP and thus is experimentally known to be less than $\approx 10^{-10}$. Gravity adds one additional parameter, Newton's constant $G_N = 1/M_{\rm Pl}^2$ or equivalently the Planck scale. Finally neutrino masses and mixing angles have been observed definitively in many recent experiments measuring solar and atmospheric neutrino oscillations and by measuring carefully reactor or accelerator neutrino fluxes. The evidence for neutrino masses and flavour violation in the neutrino sector has little controversy. It is the first strong evidence for new physics beyond the SM. We shall return to these developments later. Neutrino masses and mixing angles are described by nine new fundamental parameters—three masses, three real mixing angles and three CP violating phases.

Let us now consider the minimal supersymmetric standard model (MSSM). It is defined by the following two properties: (i) the particle spectrum and (ii) their interactions.

(i) Every matter Fermion of the SM has a bosonic superpartner. In addition, every gauge boson has a fermionic superpartner. Finally, while the SM has one Higgs doublet, the MSSM has two Higgs doublets.

$$H_u = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}, \qquad H_d = \begin{pmatrix} \bar{h}^0 \\ \bar{h}^- \end{pmatrix} \tag{3}$$

with $Y = +1, -1$. The two Higgs doublets are necessary to give mass to up quarks, and to down quarks and charged leptons, respectively. The VEVs are now given by $\langle h^0 \rangle = v \sin \beta/\sqrt{2}$, $\langle \bar{h}^0 \rangle = v \cos \beta/\sqrt{2}$, where $\tan \beta$ is a new free parameter of the MSSM.

(ii) The MSSM has the discrete symmetry called $R$ parity[1]. All SM particles are $R$ parity even, while all superpartners are $R$ odd. This has two important consequences.

- The lightest superpartner (LSP) is absolutely stable since the lightest state with odd $R$ parity cannot decay into only even $R$ parity states. Assuming that the LSP is electrically neutral, it is a weakly interacting massive particle. Hence it is a very good candidate for the dark matter of the universe.
- Perhaps more importantly, the interactions of all superpartners with SM particles is determined completely by SUSY and the observed interactions of the SM. Hence, though we cannot predict the masses of the superpartners, we know exactly how they interact with SM particles.

The MSSM has some very nice properties. It is perturbative and easily consistent with all precision electroweak data. In fact global fits of the SM and the MSSM provide equally good fits to the data (de Boer and Sander 2003). Moreover, as the SUSY particle masses increase, they decouple from low energy physics. On the other hand, their masses cannot increase indefinitely since one soon runs into problems of 'naturalness'. In the SM the Higgs Boson has a potential with a negative mass squared, of the order of the $Z^0$ mass, and an arbitrary quartic coupling. The quartic coupling stabilizes the vacuum value of the Higgs. In the MSSM the quartic coupling is fixed by SUSY in terms of the electroweak gauge couplings. As a result of this strong constraint, at tree level the light Higgs boson mass is constrained to be lighter than $M_Z$. One-loop corrections to the Higgs mass are significant. Nevertheless the Higgs mass is bounded to be lighter than about 135 GeV (Ellis *et al* 1991, Okada *et al* 1991, Casas *et al* 1995, Carena *et al* 1995,1996, Haber *et al* 1997, Zhang 1999, Espinosa and Zhang 2000a,b, Degrassi *et al* 2003). The upper bound is obtained in the limit of large $\tan \beta$.

It was shown early on that even if the tree level Higgs mass squared was positive, radiative corrections due to a large top quark Yukawa coupling are sufficient to drive the Higgs mass squared negative (Ibañez and Ross 1982, Alvarez-Gaume *et al* 1983, Ibañez and Ross 1992). Thus, radiative corrections lead naturally to electroweak symmetry breaking at a scale determined by squark and slepton SUSY breaking masses. Note that, a large top quark Yukawa coupling implies a heavy top quark. Early predictions for a top quark with mass above 50 GeV (Ibañez and Lopez 1983) were soon challenged by the announcement of the discovery of the top quark by UA1 with a mass of 40 GeV. Of course, this false discovery was followed much later by the discovery of the top quark at Fermilab with a mass of the order of 175 GeV.

[1] One may give up $R$ parity at the expense of introducing many new interactions with many new arbitrary couplings into the MSSM. These interactions violate either baryon or lepton number. Without $R$ parity the LSP is no longer stable. There are many papers that give limits on these new couplings. The strongest constraint is on the product of couplings for the dimension 4 baryon and lepton number violating operators that contribute to proton decay. We do not discuss $R$ parity violation further in this review.

If the only virtue of SUSY is to explain why the weak scale $(M_Z, m_h)$ is so much less than the Planck scale, one might ponder whether the benefits outweigh the burden of doubling the SM particle spectrum. Moreover, there are many other ideas addressing the hierarchy problem, such as Technicolor theories with new strong interactions at a teraelectronvolt scale. One particularly intriguing possibility is that the universe has more than three spatial dimensions. In these theories the fundamental Planck scale, $M_*$, is near 1 TeV, and so there is no apparent hierarchy. I say 'apparent' since in order to have the observed Newton's constant, $1/M_{\mathrm{Pl}}^2$, much smaller than $1/M_*^2$, one needs a large extra dimension such that the gravitational lines of force can probe the extra dimension. If we live on a three-dimensional brane in this higher dimensional space, then at large distances compared with the size of the $d$ extra dimensions we will observe an effective Newton's constant given by $G_N = 1/M_{\mathrm{Pl}}^2 = 1/(R^d M_*^{d+2})$ (Arkani-Hamed *et al* 1998). For example, with $d = 2$ and $M_* = 1$ TeV, we need the radius of the extra dimension $R \approx 1$ mm. If any of these new scenarios with new strong interactions at a teraelectronvolt scale[2] are true, then we should expect a plethora of new phenomena occurring at the next generation of high energy accelerators, i.e. the large hadron collider (LHC) at CERN. It is thus important to realize that SUSY does much more. It provides a framework for understanding the 16 parameters of the SM associated with gauge and Yukawa interactions and also the nine parameters in the neutrino sector. This will be discussed in the context of supersymmetric grand unified theories (SUSY GUTs) and family symmetries. As we will see, these theories are very predictive and will soon be tested at high energy accelerators or underground detectors. We will elaborate further on this below. Finally, it is also incorporated naturally into string theory, which provides a quantum mechanical description of gravity. Unfortunately, this last virtue is apparently true for all the new ideas proposed for solving the gauge hierarchy problem.

A possible subtitle for this article could be 'A Tale of Two Symmetries: SUSY GUTs'. Whereas SUSY by itself provides a framework for solving the gauge hierarchy problem, i.e. why $M_Z \ll M_{\mathrm{GUT}}$, SUSY GUTs (with the emphasis on GUTs) adds the framework for understanding the relative strengths of the three gauge couplings and for understanding the puzzle of charge and mass. It also provides a theoretical lever arm for uncovering the physics at the Planck scale with experiments at the weak scale. Without any exaggeration, it is safe to say that SUSY GUTs also address the following problems.

- They explain charge quantization since weak hypercharge $(Y)$ is imbedded in a non-abelian symmetry group.
- They explain the family structure and in particular the peculiar colour and electroweak charges of Fermions in one family of quarks and leptons.
- They predict gauge coupling unification. Thus, given the experimentally determined values of two gauge couplings at the weak scale, one predicts the value of the third. The experimental test of this prediction is the one major success of SUSY theories. It relies on the assumption of SUSY particles with mass in the 100 GeV to 1 TeV range. Hence it predicts the discovery of SUSY particles at the LHC.
- They predict Yukawa coupling unification for the third family. In $SU(5)$ we obtain $b$–$\tau$ unification, while in $SO(10)$ we have $t$–$b$–$\tau$ unification. We shall argue that the latter prediction is eminently testable at the Tevatron, the LHC or a possible next linear collider.
- With the addition of family symmetry they provide a predictive framework for understanding the hierarchy of Fermion masses.

[2] Field theories in extra dimensions are divergent and require new non-perturbative physics, perhaps string theory, at the teraelectronvolt scale.

- It provides a framework for describing the recent observations of neutrino masses and mixing. At zeroth order, the seesaw scale for generating light neutrino masses probes physics at the GUT scale.
- The LSP is one of the best motivated candidates for dark matter. Moreover back-of-the-envelope calculations of LSPs, with mass of order 100 GeV and annihilation cross-sections of the order of 11 TeV$^2$, give the right order of magnitude of their cosmological abundance for LSPs to be dark matter. More detailed calculations agree. Underground dark matter detectors will soon probe the mass/cross-section region in the LSP parameter space.
- Finally, the cosmological asymmetry of baryons versus anti-baryons can be explained via the process known as leptogenesis (Fukugita and Yanagida 1986). In this scenario an initial lepton number asymmetry, generated by the out-of-equilibrium decays of heavy Majorana neutrinos, leads to a net baryon number asymmetry today.

Grand unified theories are the natural extension of the SM. Ever since it became clear that quarks are the fundamental building blocks of all strongly interacting particles, protons, neutrons, pions, kaons, etc, and that they appear to be just as elementary as leptons, it was proposed (Pati and Salam 1973a,b, 1974) that the strong $SU(3)$ colour group should be extended to $SU(4)$ colour, with lepton number as the fourth colour.

$$\boldsymbol{G}_{\text{Pati–Salam}} \equiv \boldsymbol{SU_4}(\text{colour}) \times \boldsymbol{SU_2(L)} \times \boldsymbol{SU_2(R)}$$

$$\begin{pmatrix} u & \nu_e \\ d & e \end{pmatrix}, \qquad \begin{pmatrix} \bar{u} & \bar{\nu}_e \\ \bar{d} & \bar{e} \end{pmatrix}, \tag{4}$$

$$(H_u \quad H_d). \tag{5}$$

In the Pati–Salam (PS) model, quarks and leptons of one family are united into two irreducible representations (equation (4)). The two Higgs doublets of the MSSM sit in one irreducible representation (equation (5)). This has significant consequences for Fermion masses, as we discuss later. However, the gauge groups are not unified and there are still three independent gauge couplings, or two if one enlarges the PS with a discrete parity symmetry where $L \leftrightarrow R$. The PS must be broken spontaneously to the SM at some large scale $M_G$. Below the PS breaking scale, the three low energy couplings, $\alpha_i$, $i = 1, 2, 3$, renormalize independently. Thus with the two gauge couplings and the scale $M_G$, one can fit the three low energy couplings.

Shortly after PS, the completely unified gauge symmetry $\boldsymbol{SU_5}$ was proposed (Georgi and Glashow 1974). Quarks and leptons of one family sit in two irreducible representations.

$$\left\{ Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad e^c \quad u^c \right\} \subset \mathbf{10}, \tag{6}$$

$$\left\{ d^c \quad L = \begin{pmatrix} \nu \\ e \end{pmatrix} \right\} \subset \bar{\mathbf{5}}. \tag{7}$$

The two Higgs doublets necessarily receive colour triplet $SU(5)$ partners filling out 5, $\bar{5}$ representations.

$$\begin{pmatrix} H_u \\ T \end{pmatrix}, \quad \begin{pmatrix} H_d \\ \bar{T} \end{pmatrix} \subset \mathbf{5_H}, \quad \bar{\mathbf{5}}_{\mathbf{H}}. \tag{8}$$

As a consequence of complete unification, the three low energy gauge couplings are given in terms of only two independent parameters, the one unified gauge coupling $\alpha_G(M_G)$ and the unification (or equivalently the $SU(5)$ symmetry breaking ) scale $M_G$ (Georgi *et al* 1974). Hence there is one prediction. In addition, we now have the dramatic prediction that a proton is unstable to decay into a $\pi^0$ and a positron, $e^+$.

**Table 1.** This table gives the particle spectrum for the 16-dimensional spinor representation of $SO(10)$. The states are described in terms of the tensor product of five spin $\frac{1}{2}$ states with spin up (+) or down ($-$) and in addition having an even number of ($-$) spins. $\{r, b, y\}$ are the three colour quantum numbers, and $Y$ is weak hypercharge given in terms of the formula $\frac{2}{3}\Sigma(C) - \Sigma(W)$, where the sum ($\Sigma$) is over all colour and weak spins with values ($\pm\frac{1}{2}$). Note that, an $SO(10)$ rotation corresponds either to raising one spin and lowering another or raising (or lowering) two spins. In the table, the states are arranged in $SU(5)$ multiplets. One sees readily that the first operation of raising one spin and lowering another is an $SU(5)$ rotation, while the others are special to $SO(10)$.

| State | $Y = \frac{2}{3}\Sigma(C) - \Sigma(W)$ | Colour $C$ spins | Weak $W$ spins |
|---|---|---|---|
| $\bar{\nu}$ | 0 | $+++$ | $++$ |
| $\bar{e}$ | 2 | $+++$ | $--$ |
| $u_r$ | | $-++$ | $+-$ |
| $d_r$ | | $-++$ | $-+$ |
| $u_b$ | $\frac{1}{3}$ | $+-+$ | $+-$ |
| $d_b$ | | $+-+$ | $-+$ |
| $u_y$ | | $++-$ | $+-$ |
| $d_y$ | | $++-$ | $-+$ |
| $\bar{u}_r$ | | $+--$ | $++$ |
| $\bar{u}_b$ | $-\frac{4}{3}$ | $-+-$ | $++$ |
| $\bar{u}_y$ | | $--+$ | $++$ |
| $\bar{d}_r$ | | $+--$ | $--$ |
| $\bar{d}_b$ | $\frac{2}{3}$ | $-+-$ | $--$ |
| $\bar{d}_y$ | | $--+$ | $--$ |
| $\nu$ | $-1$ | $---$ | $+-$ |
| $e$ | | $---$ | $-+$ |

Finally, complete gauge and family unification occurs in the group $\boldsymbol{SO_{10}}$ (Georgi 1974, Fritzsch and Minkowski 1974); with one family contained in one irreducible representation,

$$\mathbf{10} + \bar{\mathbf{5}} + \bar{\nu} \subset \mathbf{16} \tag{9}$$

and the two multiplets of Higgs unified as well.

$$\mathbf{5_H}, \quad \bar{\mathbf{5}}_\mathbf{H} \subset \mathbf{10_H}, \tag{10}$$

(see table 1).

GUTs predict that protons decay with a lifetime, $\tau_p$, of order $M_G^4/\alpha_G^2 m_p^5$. The first experiments looking for proton decay were begun in the early 1980s. However at the very moment that proton decay searches began, motivated by GUTs, it was shown that SUSY GUTs naturally increase $M_G$, thus increasing the proton lifetime. Hence, if SUSY GUTs were correct, it was unlikely that the early searches would succeed (Dimopoulos and Georgi 1981, Dimopoulos *et al* 1981, Ibañez and Ross 1981, Sakai 1981, Einhorn and Jones 1982, Marciano and Senjanovic 1982). At the same time, it was shown that SUSY GUTs did not affect significantly the predictions for gauge coupling unification (for a review see (Dimopoulos *et al* 1991, Raby 2002a). At present, non-SUSY GUTs are excluded by the data for gauge coupling unification, whereas SUSY GUTs work quite well: so well in fact, that the low energy data is now probing the physics at the GUT scale. In addition, the experimental bounds on proton decay from Super-Kamiokande exclude non-SUSY GUTs, while severely testing SUSY GUTs. Moreover, future underground proton decay/neutrino observatories, such as the proposed Hyper-Kamiokande detector in Japan or UNO in the USA, will cover the entire allowed range for the proton decay rate in SUSY GUTs.

If SUSY is so great, if it is nature, then where are the SUSY particles? Experimentalists at high energy accelerators, such as the Fermilab Tevatron and the CERN LHC (now under construction), are desperately seeking SUSY particles or other signs of SUSY. At underground proton decay laboratories, such as Super-Kamiokande in Japan or Soudan II in Minnesota, USA, electronic eyes continue to look for the tell-tale signature of a proton or neutron decay. Finally, they are searching for cold dark matter, via direct detection in underground experiments such as CDMS, UKDMC or EDELWEISS, or indirectly by searching for energetic gammas or neutrinos released when two neutralino dark matter particles annihilate. In section 2 we focus on the perplexing experimental/theoretical problem of where these SUSY particles are. We then consider the status of gauge coupling unification (section 3), proton decay predictions (section 4), Fermion masses and mixing angles (including neutrinos) and the SUSY flavour problem (section 5) and SUSY dark matter (section 6). We conclude with a discussion of some remaining open questions.

## 2. Where are the supersymmetric particles?

The answer to this question depends on two interconnected theoretical issues:

(i) the mechanism for SUSY breaking, and
(ii) the scale of SUSY breaking.

The first issue is tied inextricably to the SUSY flavour problem, while the second issue is tied to the gauge hierarchy problem. We discuss these issues in sections 2.1 and 2.2.

### *2.1. SUSY breaking mechanisms*

SUSY is necessarily a local gauge symmetry since Einstein's general theory of relativity corresponds to local Poincaré symmetry and SUSY is an extension of the Poincaré group. Hence SUSY breaking must necessarily be spontaneous, in order not to cause problems with unitarity and/or relativity. In this section we discuss some of the spontaneous SUSY breaking mechanisms considered in the literature. However, from a phenomenological standpoint, any spontaneous SUSY breaking mechanism results in *soft SUSY breaking* operators with dimension 3 or less (such as quadratic or cubic scalar operators or Fermion mass terms) in the effective low energy theory below the scale of SUSY breaking (Dimopoulos and Georgi 1981, Sakai 1981, Girardello and Grisaru 1982). There are *a priori* hundreds of arbitrary soft SUSY breaking parameters (the coefficients of the soft SUSY breaking operators) (Dimopoulos and Sutter 1995). These are parameters not included in the SM but are necessary for comparing with data or making predictions for new experiments.

The general set of renormalizable soft SUSY breaking operators, preserving the solution to the gauge hierarchy problem, is given in a paper by Girardello and Grisaru (1982). These operators are assumed to be the low energy consequence of spontaneous SUSY breaking at some fundamental SUSY breaking scale $\Lambda_S \gg M_Z$. The list of soft SUSY breaking parameters includes squark and slepton mass matrices, cubic scalar interaction couplings, gaugino masses, etc. Let us count the number of arbitrary parameters (Dimopoulos and Sutter 1995). Left and right chiral scalar quark and lepton mass matrices are *a priori* independent $3 \times 3$ hermitian matrices. Each contains nine arbitrary parameters. Thus for the scalar partners of $\{Q, \bar{u}, \bar{d}, L, \bar{e}\}$, we have five such matrices or 45 arbitrary parameters. In addition, corresponding to each complex $3 \times 3$ Yukawa matrix (one for up and down quarks and charged leptons), we have a complex soft SUSY breaking tri-linear scalar coupling ($A$) of left and right chiral squarks or sleptons to Higgs doublets. This adds $3 \times 18 = 54$

additional arbitrary parameters. Finally, add to these three complex gaugino masses ($M_i$, $i = 1, 2, 3$), and the complex soft SUSY breaking scalar Higgs mass ($\mu B$), and we have a total of 107 arbitrary soft SUSY breaking parameters. In additon, the minimal supersymmetric extension of the SM requires a complex Higgs mass parameter ($\mu$) that is the coefficient of a supersymmetric term in the Lagrangian. Therefore, altogether this minimal extension has 109 arbitrary parameters. Granted, not all of these parameters are physical, just as not all 54 parameters in the three complex $3 \times 3$ Yukawa matrices for up and down quarks and charged leptons are observable. Some of them can be rotated away by unitary redefinitions of quark and lepton superfields. Consider the maximal symmetry of the kinetic term of the theory—global $SU(3)_Q \times SU(3)_{\bar{u}} \times SU(3)_{\bar{d}} \times SU(3)_L \times SU(3)_{\bar{e}} \times U(1)^5 \times U(1)_R$. Out of the total number of parameters—163 = 109 (new SUSY parameters) + 54 (SM parameters)—we can use the $SU(3)^5$ to eliminate 40 parameters and three of the $U(1)$s to remove three phases. The other three $U(1)$s, however, are symmetries of the theory corresponding to B, L and weak hypercharge Y. We are thus left with 120 observables, corresponding to the nine charged Fermion masses, four quark mixing angles and *107 new, arbitrary observable SUSY parameters.*

Such a theory, with so many arbitrary parameters, clearly makes no predictions. However, this general MSSM is a 'straw man' (one to be struck down), but fear not since it is the worst case scenario. In fact, there are several reasons why this worst case scenario cannot be correct. First, and foremost, it is constrained severely by precision electroweak data. Arbitrary $3 \times 3$ matrices for squark and slepton masses or for tri-linear scalar interactions maximally violate quark and lepton flavour. The strong constraints from flavour violation were discussed by Dimopoulos and Georgi (1981), Dimopoulos and Sutter (1995), Gabbiani *et al* (1996). In general, they would exceed the strong experimental contraints on flavour violating processes, such as $b \to s\gamma$ or $b \to sl^+l^-$, $B_s \to \mu^+\mu^-$, $\mu \to e\gamma$, $\mu - e$ conversion in nuclei, etc. For this general MSSM not to be excluded by flavour violating constraints, the soft SUSY breaking terms must be

(i) flavour independent, or
(ii) aligned with quark and lepton masses or
(iii) the first and second generation squark and slepton masses ($\tilde{m}$) should be large (i.e. greater than 1 TeV).

In the first case, squark and slepton mass matrices are proportional to the $3 \times 3$ identity matrix and the tri-linear couplings are proportional to the Yukawa matrices. In this case the squark and slepton masses and tri-linear couplings are diagonalized on the same basis that quark and lepton Yukawa matrices are diagonalized. This limit preserves three lepton numbers—$L_e$, $L_\mu$ and $L_\tau$—(neglecting neutrino masses) and gives minimal flavour violation (due to only CKM mixing) in the quark sector (Hall *et al* 1986). The second case does not require degenerate quark flavours but *approximately* diagonal squark and slepton masses and interactions, when in the diagonal quark and lepton Yukawa basis. It necessarily ties any theoretical mechanism explaining the hierarchy of Fermion masses and mixing to the hierarchy of sfermion masses and mixing. This will be discussed further in sections 5.2 and 5.4. Finally, the third case minimizes flavour violating processes since all such effects are given by effective higher dimension operators that scale as $1/\tilde{m}^2$. The theoretical issue is what SUSY breaking mechanisms are 'naturally' consistent with these conditions.

Several such SUSY breaking mechanisms exist in the literature. They are called minimal supergravity (mSugra) breaking, gauge mediated SUSY breaking (GMSB), dilaton mediated SUSY breaking (DMSB), anomaly mediated SUSY breaking (AMSB), and gaugino mediated SUSY breaking (gMSB). Consider first mSugra, which has been the benchmark for experimental searches. The mSugra model (Barbieri *et al* 1982, Chamseddine *et al* 1982, Ibañez 1982, Ovrut and Wess 1982, Hall *et al* 1983, Nilles *et al* 1983) is defined to have the minimal set of soft SUSY breaking parameters. It is motivated by the simplest (Polony 1977) hidden sector in supergravity, with the additional assumption of grand unification. This SUSY breaking scenario is also known as the constrained MSSM (CMSSM) (Kane *et al* 1994). In mSUGRA/CMSSM, there are four soft SUSY breaking parameters at $M_G$ defined by $m_0$, a universal scalar mass; $A_0$, a universal tri-linear scalar coupling; $M_{1/2}$, a universal gaugino mass; and $\mu_0 B_0$, the soft SUSY breaking Higgs mass parameter, where $\mu_0$ is the supersymmetric Higgs mass parameter. In most analyses, $|\mu_0|$ and $\mu_0 B_0$ are replaced, using the minimization conditions of the Higgs potential, by $M_Z$ and the ratio of Higgs VEVs, $\tan\beta = \langle H_u\rangle/\langle H_d\rangle$. Thus, the parameter set defining mSugra/CMSSM is given by

$$m_0, A_0, M_{1/2}, \ \mathrm{sign}(\mu_0), \tan\beta. \tag{11}$$

This scenario is an example of the first case above (with minimal flavour violation); however, it is certainly not a consequence of the most general supergravity theory and thus requires further justification. Nevertheless it is a useful framework for experimental SUSY searches.

In GMSB models, SUSY breaking is first felt by messengers carrying SM charges and then transmitted to to the superpartners of SM particles (sparticles) via loop corrections containing SM gauge interactions. Squark and slepton masses in these models are proportional to $\alpha_i \Lambda_{\mathrm{SUSY}}$, with $\Lambda_{\mathrm{SUSY}} = F/M$. In this expression, $\alpha_i$, $i = 1, 2, 3$, are the fine structure constants for the SM gauge interactions, $\sqrt{F}$ is the fundamental scale of SUSY breaking, $M$ is the messenger mass and $\Lambda_{\mathrm{SUSY}}$ is the effective SUSY breaking scale. In GMSB the flavour problem is solved naturally since all squarks and sleptons with the same SM charges are degenerate and the $A$ terms vanish to zeroth order. In addition, GMSB resolves the formidable problems of model building (Fayet and Ferrara 1977) resulting from the direct tree level SUSY breaking of sparticles. This problem derives from the supertrace theorem, valid for tree level SUSY breaking,

$$\Sigma(2J + 1)(-1)^{2J} M_J^2 = 0, \tag{12}$$

where the sum is over all particles with spin $J$ and mass $M_J$. It leads generically to charged scalars with negative mass squared (Fayet and Ferrara 1977, Dimopoulos and Georgi 1981). Fortunately the supertrace theorem is violated explicitly when SUSY breaking is transmitted radiatively[3]. Low energy SUSY breaking models (Dine *et al* 1981, Dimopoulos and Raby 1981, Witten 1981, Dine and Fischler 1982, Alvarez-Gaume *et al* 1982), with $\sqrt{F} \sim M \sim \Lambda_{\mathrm{SUSY}} \sim 100\,\mathrm{TeV}$, make dramatic predictions (Dimopoulos *et al* 1996). Following the seminal work of (Dine and Nelson 1993, Dine *et al* 1995, 1996), complete GMSB models now exist (for a review, see Giudice and Rattazzi (1999)). Of course, the fundamental SUSY breaking scale can be much larger than the weak scale. Note that SUSY breaking effects are proportional to $1/M$ and hence they decouple as $M$ increases. This is a consequence of SUSY breaking decoupling theorems (Polchinski and Susskind 1982, Banks and Kaplunovsky 1983, Dimopoulos and Raby 1983). However, when $\sqrt{F} \geqslant 10^{10}$ GeV, then supergravity becomes important.

DMSB, motivated by string theory, and AMSB and gMSB, motivated by brane models with extra dimensions, also alleviate the SUSY flavour problem. We see that there are several possible SUSY breaking mechanisms that solve the SUSY flavour problem and provide predictions for superpartner masses in terms of a few fundamental parameters. Unfortunately we do not *a priori* know which one of these (or some other) SUSY breaking mechanisms is chosen by nature. For this we need experiment.

[3] It is also violated in supergravity, where the right-hand side is replaced by $2(N-1)m_{3/2}^2$ with $m_{3/2}$, the gravitino mass, and $N$, the number of chiral superfields.

*2.2. Fine-tuning or 'Naturalness'*

Presently, the only evidence for SUSY is indirect, given by the successful prediction for gauge coupling unification. Supersymmetric particles at the weak scale are necessary for this to work; however, it is discouraging that there is yet no direct evidence. Searches for new supersymmetric particles at CERN or Fermilab have produced only lower bounds on their mass. The SM Higgs mass bound, applicable to the MSSM when the CP odd Higgs (A) is much heavier, is 114.4 GeV (LEP2 2003). In the case of an equally light A, the Higgs bound is somewhat lower, $\sim$89 GeV. Squark, slepton and gluino mass bounds are of the order of 200 GeV, while the chargino bound is 103 GeV (LEP2 2003). In addition, other indirect indications for new physics beyond the SM, such as the anomalous magnetic moment of the muon ($a_\mu$), are inconclusive. Perhaps nature does not make use of this beautiful symmetry. Or perhaps the SUSY particles are heavier than we once believed.

Nevertheless, global fits to precision electroweak data in the SM or in the MSSM give equally good $\chi^2$/dof (de Boer 2003). In fact the fit is slightly better for the MSSM due mostly to the pull of $a_\mu$. The real issue among SUSY enthusiasts is the problem of fine-tuning. If SUSY is a solution to the gauge hierarchy problem (making the ratio $M_Z/M_G \sim 10^{-14}$ 'naturally' small), then radiative corrections to the $Z$ mass should be insensitive to physics at the GUT scale, i.e. it should not require any 'unnatural' fine-tuning of GUT scale parameters. A numerical test of fine-tuning is obtained by defining the *fine-tuning parameter*, $\Delta_\alpha (= |(a_\alpha/M_Z^2)(\mathrm{d}M_Z^2/\mathrm{d}a_\alpha)|)$, the logarithmic derivative of the $Z$ mass with respect to different 'fundamental' parameters $a_\alpha = \{\mu, M_{1/2}, m_0, A_0, \ldots\}$ defined at $M_G$ (Ellis *et al* 1986, Barbieri and Giudice 1988, de Carlos and Casas 1993, Anderson *et al* 1995). Smaller values of $\Delta_\alpha$ correspond to less fine-tuning, and roughly speaking, $p = \mathrm{Max}(\Delta_\alpha)^{-1}$ is the probability that a given model is obtained in a random search over SUSY parameter space.

There are several recent analyses, including LEP2 data, by (Chankowski *et al* 1997, Barbieri and Strumia 1998, Chankowski *et al* 1999). In particular (Barbieri and Strumia 1998, Chankowski *et al* 1999) find several notable results. In their analysis (Barbieri and Strumia 1998), only consider values of $\tan\beta < 10$ and soft SUSY breaking parameters of the CMSSM or GMSB. (Chankowski *et al* 1999) also consider large $\tan\beta = 50$ and more general soft SUSY breaking scenarios. They both conclude that the value of $\mathrm{Max}(\Delta_\alpha)$ is significantly lower when one includes the one-loop radiative corrections to the Higgs potential as compared with the tree level Higgs potential used in the analysis of (Chankowski *et al* 1997). In addition they find that the experimental bound on the Higgs mass is a very strong constraint on fine-tuning. Larger values of the light Higgs mass require larger values of $\tan\beta$. Values of $\mathrm{Max}(\Delta_\alpha) < 10$ are possible for a Higgs mass $<$111 GeV (for values of $\tan\beta < 10$ used in the analysis of (Barbieri and Strumia 1998)). However, allowing for larger values of $\tan\beta$ (Chankowski *et al* 1999) allows for a heavier Higgs. With LEP2 bounds on a SM Higgs mass of 114.4 GeV, larger values of $\tan\beta > 4$ are required. It is difficult to conclude too much from these results. Note that the amount of fine-tuning is somewhat sensitive to small changes in the definition of $\Delta_\alpha$. For example, replacing $a_\alpha \to a_\alpha^2$ or $M_Z^2 \to M_Z$ will change the result by a factor of $2^{\pm 1}$. Hence factors of 2 in the result are definition dependent. Let us assume that fine-tuning by 1/10 is acceptable; then is fine-tuning by 1 part in 100 'unnatural'? Considering the fact that the fine-tuning necessary to maintain the gauge hierarchy in the SM is at least 1 part in $10^{28}$, a fine-tuning of 1 part in 100 (or even $10^3$) seems like a great success.

A slightly different way of asking the fine-tuning question is to ask what fraction of this domain is already excluded by the low energy data if I assign equal weights to all 'fundamental' parameters at $M_G$ and scan over all values within some *a priori* assigned domain. This is the analysis that (Strumia 1999) uses to argue that 95% of the SUSY parameter space is now excluded by LEP2 bounds on the SUSY spectrum and in particular by the Higgs and chargino mass bounds. This conclusion is practically insensitive to the method of SUSY breaking assumed in the analysis which included the CMSSM, GMSB or AMSB or some variations of these. One might still question whether the *a priori* domain of input parameters, upon which this analysis stands, is reasonable. Perhaps if we doubled the input parameter domain we could find acceptable solutions in 50% of parameter space. To discuss this issue in more detail, let us consider two of the parameter domains considered in (Strumia 1999). Within the context of the CMSSM, he considers the domain defined by

$$m_0, |A_0|, |M_{1/2}|, |\mu_0|, |B_0| = (0\text{–}1)\, m_{\text{SUSY}}, \tag{13}$$

where $(a\text{–}b) \equiv$ a random number between $a$ and $b$ and the overall mass scale $m_{\text{SUSY}}$ is obtained from the minimization condition for electroweak symmetry breaking. He also considered an alternative domain defined by

$$m_0 = \left(\tfrac{1}{9}\text{–}3\right) m_{\text{SUSY}}, \qquad |\mu_0|, |M_{1/2}| = \left(\tfrac{1}{3}\text{–}3\right) m_{\text{SUSY}}, \qquad A_0, B_0 = (-3\text{–}3) m_0 \tag{14}$$

with the sampling of $m_0$, $M_{1/2}$, $\mu_0$ using a flat distribution on a log scale. In both cases, he concludes that 95% of parameter space is excluded with the light Higgs and chargino mass providing the two most stringent constraints. Hence we have failed to find SUSY in 95% of the allowed region of parameter space. But perhaps we should open the analysis to other, much larger, regions of SUSY parameter space. We return to this issue in sections 2.3 and 2.4.

For now, however, let us summarize our discussion of naturalness constraints with the following quote from (Chankowski *et al* 1999), 'We re-emphasize that naturalness is [a] subjective criterion, based on physical intuition rather than mathematical rigour. Nevertheless, it may serve as an important guideline that offers some discrimination between different theoretical models and assumptions. As such, it may indicate which domains of parameter space are to be preferred. However, one should be very careful in using it to set any absolute upper bounds on the spectrum. We think it safer to use relative naturalness to compare different scenarios, as we have done in this paper.' As these authors discuss in their paper, in some cases the amount of fine-tuning can be decreased dramatically if one assumes some linear relations between GUT scale parameters. These relations may be due to some, yet unknown, theoretical relations coming from the fundamental physics of SUSY breaking, such as string theory.

In the following we consider two deviations from the simplest definitions of fine-tuning and naturalness. The first example, called focus point (FP) (Feng and Moroi 2000, Feng *et al* 2000a, b, c, Feng and Matchev 2001), is motivated by infrared fixed points of the renormalization group equations (RGEs) for the Higgs mass and other dimensionful parameters. The second had two independent motivations. In the first case it is motivated by the SUSY flavour problem, and in this incarnation it is called the radiative inverted scalar mass hierarchy (RISMH) (Bagger *et al* 1999, 2000). More recently, it was reincarnated in the context of $SO(10)$ Yukawa unification for the third generation quark and lepton (YU) (Baer and Ferrandis 2001, Dermíšek 2001, Raby 2001, Blažek *et al* 2002a, b, Auto *et al* 2003, Tobe and Wells 2003) scenarios. In both scenarios the upper limit on soft scalar masses is increased much above 1 TeV.

### *2.3. The FP region of SUSY breaking parameter space*

In the FP SUSY breaking scenario, Feng *et al* (Feng and Moroi 2000, Feng *et al* 2000a, b) consider the RGEs for soft SUSY breaking parameters, assuming a universal scalar mass, $m_0$, at $M_G$. This may be as in the CMSSM (equation (11)) or even a variation of AMSB. They show that if the top quark mass is approximately 174 GeV, then the RGEs lead to a Higgs mass that is naturally of the order of the weak scale, independent of the precise value of $m_0$, which could be as large as 3 TeV. It was also noted that the only fine-tuning in this scenario was that necessary to obtain the top quark mass, i.e. if the top quark mass is determined by other physics, then there is no additional fine-tuning needed to obtain electroweak symmetry breaking[4]. As discussed in (Feng *et al* 2000c), this scenario opens up a new window for neutralino dark matter. Cosmologically acceptable neutralino abundances are obtained even with very large scalar masses. Moreover, as discussed in Feng and Matchev (2001), the FP scenario has many virtues. In the limit of large scalar masses, gauge coupling unification requires smaller threshold corrections at the GUT scale in order to agree with low energy data. In addition, larger scalar masses ameliorate the SUSY flavour and CP problems. This is because both processes result from effective higher dimensional operators suppressed by two powers of squark and/or slepton masses. Finally, a light Higgs mass in the narrow range from about 114–120 GeV is predicted. Clearly, the FP region includes a much larger range of soft SUSY breaking parameter space than considered previously. It may also be perfectly 'natural.'

The analysis of the FP scenario was made within the context of the CMSSM. The FP region extends to values of $m_0$ up to 3 TeV. This upper bound increases from 3 to about 4 TeV as the top quark mass is varied from 174 to 179 GeV. On the other hand, as $\tan\beta$ increases from 10 to 50, the allowed range in the $m_0$–$M_{1/2}$ plane for $A_0 = 0$, consistent with electroweak symmetry breaking, shrinks. As we shall see from the following discussion, this narrowing of the FP region is most likely an artefact of the precise CMSSM boundary conditions used in the analysis. In fact the CMSSM parameter space is particularly constraining in the large $\tan\beta$ limit.

### 2.4. SO(10) Yukawa unification and the RISMH

The top quark mass $M_t \sim 174$ GeV requires a Yukawa coupling $\lambda_t \sim 1$. In the minimal $SO(10)$ SUSY model (MSO$_{10}$SM) the two Higgs doublets, $H_u$ and $H_d$, of the MSSM are contained in one 10. In addition the three families of quarks and leptons are in $16_i$, $i = 1, 2, 3$. In the MSO$_{10}$SM the third generation Yukawa coupling is given by

$$\begin{aligned}\lambda\ 16_3\ 10\ 16_3 = {} & \lambda(Q_3 H_u \bar{t} + L_3 H_u \bar{\nu}_\tau + Q_3 H_d \bar{b} + L_3 H_d \bar{\tau}) + \lambda\left(\tfrac{1}{2} Q_3 Q_3 + \bar{t}\bar{\tau}\right) T \\ & + \lambda(Q_3 L_3 + \bar{t}\bar{b})\bar{T}.\end{aligned} \tag{15}$$

Thus we obtain the unification of all third generation Yukawa couplings with

$$\lambda = \lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau}. \tag{16}$$

Of course this simple Yukawa interaction, with the constant $\lambda$ replaced by a $3\times 3$ Yukawa matrix, does not work for all three families[5]. In this discussion, we shall assume that the first and second generations obtain mass using the same 10, but via effective higher dimensional operators resulting in a hierarchy of Fermion masses and mixing. In this case, Yukawa unification for the third family (equation (16)) is a very good approximation. The question then arises whether this symmetry relation is consistent with low energy data given by

$$M_t = 174.3 \pm 5.1\,\text{GeV}, \qquad m_b(m_b) = 4.20 \pm 0.20\,\text{GeV}, \qquad M_\tau = 1.7770 \pm 0.0018, \tag{17}$$

[4] For a counter discussion of fine-tuning in the FP region, see Romanino and Strumia (2000).

[5] In such a theory there is no CKM mixing matrix and the down quark and charged lepton masses satisfy the bad prediction $1/20 \sim m_d/m_s = m_e/m_\mu \sim 1/200$.

where the error on $M_\tau$ is purely a theoretical uncertainty due to numerical errors in the analysis. Although this topic has been around for a long time, it is only recently that the analysis has included the complete one-loop threshold corrections at the weak scale (Baer and Ferrandis 2001, Dermíšek 2001, Raby 2001, Blažek *et al* 2002a, b, Auto *et al* 2003, Tobe and Wells 2003). It turns out that these corrections are very important. The corrections to the bottom mass are functions of squark and gaugino masses times a factor of $\tan\beta \sim m_t(m_t)/m_b(m_t) \sim 50$. For typical values of the parameters the relative change in the bottom mass, $\delta m_b/m_b$, is very large, of the order of 50%. At the same time, the corrections to the top and tau masses are small. For the top, the same one-loop corrections are proportional to $1/\tan\beta$, while for the tau, the dominant contribution from neutralino loops is small. These one-loop radiative corrections are determined, through their dependence on squark and gaugino masses, by the soft SUSY breaking parameters at $M_G$. In the MSO$_{10}$SM we assume the following dimensionful parameters,

$$m_{16},\ m_{10},\ \Delta m_H^2,\ A_0,\ M_{1/2},\ \mu, \tag{18}$$

where $m_{16}$ is the universal squark and slepton mass; $m_{H_{u/d}}^2 = m_{10}^2(1 \mp \Delta m_H^2)$ is the Higgs up/down mass squared; $A_0$ is the universal tri-linear Higgs–scalar coupling; $M_{1/2}$ is the universal gaugino mass; and $\mu$ is the supersymmetric Higgs mass. $\tan\beta \approx 50$ is fixed by the top, bottom and $\tau$ mass. Note that, there are two more parameters ($m_{10}$, $\Delta m_H^2$) than in the CMSSM. They are needed in order to obtain electroweak symmetry breaking solutions in the region of parameter space with $m_{16} \gg \mu$, $M_{1/2}$. We shall defer a more detailed discussion of the results of the MSO$_{10}$SM to sections 5.1 and 6. Suffice it to say here that good fits to the data are only obtained in a narrow region of SUSY parameter space given by

$$\begin{aligned}&A_0 \approx -2m_{16}, \qquad m_{10} \approx \sqrt{2}m_{16}, \qquad \Delta m_H^2 \approx 0.13,\\ &m_{16} \gg \mu \sim M_{1/2}, \qquad m_{16} > 2\,\mathrm{TeV}.\end{aligned} \tag{19}$$

Once more we are concerned about fine-tuning with $m_{16}$ so large. However, we discover a fortuitous coincidence. This region of parameter space (equation (19)) 'naturally' results in a RISMH with $\tilde{m}_{1,2}^2 \gg \tilde{m}_3^2$ (Bagger *et al* 1999, 2000), i.e. first and second generation squark and slepton masses are of order $m_{16}^2$, while the third generation scalar masses are much lighter. Since the third generation has the largest couplings to the Higgs Bosons, they give the largest radiative corrections to the Higgs mass. Hence, with lighter third generation squarks and sleptons, a light Higgs is more 'natural'. Although a detailed analysis of fine-tuning parameters is not available in this regime of parameter space, the results of several papers suggest that the fine-tuning concern is minimal (see, e.g. Dimopoulos and Giudice (1995), Chankowski *et al* (1999), Kane *et al* (2003)). While there may not be any fine-tuning necessary in the MSO$_{10}$SM region of SUSY parameter space (equation (19)), there is still one open problem. There is no known SUSY breaking mechanism that satisfies 'naturally' the conditions of equation (19). On the other hand, we conclude this section by noting that the latter two examples suggest that there is a significant region of SUSY breaking parameter space that is yet to be explored experimentally.

## 3. Gauge coupling unification

The apparent unification of the three gauge couplings at a scale of the order of $3 \times 10^{16}$ GeV is, at the moment, the only experimental evidence for low energy SUSY (Amaldi *et al* 1991, Ellis *et al* 1991, Langacker and Luo 1991). In this section, we consider the status of gauge coupling unification and the demise of minimal SUSY $SU(5)$.

The theoretical analysis of unification is now at the level requiring a two-loop renormalization group (RG) running from $M_G$ to $M_Z$. Consistency then requires including one-loop threshold corrections at both the GUT and weak scales. Once GUT threshold corrections are considered, a precise definition of the GUT scale ($M_G$) is needed. The three gauge couplings no longer meet at one scale[6], since

$$\alpha_i(\mu)^{-1} = \alpha_G^{-1} + \Delta_i(\mu), \tag{20}$$

where the corrections $\Delta_i(\mu)$ are logarithmic functions of mass for all states with GUT scale mass. In principle, the GUT scale can now be defined as the mass, $M_X$, of the $X$, $\bar{X}$ gauge Bosons mediating proton decay or as the scale where any two couplings meet. We define $M_G$ as the value of $\mu$ where

$$\alpha_1(M_G) = \alpha_2(M_G) \equiv \tilde{\alpha}_G. \tag{21}$$

Using the two-loop RGE from $M_Z$ to $M_G$, we find

$$\begin{aligned} M_G &\approx 3 \times 10^{16}\,\text{GeV}, \\ \alpha_G^{-1} &\approx 24. \end{aligned} \tag{22}$$

In addition, good fits to the low energy data require

$$\epsilon_3 \equiv \frac{(\alpha_3(M_G) - \tilde{\alpha}_G)}{\tilde{\alpha}_G} \sim -3\% \text{ to } -4\%. \tag{23}$$

Note that the exact value of the threshold correction ($\epsilon_3$) needed to fit the data, depends on the weak scale threshold corrections and in particular on the SUSY particle spectrum. We shall return to this later. On the other hand, significant contributions to the GUT threshold correction, $\epsilon_3$, arise typically from the Higgs and GUT breaking sectors of the theory. Above $M_G$, there is a single coupling constant $\alpha_G \approx \tilde{\alpha}_G$ that then runs up to some fundamental scale, $M_*$, as the string scale, where the running is cut off. The GUT symmetry, in concert with SUSY, regulates the radiative corrections. Without the GUT, $\epsilon_3$ would naturally take on a value of order 1.

Following Lucas and Raby (1996), we show that the allowed functional dependence of $\epsilon_3$ on GUT symmetry breaking (VEVs) is quite restricted. Consider a general $SO(10)$ theory with

$$(n_{16} + 3)\mathbf{16} + n_{16}\overline{\mathbf{16}} + n_{10}\mathbf{10} + n_{45}\mathbf{45} + n_{54}\mathbf{54} + n_1\mathbf{1}. \tag{24}$$

Note that the superpotential for the GUT breaking sector of the theory typically has a $U(1)^n \times R$ symmetry, which, as we shall see, has an important consequence for the threshold corrections. The one-loop threshold corrections are given by

$$\alpha_i^{-1}(M_G) = \alpha_G^{-1} - \Delta_i \tag{25}$$

with

$$\Delta_i = \frac{1}{2\pi} \sum_\xi b_i^\xi \log \left| \frac{M_\xi}{M_G} \right|. \tag{26}$$

The sum is over all super heavy particles, $\xi$, with mass $M_\xi$, and $b_i^\xi$ is the contribution a super heavy particle would make to the beta function coefficient, $b_i$, if the particle were not integrated out at $M_G$.

[6] Brodsky *et al* (2003) have argued that the three gauge couplings always meet in a GUT at a scale above the largest GUT mass. They define this to be the GUT scale. Unfortunately, this scale cannot be defined in the effective low energy theory.

As a consequence of SUSY and the $U(1)$ symmetries, Lucas and Raby proved the following theorem: $\epsilon_3$ is only a function of $U(1)$ and $R$ invariant products of powers of VEVs $\{\zeta_i\}$, i.e.

$$\epsilon_3 = F(\zeta_1, \ldots, \zeta_m) + \frac{3\tilde{\alpha}_G}{5\pi} \log \left| \frac{M_T^{\text{eff}}}{M_G} \right| + \cdots . \tag{27}$$

As an example, consider the symmetry breaking sector given by the superpotential

$$\begin{aligned} W_{\text{sym breaking}} = {} & \frac{1}{M_*} A_1'(A_1^3 + \mathcal{S}_3 S A_1 + \mathcal{S}_4 A_1 A_2) + A_2(\psi\overline{\psi} + \mathcal{S}_1 \tilde{A}) + S\tilde{A}^2 \\ & + S'(S\mathcal{S}_2 + A_1\tilde{A}) + \mathcal{S}_3 S'^{\,2}, \end{aligned} \tag{28}$$

where the fields transform as follows: $\{A_1,\ A_2,\ \tilde{A},\ A_1'\} \subset \mathbf{45}$, $\{S,\ S'\} \subset \mathbf{54}$, $\psi, \bar{\psi} \subset \mathbf{16}$, $\overline{\mathbf{16}}$ and $\{\mathcal{S}_1, \ldots, \mathcal{S}_4\} \subset \mathbf{1}$. In addition, we include the Lagrangian for the electroweak Higgs sector given by

$$L_{\text{Higgs}} = \left[ 10_1 A_1 10_2 + \mathcal{S}_5 10_2^2|_F + \frac{1}{M} \right] z^* 10_1^2|_D. \tag{29}$$

$W_{\text{sym breaking}}$ has a $[U(1)]^4 \times R$ symmetry. Since SUSY is unbroken, the potential has many F and D flat directions. One in particular (equation (30)) breaks $SO(10)$ to $SU(3) \times SU(2) \times U(1)_Y$, leaving only the states of the MSSM massless plus some non-essential SM singlets.

$$\begin{aligned} &\langle A_1 \rangle = a_1 \tfrac{3}{2}(B - L), \qquad \langle A_2 \rangle = a_2 \tfrac{3}{2} Y, \\ &\langle \tilde{A} \rangle = \tilde{a} \tfrac{1}{2} X, \qquad \langle S \rangle = s \,\text{diag}(111 - \tfrac{3}{2} - \tfrac{3}{2}) \otimes \mathbf{1}_{2\times 2}, \\ &\langle \psi \rangle = v |SU(5) \text{ singlet}\rangle, \qquad \langle \overline{\psi} \rangle = \overline{v} |SU(5) \text{ singlet}\rangle. \end{aligned} \tag{30}$$

The VEVs $\{a_1,\ a_2,\ \tilde{a},\ v,\ \overline{v},\ \mathcal{S}_4\}$ form a complete set of independent variables along the F and D flat directions. Note that since the superpotential (equation (28)) contains higher dimension operators fixed by the cutoff scale $M_* \sim M_{\text{Planck}}$, the GUT scale spectrum ranges from $10^{13}$ to $10^{20}$ GeV. Nevertheless the threshold corrections are controlled. The only invariant under a $[U(1)]^4 \times R$ rotation of the VEVs is $\zeta = (a_1^4/a_2^2\mathcal{S}_4^2)$. By an explicit calculation we find the threshold correction

$$\epsilon_3 = \frac{3\tilde{\alpha}_G}{5\pi} \left\{ \log \frac{256}{243} - \frac{1}{2} \ \log \left| \frac{(1 - 25\zeta)^4}{(1 - \zeta)} + \log \text{abs} \frac{M_T^{\text{eff}}}{M_G} \right| \right\}. \tag{31}$$

Taking reasonable values of the VEVs given by

$$a_1 = 2a_2 = 2\mathcal{S}_4 = M_G \tag{32}$$

and the effective colour triplet Higgs mass,

$$M_T^{\text{eff}} \sim 10^{19}\,\text{GeV}, \tag{33}$$

we find

$$\zeta = 16, \qquad \epsilon_3 \approx -0.030. \tag{34}$$

Note that, the large value of $M_T^{\text{eff}}$ is necessary for suppressing proton decay rates as discussed in the following section.

**Table 2.** Recent preliminary lower bounds on the proton lifetime into specific decay modes from Super-Kamiokande (Jung 2002).

| Mode | $\tau/B$ limit ($10^{33}$ yr) |
|---|---|
| $p \to \pi^0 + e^+$ | 5.7 @ 90% C.L. |
| $p \to K^+ + \bar{\nu}$ | 1.9 @ 90% C.L. |

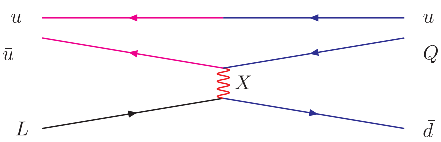


**Figure 1.** $X$ Boson exchange diagram giving the dimension 6 four Fermion operator for proton and neutron decay.

## 4. Nucleon decay: minimal $SU(5)$ SUSY GUT

Protons and neutrons (nucleons) are not stable particles; they necessarily decay in any GUT. Super-Kamiokande and Soudan II are looking for these decay products. The most recent (preliminary) Super-Kamiokande bounds on the proton lifetime (Jung 2002) are given in table 2. In the future, new detectors $\geqslant$10 times larger than Super-K have been proposed—Hyper-Kamiokande in Japan and UNO in the USA. Note, a generic, dimension 6 nucleon decay operator is given by a four Fermion operator of the form $\sim(1/\Lambda^2)\ q\ q\ q\ l$. Given the bound $\tau_p > 5 \times 10^{33}$ yr, we find $\Lambda > 4 \times 10^{15}$ GeV. This is nice since it is roughly consistent with the GUT scale and with the seesaw scale for neutrino masses.

In this section we consider nucleon decay in the minimal $SU(5)$ SUSY GUT in more detail. In minimal SUSY $SU(5)$, we have the following gauge and Higgs sectors. The gauge sector includes the gauge Bosons for $SU(5)$ that decompose, in the SM, to $SU(3) \times SU(2) \times U(1)$ plus the massive gauge Bosons $\{X, \bar{X}\}$. The $\{X, \bar{X}\}$ Bosons with charges $\{(3, \bar{2}, -\frac{5}{3}), (\bar{3}, 2, \frac{5}{3})\}$ are responsible for nucleon decay. The minimal $SU(5)$ theory has, by definition, the minimal Higgs sector. It includes a single adjoint of $SU(5)$, $\Sigma \subset 24$, for the GUT breaking sector and the electroweak Higgs sector (equation (8))

$$\begin{pmatrix} H_u \\ T \end{pmatrix}, \quad \begin{pmatrix} H_d \\ \bar{T} \end{pmatrix} \subset \mathbf{5_H}, \quad \bar{\mathbf{5}}_{\mathbf{H}}.$$

The superpotential for the GUT breaking and Higgs sectors of the model is given by Dimopoulos and Georgi (1981), Sakai (1981), Witten (1981)

$$W = \frac{\lambda}{3}\,\mathrm{Tr}\ \Sigma^3 + \frac{\lambda V}{2}\,\mathrm{Tr}\ \Sigma^2 + \bar{H}(\Sigma + 3V)H. \tag{35}$$

In general, nucleon decay can have contributions from operators with dimensions 4, 5 and 6.

### 4.1. Dimension 6 operators

The dimension 6 operators are derived from gauge Boson exchange (see figure 1). We obtain the effective dimension 6 (four Fermion) operator given by

$$\frac{g_G^2}{2M_X^2}\bar{u}^\dagger Q \bar{d}^\dagger L. \tag{36}$$

Thus the decay amplitude is suppressed by one power of $1/M_X^2$. How is $M_X$ related to the GUT scale, $M_G$, determined by gauge coupling unification? Recall that, in general we have

$$\epsilon_3 = \frac{3\alpha_G}{5\pi} \ln \frac{M_T}{M_G} + \epsilon_3(M_X, M_\Sigma). \tag{37}$$

However, in minimal $SU(5)$ we find

$$\epsilon_3(M_X, M_\Sigma) \equiv 0. \tag{38}$$

Thus, gauge coupling unification fixes the values of the three parameters, $\{\alpha_G(\equiv g_G^2/4\pi), M_G, M_T\}$. In addition, the $\alpha_1(M_G) = \alpha_2(M_G)$ condition gives the relation

$$\left(\frac{M_G}{M_T}\right)^{1/15} = \frac{M_G}{(M_X^2 M_\Sigma)^{1/3}} \sim \frac{M_G}{(g_G^2 \lambda)^{1/3} \langle \Sigma \rangle}. \tag{39}$$

In the last term we used the approximate relations

$$M_X \sim g_G \langle \Sigma \rangle, \qquad M_\Sigma \sim \lambda \langle \Sigma \rangle. \tag{40}$$

Hence the natural values for these parameters are given by

$$M_X \sim M_\Sigma \sim M_G \approx 3 \times 10^{16}\,\text{GeV}. \tag{41}$$

As a result, the proton lifetime is given by

$$\tau_p \sim 5 \times 10^{36} \left(\frac{M_X}{3 \times 10^{16}\,\text{GeV}}\right)^4 \left(\frac{0.015\,\text{GeV}^3}{\beta_{\text{lattice}}}\right)^2 \text{yr} \tag{42}$$

and the dominant decay mode is

$$p \to \pi^0 + e^+. \tag{43}$$

Note that it is not possible to enhance the decay rate by taking $M_X \ll M_G$ without spoiling perturbativity since this limit requires $\lambda \gg 1$. On the other hand, $M_X \gg M_G$ is allowed.

### *4.2. Dimension 4 and 5 operators*

The contribution of dimension 4 and 5 operators to nucleon decay in SUSY GUTs was noted by Sakai and Yanagida (1982), Weinberg (1982). Dimension 4 operators are dangerous. In SUSY GUTs they always appear in the combination

$$(U^c\, D^c\, D^c) + (Q\, L\, D^c) + (E^c\, L\, L), \tag{44}$$

leading to unacceptable nucleon decay rates. $R$ parity (Farrar and Fayet 1979) forbids all dimension 3 and 4 (and even one dimension 5) baryon and lepton number violating operators. It is thus a necessary ingredient of any 'natural' SUSY GUT.

Dimension 5 operators are obtained when integrating out heavy colour triplet Higgs fields.

$$W \supset H_u Q Y_u \overline{U} + H_d(Q Y_d \overline{D} + L Y_e \overline{E}) + T\left(Q \tfrac{1}{2} c_{\text{qq}} Q + \overline{U} c_{\text{ue}} \overline{E}\right) + \bar{T}(Q c_{\text{ql}} L + \overline{U} c_{\text{ud}} \overline{D}). \tag{45}$$

If the colour triplet Higgs fields in equation (45) $\{T, \bar{T}\}$ have an effective mass term $M_T^{\text{eff}} \bar{T}\, T$, we obtain the dimension 5 operators

$$\left(\frac{1}{M_T^{\text{eff}}}\right) \left[Q \tfrac{1}{2} c_{\text{qq}} Q Q c_{\text{ql}} L + \overline{U} c_{\text{ud}} \overline{D} \overline{U} c_{\text{ue}} \overline{E}\right], \tag{46}$$

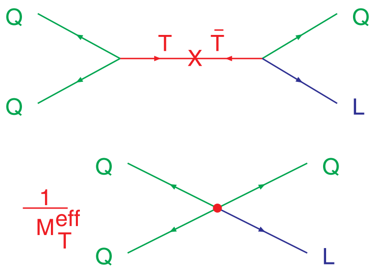


**Figure 2.** Colour triplet Higgs exchange diagram giving the dimension 5 superpotential operator for proton and neutron decay.

denoted *LLLL* and *RRRR* operators, respectively (figure 2). Nucleon decay via dimension 5 operators was considered by Dimopoulos *et al* (1982), Ellis *et al* (1982), Sakai and Yanagida (1982). The proton decay amplitude is then given generically by the expression

$$T(p \to K^+ + \bar{\nu}) \propto \frac{c^2}{M_T^{\rm eff}}(\text{Loop Factor})(RG)\langle K^+\bar{\nu}|qqql|p\rangle$$
$$\sim \frac{c^2}{M_T^{\rm eff}}(\text{Loop Factor})(RG)\frac{\beta_{\rm lattice}}{f_\pi}m_p. \tag{47}$$

The last step used a chiral Lagrangian analysis to remove the $K^+$ state in favour of the vacuum state. Now we only need calculate the matrix element of a three quark operator between the proton and vacuum states. This defines the parameter $\beta_{\rm lattice}$ using lattice QCD calculations. The decay amplitude includes four independent factors:

(i) $\beta_{\rm lattice}$, the three quark matrix element,
(ii) $c^2$, a product of two $3 \times 3$ dimensionless coupling constant matrices,
(iii) a Loop Factor, which depends on the SUSY breaking squark, slepton and gaugino masses, and
(iv) $M_T^{\rm eff}$, the effective colour triplet Higgs mass, which is subject to the GUT breaking sector of the theory.

Let us now consider each of these factors in detail.

*4.2.1.* $\beta_{\rm lattice}$. The strong interaction matrix element of the relevant three quark operators taken between the nucleon and the appropriate pseudo-scalar meson may be obtained directly using lattice techniques. However, these results have only been obtained recently (Aoki *et al* (JLQCD) 2000, Aoki *et al* (RBC) 2002). Alternatively, chiral Lagrangian techniques (Chadha *et al* 1983) are used to replace the pseudo-scalar meson by the vacuum. Then the following three quark matrix elements are needed.

$$\beta U(\boldsymbol{k}) = \epsilon_{\alpha\beta\gamma}\langle 0|(u^\alpha d^\beta)u^\gamma|\text{proton}(\boldsymbol{k})\rangle, \tag{48}$$

$$\alpha U(\boldsymbol{k}) = \epsilon_{\alpha\beta\gamma}\langle 0|(\overline{u}^{*\alpha}\overline{d}^{*\beta})u^\gamma|\text{proton}(\boldsymbol{k})\rangle \tag{49}$$

and $U(\boldsymbol{k})$ is the left-handed component of the proton's wavefunction. It has been known for some time that $|\beta| \approx |\alpha|$ (Brodsky *et al* 1984, Gavela *et al* 1989) and that $|\beta|$ ranges from 0.003 to 0.03 GeV$^3$ (Brodsky *et al* 1984, Hara *et al* 1986, Gavela *et al* 1989). Until quite recently, lattice calculations did not reduce the uncertainty in $|\beta|$; lattice calculations have reported $|\beta|$ as low as 0.006 GeV$^3$ (Gavela *et al* 1989) and as high as 0.03 GeV$^3$ (Hara *et al* 1986). Additionally, the phase between $\alpha$ and $\beta$ satisfies $\beta \approx -\alpha$ (Gavela *et al* 1989).

As a consequence, when calculating nucleon decay rates most authors have chosen to use a conservative lower bound with $|\alpha| \sim |\beta| = 0.003\,\mathrm{GeV}^3$ and an arbitrary relative phase.

Recent lattice calculations (Aoki *et al* (JLQCD) 2000, Aoki *et al* (RBC) 2002) have obtained significantly improved results. In addition, they have compared the direct calculation of the three quark matrix element between the nucleon and pseudo-scalar meson with the indirect chiral Lagrangian analysis with the three quark matrix element between the nucleon and vacuum. Aoki *et al* (JLQCD) (2000) find

$$\beta_{\text{lattice}} = \langle 0|qqq|N\rangle = 0.015(1)\,\mathrm{GeV}^3. \tag{50}$$

Also Aoki *et al* (RBC) (2002), in preliminary results reported in conference proceedings, obtained

$$\beta_{\text{lattice}} = 0.007(1)\,\mathrm{GeV}^3. \tag{51}$$

They both find

$$\alpha_{\text{lattice}} \approx -\beta_{\text{lattice}}. \tag{52}$$

Several comments are in order. The previous theoretical range, $0.003\,\mathrm{GeV}^3 < \beta_{\text{lattice}} < 0.03\,\mathrm{GeV}^3$, has been reduced significantly and the relative phase between $\alpha$ and $\beta$ has been confirmed. The JLQCD central value is five times larger than the previous 'conservative lower bound', although the new, preliminary, RBC result is a factor of 2 smaller than that of JLQCD. We will have to wait for further results. What about the uncertainties? The error bars listed are only statistical. Systematic uncertainties (quenched + chiral Lagrangian) are likely to be of the order of $\pm 50\%$ (my estimate). This stems from the fact that errors due to quenching are characteristically of the order of 30%, while the comparison of the chiral Lagrangian results to the direct calculation of the decay amplitudes agree to within about 20%, depending on the particular final state meson.

*4.2.2. $c^2$—Model dependence.* Consider the quark and lepton Yukawa couplings in $SU(5)$,

$$\lambda(\langle\Phi\rangle) 10\ 10\ 5_H + \lambda'(\langle\Phi\rangle) 10\ \bar{5}\ \bar{5}_H, \tag{53}$$

or in $SO(10)$,

$$\lambda(\langle\Phi\rangle) 16\ 16\ 10_H. \tag{54}$$

The Yukawa couplings

$$\lambda(\langle\Phi\rangle), \quad \lambda'(\langle\Phi\rangle) \tag{55}$$

are effective higher dimensional operators, functions of adjoint ($\Phi$) (or higher dimensional) representations of $SU(5)$ (or $SO(10)$). The adjoint representations are necessarily there in order to correct the unsuccessful predictions of minimal $SU(5)$ (or $SO(10)$) and to generate a hierarchy of Fermion masses[7]. Once the adjoint (or higher dimensional) representations obtain VEVs ($\langle\Phi\rangle$), we find the Higgs Yukawa couplings,

$$H_u Q Y_u \overline{U} + H_d (Q Y_d \overline{D} + L Y_e \overline{E}) \tag{56}$$

and also the effective dimension 5 operators,

$$\left(\frac{1}{M_T^{\text{eff}}}\right)\left[Q\,\tfrac{1}{2}c_{\text{qq}}\,Q\,Q\,c_{\text{ql}}\,L + \overline{U}\,c_{\text{ud}}\,\overline{D}\,\overline{U}\,c_{\text{ue}}\,\overline{E}\right]. \tag{57}$$

[7] Effective higher dimensional operators may be replaced by Higgs in higher dimensional representations, such as 45 of $SU(5)$ or 120 and 126 or $SO(10)$. Using these Higgs representations, however, does not by itself address the Fermion mass hierarchy.

Note that because of the Clebsch relations due to the VEVs of the adjoint representations, etc we have

$$Y_u \neq c_{\mathrm{qq}} \neq c_{\mathrm{ue}} \tag{58}$$

and

$$Y_d \neq Y_e \neq c_{\mathrm{ud}} \neq c_{\mathrm{ql}}. \tag{59}$$

Hence, the $3 \times 3$ complex matrices entering nucleon decay are not the same $3 \times 3$ Yukawa matrices entering Fermion masses. Is this complication absolutely necessary and how large can the difference be? Consider the $SU(5)$ relation,

$$\lambda_b = \lambda_\tau \tag{60}$$

(Einhorn and Jones 1982, Inoue *et al* 1982, Ibañez and Lopez 1984). It is known to work quite well for small $\tan\beta \sim 1$–2 or large $\tan\beta \sim 50$ (Dimopoulos *et al* 1992, Barger *et al* 1993). For a recent discussion (see Barr and Dorsner (2003)). On the other hand, the same relation for the first two families gives

$$\begin{aligned} \lambda_s &= \lambda_\mu, \\ \lambda_d &= \lambda_e, \end{aligned} \tag{61}$$

leading to the unsuccessful relation

$$20 \sim \frac{m_s}{m_d} = \frac{m_\mu}{m_e} \sim 200. \tag{62}$$

This bad relation can be corrected using Higgs multiplets in higher dimensional representations (Georgi and Jarlskog 1979, Georgi and Nanopoulos 1979, Harvey *et al* 1980, 1982) or using effective higher dimensional operators (Anderson *et al* 1994). Clearly the corrections to the simple $SU(5)$ relation for Yukawa and $c$ matrices can be an order of magnitude. Nevertheless, in predictive SUSY GUTs the $c$ matrices are obtained once the Fermion masses and mixing angles are fit (Kaplan and Schmaltz 1994, Babu and Mohapatra 1995, Frampton and Kong 1996, Lucas and Raby 1996, Allanach *et al* 1997, Barbieri and Hall 1997, Barbieri *et al* 1997, Blažek *et al* 1997, Berezhiani 1998, Babu *et al* 2000, Blažek *et al* 1999, 2000, Dermíšek and Raby 2000, Shafi and Tavartkiladze 2000, Altarelli *et al* 2000, Albright and Barr 2000, 2001, Berezhiani and Rossi 2001, Kitano and Mimura 2001, Maekawa 2001, Aulakh *et al* 2003, Goh *et al* 2003, Chen and Mahanthappa 2003, King and Ross 2003, Raby 2003, Ross and Velasco-Sevilla 2003). In spite of the above cautionary remarks, we still find the inexact relations

$$c_{\mathrm{qq}} c_{\mathrm{ql}}, \qquad c_{\mathrm{ud}} c_{\mathrm{ue}} \propto m_u m_d \tan\beta. \tag{63}$$

In addition, family symmetries can affect the texture of $\{c_{\mathrm{qq}}, c_{\mathrm{ql}}, c_{\mathrm{ud}}, c_{\mathrm{ue}}\}$, just as it will affect the texture of Yukawa matrices.

In order to make predictions for nucleon decay, it is necessary to follow these simple steps. Vary the GUT scale parameters, $\tilde{\alpha}_G$, $M_G$, $Y_u$, $Y_d$, $Y_e$ and soft SUSY breaking parameters, until one obtains a good fit to the precision electroweak data, whereby we now explicitly include Fermion masses and mixing angles in the category of precision data. Once these parameters are fit, then in any predictive SUSY GUT the matrices $c_{\mathrm{qq}}$, $c_{\mathrm{ql}}$, $c_{\mathrm{ud}}$ and $c_{\mathrm{ue}}$ at $M_G$ are also fixed. Now renormalize the dimension 5 baryon and lepton number violating operators from $M_G \to M_Z$ in the MSSM; evaluate the Loop Factor at $M_Z$ and renormalize the dimension 6 operators from $M_Z \to 1$ GeV. The latter determines the renormalization constant $A_3 \sim 1.3$ (Dermíšek *et al* 2001). (Note that this should not be confused with the renormalization factor $A_L \sim 0.22$ (Ellis *et al* 1982), which is used when one does not have a theory for Yukawa matrices. The

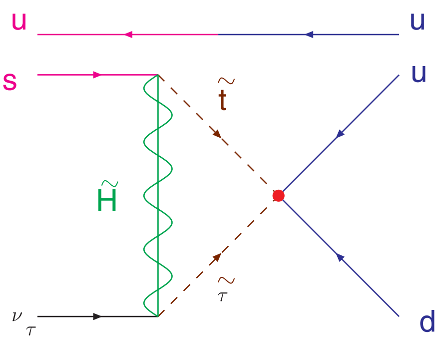


**Figure 3.** Higgsino and third generation stop–stau loop giving the dominant loop contribution to dimension 5 nucleon decay.

latter RG factor takes into account the combined renormalization of the dimension 6 operator from the weak scale to 1 GeV and also the renormalization of Fermion masses from 1 GeV to the weak scale.) Finally calculate decay amplitudes using a chiral Lagrangian approach or direct lattice gauge calculation.

Before leaving this section we should remark that we have assumed that the electroweak Higgs in $SO(10)$ models is contained solely in a 10. If however the electroweak Higgs is a mixture of weak doublets from $16_H$ and $10_H$ and, in addition, we include the higher dimensional operator $(1/M)(16\ 16\ 16_H\ 16_H)$, useful for neutrino masses, then there are additional contributions to the dimension 5 operators considered in (equation (46)) (Babu *et al* 2000). However, these additional terms are not required for neutrino masses (Blažek *et al* 1999, 2000).

### *4.3. Loop factor*

The dimension 5 operators are a product of two Fermion and two scalar fields. The scalar squarks and/or sleptons must be integrated out of the theory below the SUSY breaking scale. There is no consensus on the best choice for an appropriate SUSY breaking scale. Moreover, in many cases there is a hierarchy of SUSY particle masses. Hence we take the simplest assumption, integrating out all SUSY particles at $M_Z$. When integrating out the SUSY particles in loops, the effective dimension 5 operators are converted to effective dimension 6 operators. This results in a loop factor that depends on the sparticle masses and mixing angles.

Consider the contribution to the loop factor for the process $p \rightarrow K^+ + \bar{\nu}_\tau$ in figure 3. This graph is due to the *RRRR* operators and gives the dominant contribution at large tan $\beta$ and a significant contribution for all values of tan $\beta$ (Lucas and Raby 1997, Babu and Strassler 1998, Goto and Nihei 1999, Murayama and Pierce 2002). Although the loop factor is a complicated function of the sparticle masses and mixing angles, it nevertheless has the following simple dependence on the overall gaugino and scalar masses,

$$\frac{\lambda_t \lambda_\tau}{16\pi^2} \frac{\sqrt{\mu^2 + M_{1/2}^2}}{m_{16}^2}. \tag{64}$$

Thus in order to minimize this factor one needs

$$\mu,\ M_{1/2} \quad \text{SMALL} \tag{65}$$

and

$$m_{16} \quad \text{large}. \tag{66}$$

**Table 3.** GUT threshold corrections in three different theories. The first column is the minimal $SU(5)$ SUSY theory (Dimopoulos and Georgi 1981, Sakai 1982, Goto and Nihei 1999, Murayama and Pierce 2002), the second column is $SU(5)$ with 'natural' Higgs doublet–triplet splitting (Grinstein 1982, Masiero *et al* 1982, Altarelli *et al* 2000) and the third column is the minimal $SO(10)$ SUSY model (Blažek *et al* 1999, 2000, Dermíšek *et al* 2001, Blažek *et al* 2002a, b).

| | Minimal $SU(5)$ | $SU(5)$ 'Natural' $D/T$ | Minimal $SO(10)$ |
|---|---|---|---|
| $\epsilon_3^{\text{GUT breaking}}$ (%) | 0 | −7.7 | −10 |
| $\epsilon_3^{\text{Higgs}}$ (%) | −4 | +3.7 | +6 |
| $M_T^{\text{eff}}$ (GeV) | $2 \times 10^{14}$ | $3 \times 10^{18}$ | $6 \times 10^{19}$ |

### 4.4. $M_t^{\text{eff}}$

The largest uncertainty in the nucleon decay rate is due to the colour triplet Higgs mass parameter, $M_T^{\text{eff}}$. As $M_T^{\text{eff}}$ increases, the nucleon lifetime increases. Thus it is useful to obtain an upper bound on the value of $M_T^{\text{eff}}$. This constraint comes from imposing perturbative gauge coupling unification (Lucas and Raby 1997, Goto and Nihei 1999, Babu *et al* 2000, Altarelli *et al* 2000, Dermíšek *et al* 2001, Murayama and Pierce 2002). Recall that in order to fit the low energy data a GUT scale threshold correction

$$\epsilon_3 \equiv \frac{(\alpha_3(M_G) - \tilde{\alpha}_G)}{\tilde{\alpha}_G} \sim -3\% \text{ to } -4\%, \tag{67}$$

is needed. $\epsilon_3$ is a logarithmic function of particle masses of order $M_G$, with contributions from the electroweak Higgs and GUT breaking sectors of the theory.

$$\epsilon_3 = \epsilon_3^{\text{Higgs}} + \epsilon_3^{\text{GUT breaking}} + \cdots, \tag{68}$$

$$\epsilon_3^{\text{Higgs}} = \frac{3\alpha_G}{5\pi} \ln\left(\frac{M_T^{\text{eff}}}{M_G}\right). \tag{69}$$

In table 3 we have analysed three different GUT theories—the minimal $SU(5)$ model, an $SU(5)$ model with natural doublet–triplet splitting and minimal $SO(10)$ (which also has natural doublet–triplet splitting). We have assumed that the low energy data, including weak scale threshold corrections, requires $\epsilon_3 = -4\%$. We have then calculated the contribution to $\epsilon_3$ from the GUT breaking sector of the theory in each case.

The minimal $SU(5)$ is defined by its minimal GUT breaking sector with one $SU(5)$ adjoint $\Sigma$. The one-loop contribution from this sector to $\epsilon_3$ vanishes. Hence, the −4% must come from the Higgs sector alone, requiring the colour triplet Higgs mass $M_T^{\text{eff}} \sim 2 \times 10^{14}$ GeV. Note since the Higgs sector is also minimal, with the doublet masses fine-tuned to vanish, we have $M_T^{\text{eff}} \equiv M_T$. By varying the SUSY spectrum at the weak scale, we may be able to increase $\epsilon_3$ to −3% or even −2%, but this cannot save minimal $SU(5)$ from disaster due to rapid proton decay from Higgsino exchange.

In the other theories, Higgs doublet–triplet splitting is obtained without fine-tuning. This has two significant consequences. First, the GUT breaking sectors are more complicated, leading in these theories to large negative contributions to $\epsilon_3$. The maximum value, $|\epsilon_3| \sim 10\%$, in minimal $SO(10)$ is fixed by perturbativity bounds (Dermíšek *et al* 2001). Second, the effective colour triplet Higgs mass, $M_T^{\text{eff}}$, does not correspond to the mass of any particle in the theory. In fact, in both cases with 'natural' doublet–triplet splitting, the colour triplet Higgs mass is of the order of $M_G$, even though $M_T^{\text{eff}} \gg M_G$. The values for $M_T^{\text{eff}}$ in table 3 are fixed by the value of $\epsilon_3^{\text{Higgs}}$ needed to obtain $\epsilon_3 = -4\%$.

Before discussing the bounds on the proton lifetime due to the exchange of colour triplet Higgsinos, let us elaborate on the meaning of $M_T^{\text{eff}}$. Consider a simple case with two pairs of

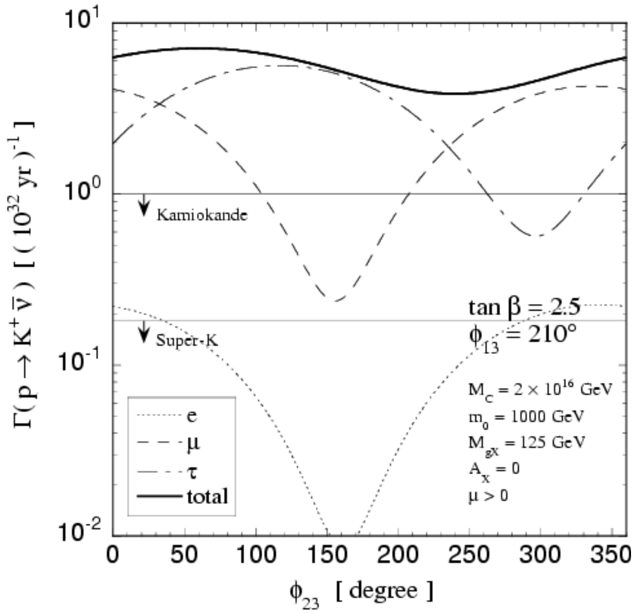


**Figure 4.** (Figure 2. Goto and Nihei (1999)). Decay rates $\Gamma(p \to K^+\bar{\nu}_i)$ ($i = e$, $\mu$ and $\tau$) as functions of the phase $\phi_{23}$ for $\tan\beta = 2.5$. The soft SUSY breaking parameters are fixed at $m_0 = 1$ TeV, $M_{\tilde{g}} = 125$ GeV and $A_0 = 0$ with $\mu > 0$. $M_T$ and $M_\Sigma$ are taken as $M_T = M_\Sigma = 2 \times 10^{16}$ GeV. The horizontal lower line corresponds to the Super-Kamiokande limit $\tau(\mathrm{p} \to K^+\bar{\nu}) > 5.5 \times 10^{32}$ yr, and the horizontal upper line corresponds to the Kamiokande limit $\tau(\mathrm{p} \to K^+\bar{\nu}) > 1.0 \times 10^{32}$ yr. The new Super-Kamiokande bound is $\tau(\mathrm{p} \to K^+\bar{\nu}) > 1.9 \times 10^{33}$ yr.

$SU(5)$ Higgs multiplets, $\{\bar{5}^i_H,\ 5^i_H\}$ with $i = 1, 2$. In addition, also assume that only $\{\bar{5}^1_H,\ 5^1_H\}$ couples to quarks and leptons. Then $M_T^{\mathrm{eff}}$ is defined by the expression

$$\frac{1}{M_T^{\mathrm{eff}}} = (M_T^{-1})_{11}, \tag{70}$$

where $M_T$ is the colour triplet Higgs mass matrix. In the cases with 'natural' doublet–triplet splitting, we have

$$M_T = \begin{pmatrix} 0 & M_G \\ M_G & X \end{pmatrix} \tag{71}$$

with

$$\frac{1}{M_T^{\mathrm{eff}}} \equiv \frac{X}{M_G^2}. \tag{72}$$

Thus for $X \ll M_G$ we have $M_T^{\mathrm{eff}} \gg M_G$ and *no* particle has mass greater than $M_G$ (Babu and Barr 1993). The large Higgs contribution to the GUT threshold correction is in fact due to an extra pair of light Higgs doublets with mass of the order of $X$.

Due to the light colour triplet Higgsino, it has been shown that minimal SUSY $SU(5)$ is ruled out by the combination of proton decay constrained by gauge coupling unification (Goto and Nihei 1999, Murayama and Pierce 2002)!! In figures 4 and 5 we reprint the figures from the paper by (Goto and Nihei 1999). In figure 4, the decay rate for $p \to K^+\bar{\nu}_i$ for any one of the three neutrinos ($i = e$, $\mu$ or $\tau$) is plotted for fixed soft SUSY breaking parameters as a function of the relative phase, $\phi_{23}$, between two $LLLL$ contributions to the decay amplitude. The phase

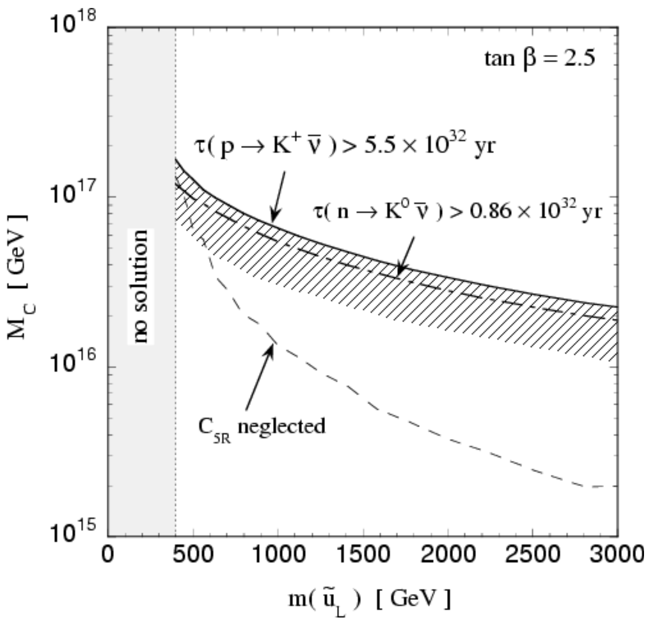


**Figure 5.** (Figure 4. Goto and Nihei 1999). Lower bound on the coloured Higgs mass, $M_T$, as a function of the left-handed scalar up quark mass, $m_{\tilde{u}_L}$. The soft SUSY breaking parameters $m_0$, $M_{\tilde{g}}$ and $A_0$ are scanned within the range $0 < m_0 < 3$ TeV, $0 < M_{\tilde{g}} < 1$ TeV and $-5 < A_0 < 5$, with $\tan\beta$ fixed at 2.5. Both signs of $\mu$ are considered. The whole parameter region of the two phases $\phi_{13}$ and $\phi_{23}$ is examined. The solid curve represents the bound derived from the Super-Kamiokande limit $\tau(p \to K^+\bar{\nu}) > 5.5\times 10^{32}$ yr, and the dashed curve represents the corresponding result without the *RRRR* effect. The left-hand side of the vertical dotted line is excluded by other experimental constraints. The dash–dotted curve represents the bound derived from the Kamiokande limit on the neutron partial lifetime $\tau(n \to K^0\bar{\nu}) > 0.86 \times 10^{32}$ yr.

$\phi_{13}$ is the relative phase between one of the *LLLL* contributions and the *RRRR* contribution. The latter contributes predominantly to the $\bar{\nu}_\tau$ final state since it is proportional to the up quark and charged lepton Yukawa couplings. As noted by Goto and Nihei (1999), the partial cancellation between *LLLL* contributions to the decay rate is filled completely by the *RRRR* contribution. It is this result that provides the stringent limit on minimal SUSY $SU(5)$. As one sees from figure 4, for the colour triplet Higgs mass $M_T = 2 \times 10^{16}$ GeV ($\equiv M_C$ in the notation of Goto and Nihei (1999)), the universal scalar mass $m_0 = 1$ TeV and $\tan\beta = 2.5$, there is no value of the phase $\phi_{23}$ that is consistent with Super-Kamiokande bounds. Note that the proton decay rate scales as $\tan\beta^2$; hence the disagreement with data only gets worse as $\tan\beta$ increases. In figure 5 the contour of constant proton lifetime is plotted in the $M_T(\equiv M_C) - m(\tilde{u}_L)$ plane, where $m(\tilde{u}_L)$ is the mass of the left-handed up squark for $\tan\beta = 2.5$. Again, there is no value of $m(\tilde{u}_L) < 3$ TeV for which the colour triplet Higgs mass is consistent with gauge coupling unification. In Goto and Nihei (1999) the up squark mass was increased by increasing $m_0$. Hence, all squarks and slepton masses increased.

One may ask whether one can suppress the proton decay rate by increasing the mass of the squarks and sleptons of the first and second generation, while keeping the third generation squarks and sleptons light (in order to preserve 'naturalness'). This is the question addressed by Murayama and Pierce (2002). They took the first and second generation scalar masses of order 10 TeV, with the third generation scalar masses less than 1 TeV. They showed that since the *RRRR* contribution does not decouple in this limit, and moreover since any possible cancellation between the *LLLL* and *RRRR* diagrams vanishes in this limit, one finds that minimal SUSY $SU(5)$ cannot be saved by decoupling the first two generations of squarks and sleptons.

Thus minimal SUSY $SU(5)$ is dead. Is this something we should be concerned about? In my opinion, the answer is No, although others may disagree (Bajc *et al* 2002). Minimal SUSY $SU(5)$ has two *a priori* unsatisfactory features:

- It requires fine-tuning for Higgs doublet–triplet splitting.
- Renormalizable Yukawa couplings due to $5_H$, $\bar{5}_H$ alone are not consistent with Fermion masses and mixing angles.

Thus, it was clear from the beginning that two crucial ingredients of a realistic theory were missing. The theories that work much better have 'natural' doublet–triplet splitting and fit Fermion masses and mixing angles.

### 4.5. *Summary of nucleon decay in four dimensions*

Minimal SUSY $SU(5)$ is excluded by the concordance of experimental bounds on proton decay and gauge coupling unification. We discussed the different factors entering the proton decay amplitude due to dimension 5 operators.

$$T(p \to K^+ + \bar{\nu}) \sim \frac{c^2}{M_T^{\mathrm{eff}}}(\text{Loop Factor})\frac{\beta_{\mathrm{lattice}}}{f_\pi} m_p. \tag{73}$$

We find the following.

- $c^2$: model dependent but constrained by Fermion masses and mixing angles.
- $\beta_{\mathrm{lattice}}$: JLQCD central value is five times larger than the previous 'conservative lower bound'. However, one still needs to reduce the systematic uncertainties of quenching and chiral Lagrangian analyses. Moreover, the new RBC result is a factor of 2 smaller than JLQCD.
- Loop Factor: $\propto (\lambda_t\, \lambda_\tau/16\pi^2)(\sqrt{(\mu^2 + M_{1/2}^2)}/m_{16}^2)$. It is minimized by taking gauginos light and the 1st and 2nd generation squarks and sleptons heavy ($>$ TeV). However, 'naturalness' requires that the stop, sbottom and stau masses remain less than of the order of 1 TeV.
- $M_T^{\mathrm{eff}}$: constrained by gauge coupling unification and GUT breaking sectors.

The *bottom line* we find for dimension 6 operators (Lucas and Raby 1997, Murayama and Pierce 2002):

$$\tau(p \to \pi^0 + e^+) \approx 5 \times 10^{36} \left(\frac{M_X}{3 \times 10^{16}\,\mathrm{GeV}}\right)^4 \left(\frac{0.015\,\mathrm{GeV}^3}{\beta_{\mathrm{lattice}}}\right)^2 \mathrm{yr}. \tag{74}$$

Note that it has been shown recently (Klebanov and Witten 2003) that string theory can possibly provide a small enhancement of the dimension 6 operators. Unfortunately the enhancement is very small. Thus it is very unlikely that these dimension 6 decay modes, $p \to \pi^0 + e^+$, will be observed.

On the other hand, for dimension 5 operators in realistic SUSY GUTs we obtain rough upper bounds on the proton lifetime coming from gauge coupling unification and perturbativity (Altarelli *et al* 2000, Babu *et al* 2000, Dermíšek *et al* 2001)

$$\tau(p \to K^+ + \bar{\nu}) < \left(\frac{1}{3} - 3\right) \times 10^{34} \left(\frac{0.015\,\mathrm{GeV}^3}{\beta_{\mathrm{lattice}}}\right)^2 \mathrm{yr}. \tag{75}$$

Note that in general

$$\tau(\mathrm{n} \to K^0 + \bar{\nu}) < \tau(\mathrm{p} \to K^+ + \bar{\nu}). \tag{76}$$

Moreover, other decay modes may be significant, *but they are very model dependent*, for example (Carone *et al* 1996, Babu *et al* 2000)

$$\mathrm{p} \to \pi^0 + \mathrm{e}^+, \qquad K^0 + \mu^+. \tag{77}$$

*4.6. Proton decay in more than four dimensions*

We should mention that there has been a recent flurry of activity on SUSY GUTs in extra dimensions beginning with the work of Kawamura (2001a, b). However, the study of extra dimensions on orbifolds goes back to the original work of Dixon *et al* (1985, 1986) in string theory. Although this interesting topic would require another review, let me just mention some pertinent features here. In these scenarios, grand unification is only a symmetry in extra dimensions that are then compactified at scales of the order of $1/M_G$. The effective four-dimensional theory, obtained by orbifolding the extra dimensions, has only the SM gauge symmetry or at most a PS symmetry that is then broken by the standard Higgs mechanism. In these theories, it is possible to eliminate completely the contribution of dimension 5 operators to nucleon decay. This may be a consequence of global symmetries as shown by Dine *et al* (2002), Witten (2002) or a continuous $U(1)_R$ symmetry (with $R$ parity a discrete subgroup) (Hall and Nomura 2002). (Note that it is also possible to eliminate the contribution of dimension 5 operators in four-dimensional theories with extra symmetries (Babu and Barr 2002), but these four-dimensional theories are quite convoluted. Thus it is difficult to imagine that nature takes this route. On the other hand, in one or more small extra dimensions the elimination of dimension 5 operators is very natural.) Thus, at first glance, nucleon decay in these theories may be extremely difficult to see. However, this is not necessarily the case. Once again we must consider the consequences of grand unification in extra dimensions and gauge coupling unification.

Extra dimensional theories are non-renormalizable and therefore require an explicit cutoff scale, $M_*$, assumed to be larger than the compactification scale, $M_c$. The Kałuza–Klein excitations above $M_c$ contribute to threshold corrections to gauge coupling unification evaluated at the compactification scale. The one-loop renormalization of gauge couplings is given by

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha(M_*)} + b_i \log\left(\frac{M_c}{\mu}\right) + \Delta_i, \tag{78}$$

where $\Delta_i$ are the threshold corrections due to all the KK modes from $M_c$ to $M_*$ and can be expressed as $\Delta_i = b_i^{\text{eff}} \log(M_*/M_c)$ (Nomura *et al* 2001, Hall and Nomura 2001, 2002a, Contino *et al* 2002, Nomura 2002). In a five-dimensional $SO(10)$ model, broken to PS by orbifolding and to the MSSM via Higgs VEVs on the brane, it was shown (Kim and Raby 2003) that the KK threshold corrections take a particularly simple form,

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha(M_*)} + b_i^{\text{MSSM}} \log\left(\frac{\hat{M}_c}{\mu}\right) + \hat{\Delta}_i \tag{79}$$

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha(M_*)} + b_i^{\text{MSSM}} \log\left(\frac{\hat{M}_c}{\mu}\right) + \frac{2}{3} b_i^{\text{SM}}(V) \log\left(\frac{M_*}{\hat{M}_c}\right) \tag{80}$$

with

$$\hat{\Delta}_{\text{gauge}} = \frac{2}{3} b_i^{\text{SM}}(V) \log\left(\frac{M_*}{\hat{M}_c}\right), \tag{81}$$

$$\hat{\Delta}_{\text{Higgs}} = 0 \tag{82}$$

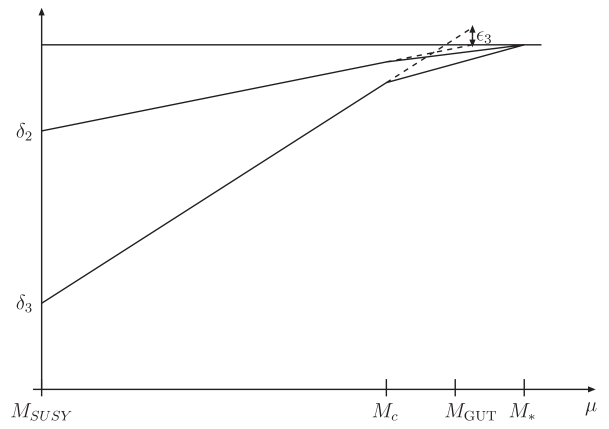


**Figure 6.** Differential running of $\delta_2 = 2\pi(1/\alpha_2 - 1/\alpha_1)$ and $\delta_3 = 2\pi(1/\alpha_3 - 1/\alpha_1)$.

and $\hat{M}_c \equiv (\pi R/2)^{-1}$. Here $b^{\rm MSSM}$ includes the gauge sector and the Higgs sector together, and $b^{\rm SM}(V)$ includes the gauge sector only. The running equation is very simple and permits us to compare directly with well known four-dimensional SUSY GUTs. In the minimal four-dimensional $SU(5)$ SUSY model, the running equation is given by

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha(M_{\rm GUT})} + b_i^{\rm MSSM}\log\left(\frac{M_T}{\mu}\right) + b_i^{\rm SM}(V)\log\left(\frac{M_{\rm GUT}}{M_T}\right), \tag{83}$$

where $M_T$ is the colour triplet Higgs mass. As discussed earlier, we can achieve unification by adjusting the colour triplet Higgs mass $M_T = 2 \times 10^{14}$ GeV (see table 3).

Comparing equations (80) and (83), we observe that if

$$\hat{M}_c = M_T, \qquad \frac{M_*}{\hat{M}_c} = \left(\frac{M_{\rm GUT}}{M_T}\right)^{3/2},$$

the same unification is achieved here. Therefore we get $\hat{M}_c = 2 \times 10^{14}$ GeV and $M_* = 3.7 \times 10^{17}$ GeV by using $M_{\rm GUT} = 3 \times 10^{16}$ GeV. A few remarks are in order. There is no problem with proton decay due to dimension 5 operators, even though the colour triplet Higgs mass is of the order of $10^{14}$ GeV, since these operators are excluded by $R$ symmetry. In a five-dimensional model, the four-dimensional GUT scale has no fundamental significance. The couplings unify at the cutoff scale and there is no scale above which we have a perturbative $SO(10)$ GUT.

In figure 6 we show the differential running of couplings for two independent cases. We show the couplings

(i) for four-dimensional gauge theories with GUT scale thresholds, in which the GUT scale is defined as the point where $\alpha_1$ and $\alpha_2$ meet and $\epsilon_3$ is the relative shift in $\alpha_3$ due to threshold corrections, and
(ii) for a five-dimensional $SO(10)$ model where the three couplings meet at the cutoff scale and the threshold corrections due to the KK tower is defined at the compactification scale.

In both cases, the running of the gauge couplings below the compactification scale must be the same. Thus we can use the low energy fits from four-dimensional theories to constrain a five-dimensional theory.

Note that since the KK modes of the baryon number violating $\{X, \bar{X}\}$ gauge Bosons have mass starting at the compactification scale $M_c \approx 10^{14}$ GeV, we must worry whether proton decay due to dimension 6 operators is safe. It has been shown (Altarelli and Feruglio 2001, Hebecker and March-Russell 2002, Hall and Nomura 2002b) that this depends on where in the extra dimensions the quarks and leptons reside. If they are on symmetric orbifold fixed points, i.e. symmetric under the GUT symmetry, then this leads to the standard dimension 6 proton decay operators, which is ruled out for the first and second families of quarks and leptons. Hence the first two families must be either in the bulk or on broken symmetry fixed points. If they are in the bulk, then the $\{X, \bar{X}\}$ mediated processes take massless modes to KK excitations, which is not a problem. Otherwise, if the first two families are on the broken symmetry FP, the wave functions for the $\{X, \bar{X}\}$ Bosons vanish there. However, certain effective higher dimensional operators on the broken symmetry orbifold fixed points can allow the $\{X, \bar{X}\}$ Bosons to couple to the first two families. These operators are allowed by symmetries, and they lead naturally to proton lifetimes for $p \to e^{+}\pi^{0}$ of the order of $10^{34\pm2}$ yr. The large uncertainty is due to the order of magnitude of uncertainty in the coefficient of these new effective operators.

Before closing this section, we should make a few comments on theories with large extra dimensions of the order of 1 TeV or as large as 1 mm. In some of these theories, only gravity lives in higher dimensions, while the ordinary matter and gauge interactions reside typically on a three-dimensional brane (Arkani-Hamed *et al* 1998), while if the extra dimensions are no larger than 1 TeV, all matter may live in the bulk. Such theories replace SUSY with new and fundamental non-perturbative physics at the 1–10 TeV scale. These theories must address the question of why dimension 6 proton decay operators, suppressed only by 1 TeV$^{2}$, are not generated. There are several ideas in the literature with suggested resolutions to this problem. They include the following:

- conserved baryon number on the brane with anomaly cancelling Chern–Simons terms in the bulk, or
- displacing quarks from leptons on a 'fat' brane with a gaussian suppression of the overlap of the quark/lepton wave functions.

A novel solution to the problem of proton decay is found in fix space–time dimensional theories with all matter spanning the two extra dimensions. It has been shown that in such theories (Appelquist *et al* 2001) a $Z(8)$ remnant of the six-dimensional Lorentz group is sufficient for suppressing proton decay to acceptable levels. These theories predict very high dimensional proton decay operators with multi-particle final states.

## 5. Fermion masses and mixing

Low energy SUSY provides a natural framework for solving the gauge hierarchy problem, while SUSY GUTs make the successful prediction for gauge coupling unification and the, still unverified, prediction for proton decay. But these successes affect only a small subset of the unexplained arbitrary parameters in the SM having to do with the $Z$ and Higgs masses (i.e. the weak scale) and the three gauge coupling constants. On the other hand, the sector of the SM with the largest number of arbitrary parameters has to do with Fermion masses and mixing angles. Grand unification also provides a natural framework for discussing the problem of Fermion masses since it arranges quarks and leptons naturally into a few irreducible multiplets, thus explaining their peculiar pattern of gauge charges, i.e. charge quantization and the family structure. Moreover, it has been realized for some time that the masses and mixing angles of quarks and leptons are ordered with respect to their generation (or family) number. The first generation of quarks and leptons, $\{u, d, e, \nu_e\}$, are the lightest; the second generation, $\{c, s, \mu, \nu_\mu\}$, are all heavier than the first, and the third generation, $\{t, b, \tau, \nu_\tau\}$, are the

**Table 4.** Quark and lepton masses in units of MeV c$^2$.

| | $\nu_e$ | $e$ | $u$ | $d$ |
|---|---|---|---|---|
| 1st | $\leqslant 10^{-7}$ | $\frac{1}{2}$ | 2 | 5 |
| 2nd | $\leqslant 10^{-7}$ | 105.6 | 1300 | 120 |
| 3rd | $\leqslant 10^{-7}$ | 1777 | 174 000 | 4500 |

heaviest (see table 4). In addition, the first two generations have a weak mixing angle given by the Cabibbo angle, $\theta_C$. If we define $\lambda \equiv \sin\theta_C \sim 0.22$, then the mixing of the second and third generations is of the order of $\lambda^2$, and that of the first and third is the weakest mixing, of the order of $\lambda^3$. This pattern is very elegantly captured in the Wolfenstein representation of the CKM matrix given by

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \approx \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & (\rho - \mathrm{i}\eta)\, A\, \lambda^3 \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\, \lambda^2 \\ (1 - \rho - \mathrm{i}\eta)\, A\, \lambda^3 & -A\, \lambda^2 & 1 \end{pmatrix}, \tag{84}$$

with $\lambda \sim \frac{1}{5}$, $A \sim 1$ and $|\rho - \mathrm{i}\eta| \sim \frac{1}{2}$.

Although the fundamental explanation for three families is still wanting, it is natural to assume that the families transform under some family symmetry. Such a possibility is consistent with weakly coupled heterotic string theory, where, for example, $E(8) \times E(8)$ and $SO(32)$ are both large enough to contain a GUT group $\times$ a family symmetry. Such a family symmetry has many potential virtues.

- A spontaneously broken family symmetry can explain the hierarchy of Fermion masses and mixing angles.
- In a SUSY theory, the family symmetry acts on both Fermions and sfermions, thus aligning the quark and squark, and the lepton and slepton mass matrices. This suppresses flavour violating processes.
- The combination of a family and a GUT symmetry can reduce the number of fundamental parameters in the theory, hence allowing for a predictive theory.

In the following sections, we consider several important issues. In section 5.1, we discuss the simplest case, of the third generation, only. Here we discuss the status of $SU(5)$ ($\lambda_b = \lambda_\tau$) and $SO(10)$ ($\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau}$) Yukawa unification. In section 5.2, we consider several different analyses in the literature for three family models with either $U(1)$ or non-abelian family symmetry. In section 5.3 we study the relation between charged Fermion and neutrino masses in SUSY GUTs and consider some examples giving bi-large neutrino mixing consistent with the data. Finally, in section 5.4 we discuss some experimental consequences of SUSY theories of Fermion masses. In particular, we consider $b \to s\gamma$, $(g-2)_\mu$, $B_s \to \mu^+\mu^-$, $\mu \to e\gamma$ and the electric dipole moments $d_n$ and $d_e$.

### *5.1. Yukawa unification*

Let us first discuss the most stringent case of $SO(10)$ Yukawa unification. It has been shown (Dermíšek 2001, Baer and Ferrandis 2001, Raby 2001, Blažek *et al* 2002a, b, Auto *et al* 2003, Tobe and Wells 2003) that $SO(10)$ boundary conditions at the GUT scale, for soft SUSY breaking parameters as well as for the Yukawa couplings of the third generation, are consistent

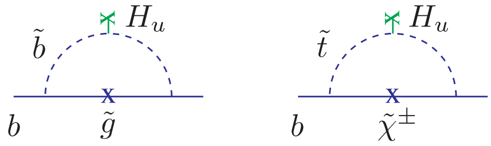


**Figure 7.** The one-loop gluino (left) and chargino (right) corrections to the bottom quark mass proportional to $\alpha_s$ (left) and $\lambda_t$ (right) and to $\tan\beta$.

with the low energy data, including $M_t$, $m_b(m_b)$, $M_\tau$, ONLY in a narrow region of SUSY breaking parameter space. Moreover, this region is also preferred by constraints from CP and flavour violation, as well as by the non-observation of proton decay. Finally we discuss the consequences for the Higgs and SUSY spectrum.

Recall that in $SO(10)$ we have the compelling unification of all quarks and leptons of one family into one irreducible representation such that $\mathbf{10} + \bar{\mathbf{5}} + \bar{\nu}_{\text{sterile}} \subset \mathbf{16}$ and the two Higgs doublets are also unified with $\mathbf{5_H}, \bar{\mathbf{5}}_{\mathbf{H}} \subset \mathbf{10_H}$. Hence, minimal $SO(10)$ also predicts Yukawa unification for the third family of quarks and leptons with $\lambda_b = \lambda_t = \lambda_\tau = \lambda_{\nu_\tau} = \lambda$ at the GUT scale (Banks 1988, Olechowski and Pokorski 1988, Pokorski 1990, Shafi and Ananthanarayan 1991, Ananthanarayan *et al* 1991, 1993, 1994, Anderson *et al* 1993, 1994).

Ignoring threshold corrections, one can use the low energy value for $m_b/m_\tau$ to fix the universal Yukawa coupling, $\lambda$. RG running from $M_G$ to $M_Z$ then gives $\lambda_\tau(M_Z)$. Then, given $m_\tau = \lambda_\tau(v/\sqrt{2})\cos\beta$, we obtain $\tan\beta \approx 50$. Finally, a prediction for the top quark mass is given with $m_t = \lambda_t(v/\sqrt{2})\sin\beta \sim 170 \pm 20\,\text{GeV}$ (see Anderson *et al* (1993)).

Note that in this case there are insignificant GUT threshold corrections from gauge and Higgs loops. Nevertheless, the previous discussion is essentially a *straw man* since there are *huge* threshold corrections at the weak scale (Carena *et al* 1994, Hempfling 1994, Hall *et al* 1994, Blažek *et al* 1995). The dominant contributions are from gluino and chargino loops plus an overall logarithmic contribution due to finite wave function renormalization given by $\delta m_b/m_b = \Delta m_b^{\tilde{g}} + \Delta m_b^{\tilde{\chi}} + \Delta m_b^{\log} + \cdots$ (see figure 7). These contributions are approximately of the form

$$\Delta m_b^{\tilde{g}} \approx \frac{2\alpha_3}{3\pi}\,\frac{\mu m_{\tilde{g}}}{m_{\tilde{b}}^2}\tan\beta, \tag{85}$$

$$\Delta m_b^{\tilde{\chi}^+} \approx \frac{\lambda_t^2}{16\pi^2}\,\frac{\mu A_t}{m_{\tilde{t}}^2}\tan\beta, \tag{86}$$

$$\Delta m_b^{\log} \approx \frac{\alpha_3}{4\pi}\log\left(\frac{\tilde{m}^2}{M_Z^2}\right) \sim 6\% \tag{87}$$

with $\Delta m_b^{\tilde{g}} \sim -\Delta m_b^{\tilde{\chi}} > 0$ for $\mu > 0$ (with our conventions). These corrections can easily be of the order of $\sim 50\%$. However, good fits require $-4\% < \delta m_b/m_b < -2\%$.

Note that, the data favour $\mu > 0$. First consider the process $b \to s\gamma$. The chargino loop contribution dominates typically and has opposite sign to the SM and charged Higgs contributions for $\mu > 0$, thus reducing the branching ratio. This is desirable since the SM contribution is a little too large. Hence, $\mu < 0$ is problematic when trying to fit the data. Second, the recent measurement of the anomalous magnetic moment of the muon suggests a contribution due to NEW physics given by $a_\mu^{\text{NEW}} = 22.1(11.3) \times 10^{-10}$ or $7.4(10.5) \times 10^{-10}$ (Muon g - 2 Collaboration 2002, Davier *et al* 2003), depending on whether one uses $e^+e^-$ or $\tau$ hadronic decay data to evaluate the leading order hadronic contributions.

For other recent theoretical analyses and references to previous work (Hagiwara *et al* 2003, Melnikov and Vainshtein 2003). However, in SUSY the sign of $a_\mu^{\rm NEW}$ is correlated with the sign of $\mu$ (Chattopadhyay and Nath 1996). Once again the data favour $\mu > 0$.

Before discussing the analysis of Yukawa unification, specifically that of Blažek *et al* (2002a, b), we need to consider one important point. $SO(10)$ Yukawa unification with the minimal Higgs sector necessarily predicts large $\tan\beta \sim 50$. In addition, it is much easier to obtain EWSB with large $\tan\beta$ when the Higgs up/down masses are split ($m_{H_u}^2 < m_{H_d}^2$) (Matalliotakis and Nilles 1995, Olechowski and Pokorski 1995, Polonsky and Pomarol 1995, Murayama *et al* 1996, Rattazzi and Sarid 1996). In the following analysis we consider two particular Higgs splitting schemes we refer to as Just So and D term splitting[8]. In the first case the third generation squark and slepton soft masses are given by the universal mass parameter, $m_{16}$, and only Higgs masses are split: $m_{(H_u,H_d)}^2 = m_{10}^2(1 \mp \Delta m_H^2)$. In the second case we assume D term splitting, i.e. that the D term for $U(1)_X$ is non-zero, where $U(1)_X$ is obtained in the decomposition of $SO(10) \to SU(5) \times U(1)_X$. In this second case, we have $m_{(H_u,H_d)}^2 = m_{10}^2 \mp 2D_X$, $m_{(Q,\bar{u},\bar{e})}^2 = m_{16}^2 + D_X$, $m_{(\bar{d},{\rm L})}^2 = m_{16}^2 - 3D_X$. The Just So case does not at first sight appear to be very well motivated. However, we now argue that it is quite natural (Blažek *et al* 2002a, b). In $SO(10)$, neutrinos necessarily have a Yukawa term coupling active neutrinos to the 'sterile' neutrinos present in the **16**. In fact for $\nu_\tau$ we have $\lambda_{\nu_\tau}\ \bar{\nu}_\tau\ L\ H_u$ with $\lambda_{\nu_\tau} = \lambda_t = \lambda_b = \lambda_\tau \equiv \lambda$. In order to obtain a tau neutrino with mass $m_{\nu_\tau} \sim 0.05$ eV (consistent with atmospheric neutrino oscillations), the 'sterile' $\bar{\nu}_\tau$ must obtain a Majorana mass $M_{\bar{\nu}_\tau} \geqslant 10^{13}$ GeV. Moreover, since neutrinos couple to $H_u$ (and not to $H_d$) with a fairly large Yukawa coupling (of order 0.7), they naturally distinguish the two Higgs multiplets. With $\lambda = 0.7$ and $M_{\bar{\nu}_\tau} = 10^{14}$ GeV, we obtain a significant GUT scale threshold correction with $\Delta m_H^2 \approx 7\%$, about 1/2 the value needed to fit the data. At the same time, we obtain a small threshold correction to Yukawa unification $\approx 1.75\%$.

*5.1.1. $\chi^2$ Analysis (Blažek et al 2002a, b).* Our analysis is a top-down approach with 11 input parameters, defined at $M_G$, varied to minimize a $\chi^2$ function composed of nine low energy observables. The 11 input parameters are the following: $M_G$, $\alpha_G(M_G)$, $\epsilon_3$ and the Yukawa coupling, $\lambda$, and the seven soft SUSY breaking parameters, $\mu$, $M_{1/2}$, $A_0$, $\tan\beta$, $m_{16}^2$, $m_{10}^2$ and $\Delta m_H^2$ ($D_X$) for the Just So (D term) case. We use two- (one-)loop (RG) running for dimensionless (dimensionful) parameters from $M_G$ to $M_Z$ and complete one-loop threshold corrections at $M_Z$ (Pierce *et al* 1997). We require electroweak symmetry breaking using an improved Higgs potential, including $m_t^4$ and $m_b^4$ corrections in an effective two Higgs doublet model below $M_{\rm stop}$ (Haber and Hempfling 1993, Carena *et al* 1995, 1996). Note that in the figures we have chosen to keep three input parameters, $\mu$, $M_{1/2}$, $m_{16}$, fixed, minimizing $\chi^2$ with respect to the remaining eight parameters only. The $\chi^2$ function includes the nine observables: six precision electroweak data $\alpha_{\rm EM}$, $G_\mu$, $\alpha_s(M_Z) = 0.118\ (0.002)$, $M_Z$, $M_W$, $\rho_{\rm NEW}$ and the three Fermion masses $M_{\rm top} = 174.3\ (5.1)$, $m_b(m_b) = 4.20\ (0.20)$ and $M_\tau$. Figure 8 (left) shows the constant $\chi^2$ contours for $m_{16} = 1.5$ TeV in the case of Just So squark and slepton masses. We find acceptable fits ($\chi^2 < 3$) for $A_0 \sim -1.9m_{16}$, $m_{10} \sim 1.4m_{16}$ and $m_{16} \geqslant 1.2$ TeV. The best fits are for $m_{16} \geqslant 2$ TeV with $\chi^2 < 1$. Figure 1 (right) shows the constant $\chi^2$ contours for $m_{16} = 2$ TeV. Figure 9 gives the constant $m_b(m_b)$ and $\delta m_b/m_b$ contours for $m_{16} = 2$ TeV. We see that the best fits, near the central value, are found with $-4\% \leqslant \delta m_b/m_b \leqslant -2\%$. The chargino contribution (equation (86)) is typically opposite in sign to the gluino (equation (85)) since $A_t$ runs to an infrared fixed point $\propto -M_{1/2}$ (see, e.g. Carena *et al* (1994)). Hence, in order

[8] Just So Higgs splitting has also been referred to as non-universal Higgs mass splitting or NUHM (Berezinsky *et al* 1996, Blažek *et al* 1997a, b, Nath and Arnowitt 1997).

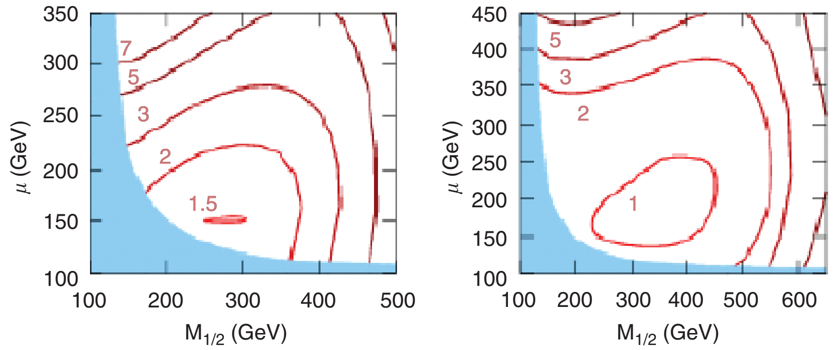


**Figure 8.** $\chi^2$ contours for $m_{16} = 1.5$ TeV (left) and $m_{16} = 2$ TeV (right). The shaded region is excluded by the chargino mass limit $m_{\tilde{\chi}^+} > 103$ GeV.

to cancel the positive contribution of both the log (equation (87)) and gluino contributions, a large negative chargino contribution is needed. This can be accomplished for $-A_t > m_{\tilde{g}}$ and $m_{\tilde{t}_1} \ll m_{\tilde{b}_1}$. The first condition can be satisfied for $A_0$ large and negative, which helps pull $A_t$ away from its infrared fixed point. The second condition is also aided by large $A_t$. However, in order to obtain a large enough splitting between $m_{\tilde{t}_1}$ and $m_{\tilde{b}_1}$, large values of $m_{16}$ are needed. Note that for Just So scalar masses the lightest stop is typically lighter than the sbottom. We typically find $m_{\tilde{b}_1} \sim 3m_{\tilde{t}_1}$. On the other hand, $D$ term splitting with $D_X > 0$ gives $m_{\tilde{b}_1} \leqslant m_{\tilde{t}_1}$. As a result, in the case of Just So boundary conditions excellent fits are obtained for top, bottom and tau masses, while for $D$ term splitting the best fits give $m_b(m_b) \geqslant 4.59$ GeV [9].

The bottom line is that Yukawa unification is only possible in a narrow region of SUSY parameter space with

$$A_0 \sim -1.9m_{16}, \qquad m_{10} \sim 1.4m_{16}, \tag{88}$$

$$(\mu,\ M_{1/2}) \sim 100\text{–}500\,\text{GeV} \qquad \text{and} \qquad m_{16} \geqslant 1.2\,\text{TeV}. \tag{89}$$

It would be nice to have some *a priori* reason for the fundamental SUSY breaking mechanism to give these soft SUSY breaking parameters. However, without such an *a priori* explanation, it is all the more interesting and encouraging to recognize two additional reasons for wanting to be in this narrow region of parameter space.

(i) One mechanism for suppressing large flavour violating processes in SUSY theories is demanding heavy first and second generation squarks and sleptons (with mass $\gg$1 TeV) and the third generation scalars lighter than 1 TeV. Since the third generation scalars couple

[9] Note that Auto *et al* (2003), Tobe and Wells (2003) use a bottom-up approach in their analysis. The results of Auto *et al* (2003) are in significant agreement with those of Blažek *et al* (2002a, b), except for the fact that they only find Yukawa unification for larger values of $m_{16}$ of the order of 8 TeV and higher. The likely reason for this discrepancy has been explained by Tobe and Wells (2003). They show that (and I quote them) 'Yukawa couplings at the GUT scale are very sensitive to the low-energy SUSY corrections. An O(1%) correction at low energies can generate close to a O(10%) correction at the GUT scale. This extreme IR sensitivity is one source of the variance in conclusions in the literature. For example, coarse-grained scatter plot methods, which are so useful in other circumstances, lose some of their utility when IR sensitivity is so high. Furthermore, analyses that use only central values of measured Fermion masses do not give a full picture of what range of SUSY parameter space enables third family Yukawa unification, since small deviations in low-scale parameters can mean so much to the high-scale theory viability'. It should also be noted that (Tobe and Wells 2003) suggest a different soft SUSY breaking solution consistent with Yukawa unification. In particular, they suggest an extension of AMSB with the addition of a large universal scalar mass $m_0 \geqslant 2$ TeV.

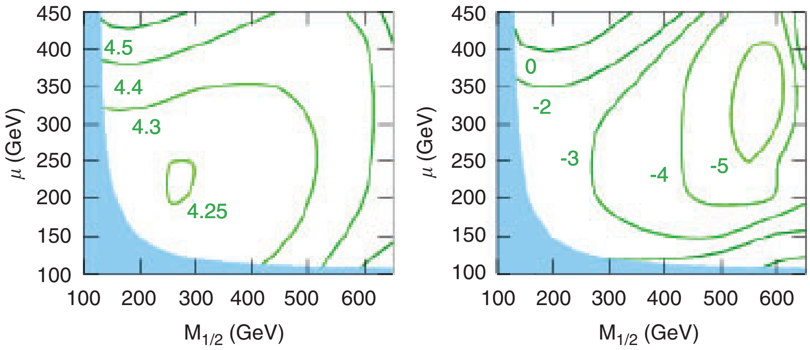


**Figure 9.** Contours of constant $m_b(m_b)$ (GeV) (left) and $\Delta m_b$ in (right) for $m_{16} = 2$ TeV.

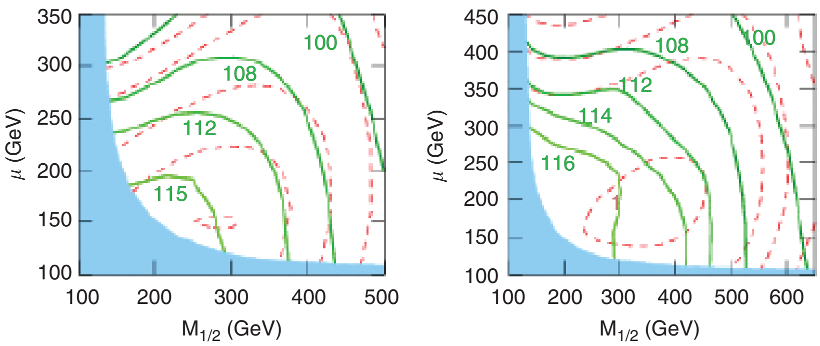


**Figure 10.** Contours of constant $m_h$ (GeV) (——) with $\chi^2$ contours from figure 1 (······) for $m_{16} = 1.5$ TeV (left) and $m_{16} = 2$ TeV (right).

most strongly to the Higgs, this limit can still leave a 'naturally' light Higgs (Dimopoulos and Giudice 1995). It was shown that this inverted scalar mass hierarchy can be obtained 'naturally,' i.e. purely as a consequence of RG running from $M_G$ to $M_Z$, with suitably chosen soft SUSY breaking boundary conditions at $M_G$ (Bagger *et al* 1999, 2000). All that is needed is $SO(10)$ boundary conditions for the Higgs mass (i.e. $m_{10}$), squark and slepton masses (i.e. $m_{16}$) and a universal scalar coupling, $A_0$. In addition, they must be in the ratio (Bagger *et al* 2000)

$$A_0^2 = 2\,m_{10}^2 = 4\,m_{16}^2 \qquad \text{with } m_{16} \gg 1\,\text{TeV}. \tag{90}$$

(ii) In order to suppress the rate for proton decay due to dimension 5 operators, one must also demand (Dermíšek *et al* 2001)

$$(\mu,\ M_{1/2}) \ll m_{16}, \qquad \text{with } m_{16} > \text{a few tetraelectronvolts}. \tag{91}$$

*5.1.2. Consequences for Higgs and SUSY Searches.* In figure 10, we show the constant light Higgs mass contours for $m_{16} = 1.5$ and 2 TeV (solid lines) with the constant $\chi^2$ contours overlayed (dotted lines). Yukawa unification for $\chi^2 \leqslant 1$ prefers clearly a light Higgs with mass in a narrow range, 112–118 GeV.

In this region the CP odd $A$, the heavy CP even Higgs $H$ and the charged Higgs Bosons $H^{\pm}$ are also quite light. In addition we find the mass of $\tilde{t}_1 \sim (150\text{–}250)$ GeV, $\tilde{b}_1 \sim (450\text{–}650)$ GeV, $\tilde{\tau}_1 \sim (200\text{–}500)$ GeV, $\tilde{g} \sim (600\text{–}1200)$ GeV, $\tilde{\chi}^+ \sim (100\text{–}250)$ GeV and $\tilde{\chi}^0 \sim (80\text{–}170)$ GeV.

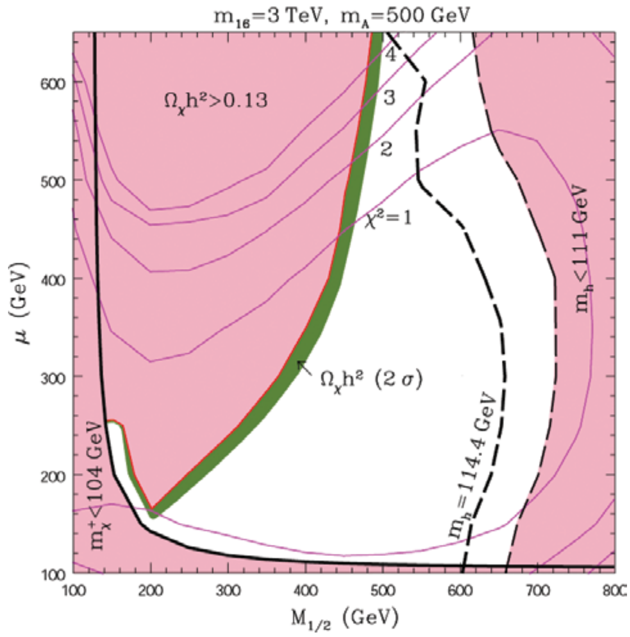


**Figure 11.** Constant $\chi^2$ contours as a function of $\mu$, $M_{1/2}$ for $m_{16} = 3$ TeV. Note the much larger range of parameters with $\chi^2 < 1$ for this larger value of $m_{16}$. The green (dark grey) shaded region is consistent with the recent WMAP data for dark matter abundance of the neutralino LSP. The light shaded region in the lower left-hand corner (separated by the solid line) is excluded by chargino mass limits, while the light shaded region in the upper left (right side) is excluded by a cosmological dark matter abundance that is too large (Higg mass that is too light). To the left of the vertical contour for a light Higgs with mass at the experimental lower limit, the light Higgs mass increases up to a maximum value of about 121 GeV at the lower left-hand acceptable boundary.

All first and second generation squarks and sleptons have mass of order $m_{16}$. The light stop and chargino may be visible at the Tevatron. With this spectrum we expect $\tilde{t}_1 \to \tilde{\chi}^+ b$ with $\tilde{\chi}^+ \to \tilde{\chi}_1^0 \bar{l} \nu$ to be dominant. Lastly $\tilde{\chi}_1^0$ is the LSP and possibly a good dark matter candidate (see, e.g. Roszkowski *et al* (2001) and figure 11).

Our analysis thus far has only included third generation Yukawa couplings and hence no flavour mixing. If we now include the second family and 2–3 family mixing, consistent with $V_{cb}$, we obtain new and significant constraints on $m_{\tilde{t}_1}$ and $m_A$. The stop mass is constrained by $B(b \to s\gamma)$ to satisfy $m_{\tilde{t}}^{\mathrm{MIN}} > 450$ GeV (unfortunately increasing the bottom quark mass). In addition, as shown by Babu and Kolda (2000), Dedes *et al* (2001), Isidori and Retico (2001) the one-loop SUSY corrections to CKM mixing angles (see Blažek *et al* (1995)) result in flavour violating neutral Higgs couplings. As a consequence the CDF bound on the process $B_s \to \mu^+\mu^-$ places a lower bound on $m_A \geqslant 200$ GeV (Babu and Kolda 2000, Dedes *et al* 2001, Isidori and Retico 2001). $\chi^2$, on the other hand, increases as $m_{A^0}$ increases. However, the increase in $\chi^2$ is less than 60% for $m_A < 400$ GeV. Note that the $H^\pm$, $H^0$ masses increase linearly with $m_A$.

*5.1.3. $SU(5)$ Yukawa unification.* Now consider Yukawa unification in $SU(5)$. In this case we only have the GUT relation $\lambda_b = \lambda_\tau$. The RG running of the ratio $\lambda_b/\lambda_\tau$ to low energies then increases (decreases) due to QCD (Yukawa) interactions. In addition, neglecting Yukawa interactions, this ratio is too large at the weak scale. For a top quark mass $M_t \sim 175$ GeV, a good fit is obtained for small tan $\beta \sim 1$ (Dimopoulos *et al* 1992, Barger *et al* 1993). In this case, only the top quark Yukawa coupling is important, while for large tan $\beta \sim 50$ we recover the results of $SO(10)$ Yukawa unification. For a recent analysis, see Barr and Dorsner (2003).

### *5.2. Fermion mass hierarchy and family symmetry*

In both the SM and the MSSM, the observed pattern of Fermion masses and mixing angles has its origin in the Higgs–quark and Higgs–lepton Yukawa couplings. In the SM these complex $3 \times 3$ matrices are arbitrary parameters that are under-constrained by the 13 experimental observables (nine charged Fermion masses and four quark mixing angles). (We consider neutrino masses and mixing angles in the following section.) In the MSSM, more of the Yukawa parameters are in principle observable, since left- and right-handed Fermion mixing angles affect squark and slepton masses and mixing. (We consider this further in section 5.4.) What can SUSY say about Fermion masses? SUSY alone constrains the Yukawa sector of the theory simply by requiring that all terms in the superpotential are holomorphic. Combined with flavour symmetries, the structure of Fermion masses can be constrained severely. On the other hand, the only information we have about these flavour symmetries is the Fermion masses and mixing angles themselves, as well as the multitude of constraints on flavour violating interactions. There are, perhaps, many different theories with different family symmetries that fit the precision low energy data (including Fermion masses and mixing angles). The goal is to find a set of predictive theories, i.e with fewer arbitrary parameters than data, that fit this data. The more predictive the theory, the more testable it will be[10]. Within the context of the MSSM, theories have been constructed with $U(1)$ family symmetries (Binetruy *et al* 1996, Elwood *et al* 1997, 1998, Faraggi and Pati 1998, Irges *et al* 1998, Kakizaki and Yamaguchi 2002, Dreiner *et al* 2003), with discrete family symmetries (Frampton and Kephart 1995a, b, Hall and Murayama 1995, Carone *et al* 1996, Carone and Lebed 1999, Frampton and Rasin 2000, Aranda *et al* 2000) or non-abelian family symmetries (Hall and Randall 1990, Dine *et al* 1993, Nir and Seiberg 1993, Pouliot and Seiberg 1993, Leurer *et al* 1993,1994, Pomarol and Tommasini 1995, Hall and Murayama 1995, Dudas *et al* 1995,1996, Barbieri *et al* 1996, Arkani-Hamed *et al* 1995,1996, Barbieri *et al* 1997, Eyal 1998). However, the most predictive theories combine both grand unified and family symmetries (Kaplan and Schmaltz 1994, Babu and Mohapatra 1995, Lucas and Raby 1996, Frampton and Kong 1996, Allanach *et al* 1997, Barbieri and Hall 1997, Barbieri *et al* 1997, Blažek *et al* 1997, Berezhiani 1998, Blažek *et al* 1999, 2000, Dermíšek and Raby 2000, Shafi and Tavartkiladze 2000, Albright and Barr 2000, 2001, Altarelli *et al* 2000, Babu *et al* 2000, Berezhiani and Rossi 2001, Kitano and Mimura 2001, Maekawa 2001, King and Ross 2003, Chen and Mahanthappa 2003, Raby 2003, Ross and Velasco–Sevilla 2003, Goh *et al* 2003, Aulakh *et al* 2003). The Yukawa couplings in a predictive theory are defined completely in terms of the states and symmetries of the theory. The ultimate goal of this programme is to construct one (or more) of these predictive theories, providing good fits to the data, in terms of a more fundamental theory, such as M theory. Only then will higher order corrections to the theory be under full control. It is important to remark at this stage that any theory, derived from some fundamental theory, includes non-renormalizable higher dimension operators. The higher dimension operators are suppressed by the fundamental scale (e.g. the string scale, $M_S$), which is assumed to be greater than the GUT scale, $M_G$. As we shall now see, these higher dimension operators are useful in explaining the hierarchy of Fermion masses.

[10] Of course, if SUSY or proton decay is observed, then there will be much more low energy data available for testing these theories.

**Table 5.** $U(1)$ charge, $Q$, of Higgs and matter fields in the (1st, 2nd, 3rd) generations.

| Field | $H_u$ | $H_d$ | $10 = \{Q, \bar{u}, \bar{e}\}$ | $\bar{5} = \{\bar{d},\ \mathrm{L}\}$ | $\bar{\nu}$ |
|---|---|---|---|---|---|
| $Q$ | $-2$ | 1 | (4, 3, 1) | (4, 2, 2) | (1, −1, 0) |

The $3 \times 3$ up, down and charged lepton mass matrices are given by the mass terms:

$$\mathcal{L}_{\text{mass}} = uY_u\bar{u}\langle H_u\rangle + dY_d\bar{d}\langle H_d\rangle + eY_e\bar{e}\langle H_d\rangle. \tag{92}$$

Empirical descriptions of the quark mass matrices have been discussed in all the papers referred to above. As an example, consider the following theory incorporating the hierarchy of masses and mixing angles in an $SU(5)$ SUSY GUT with $U(1)$ family symmetry by Altarelli *et al* (2000)

$$Y_u = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \qquad Y_d = Y_e^T = \begin{pmatrix} \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \end{pmatrix} \lambda^4. \tag{93}$$

Order 1 coefficients of the matrix elements are implicit. We then obtain the rough empirical relations.

$$\begin{aligned} \frac{m_c}{m_t} &\sim \frac{m_u}{m_c} \sim V_{\text{cb}}^2 \approx \lambda^4 \\ \frac{m_s}{m_b} &\sim \frac{m_d}{m_s} \sim V_{\text{us}}^2 \approx \lambda^2. \end{aligned} \tag{94}$$

In addition, the Yukawa matrices for down quarks and charged leptons satisfy the $SU(5)$ relations

$$\lambda_b = \lambda_\tau, \qquad \lambda_s = \lambda_\mu, \qquad \lambda_d = \lambda_e. \tag{95}$$

This works for the third generation, as discussed in section 5.1.3; however, it clearly does not work for the first and second generations, where it gives the unacceptable prediction

$$20 \approx \frac{m_s}{m_d} = \frac{m_\mu}{m_e} \approx 200. \tag{96}$$

Hence in this $SU(5)$ model with $U(1)$ family symmetry, an additional Higgs in the 75-dimensional representation (with $U(1)$ charge zero) also contributes to down quark and charged lepton Yukawa matrices (Altarelli *et al* 2000). The arbitrary, order 1 coefficients for each term in the Yukawa matrix are then fit to the quark masses and mixing angles and charged lepton masses. Note that in this theory there are more arbitrary parameters than Fermion mass observables; hence there are no predictions for Fermion masses and mixing angles. Nevertheless, predictions for proton decay are now obtained. Moreover, given a model for soft SUSY breaking terms like the CMSSM, one can also predict rates for flavour violating processes.

The structure for these Yukawa matrices is determined by the $U(1)$ family symmetry, spontaneously broken by a scalar field, $\phi$, with $U(1)$ charge $-1$. The symmetry breaking field, $\phi$, is inserted into each element of the Yukawa matrix in order to obtain a $U(1)$ invariant interaction. This results in effective higher dimensional operators suppressed by a scale $M$ with $\lambda \sim \langle\phi\rangle/M$. This is the Froggatt–Nielsen mechanism (Froggatt and Nielsen 1979, Berezhiani 1983, 1985, Dimopoulos 1983, Bagger *et al* 1984). For a review, see Raby (1995). Given the $U(1)$ charge assignments in table 5 (Altarelli *et al* 2000), we obtain the Yukawa matrices in equation (93). Similar mass matrices using analogous $U(1)$ family symmetry arguments have also been considered.

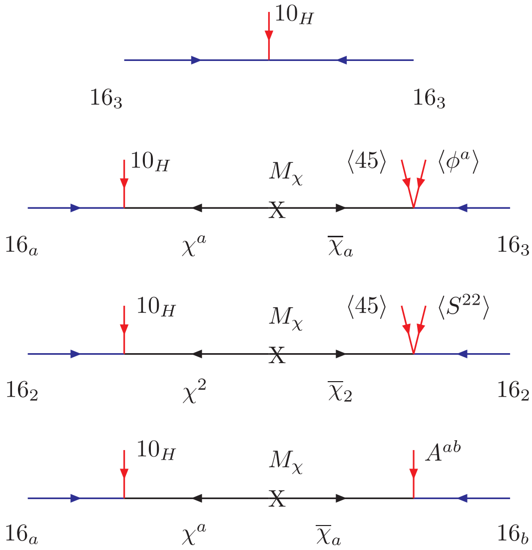


**Figure 12.** Effective Fermion mass operators. The fields $\phi^a$, $S^{ab} = S^{ba}$, $A^{ab} = -A^{ba}$ break spontaneously the $SU(2) \times U(1)^n$ family symmetry with $\epsilon \propto \langle\phi^2\rangle$, $\tilde{\epsilon} \propto \langle S^{22}\rangle$ and $\epsilon' \propto \langle A^{12}\rangle$.

Using a non-abelian family symmetry, such as $SU(2) \times U(1)$ or $SU(3)$ or discrete subgroups of $SU(2)$, models with fewer arbitrary parameters in the Yukawa sector have been constructed. An example of a very predictive $SO(10)$ SUSY GUT with $SU(2) \times U(1)^n$ family symmetry is given by Barbieri *et al* (1997a, b, 1999), Blažek *et al* (1999, 2000). An analogous model can be obtained by replacing the $SU(2)$ family symmetry with a discrete subgroup $D(3)$ (Dermíšek and Raby 2000). The model incorporates the Froggatt–Nielsen mechanism with a hierarchy of symmetry breaking VEVs explaining the hierarchy of Fermion masses. The effective Fermion mass operators are given in figure 12. In particular, the family symmetry breaking pattern

$$SU_2 \times U_1 \quad \longrightarrow \quad U_1 \quad \longrightarrow \quad \text{nothing} \tag{97}$$

with small parameters $\epsilon \approx \tilde{\epsilon}$ and $\epsilon'$, respectively, gives the hierarchy of masses with the 3rd family $\gg$ 2nd family $\gg$ 1st family. It includes the Georgi–Jarlskog (Georgi and Jarlskog 1979) solution to the unacceptable $SU(5)$ relation (equation (96)) with the improved relation

$$m_s \sim \tfrac{1}{3} m_\mu, \qquad m_d \sim 3 m_e. \tag{98}$$

This is obtained naturally using the VEV

$$\langle 45\rangle = (B - L) M_G. \tag{99}$$

In addition, it gives the $SO(10)$ relation for the third generation

$$\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} = \lambda \tag{100}$$

and it uses symmetry arguments to explain why $m_u < m_d$ even though $m_t \gg m_b$. Finally, the $SU_2$ family symmetry suppresses flavour violation such as $\mu \to e\gamma$. When $SO(10) \times SU(2) \times U(1)^n$ is broken to the MSSM the effective Yukawa couplings (equation (102)) are obtained. The superpotential for this simple model is given by

$$W \supset 16_3\ 10\ 16_3 + 16_a\ 10\ \chi^a + \bar{\chi}_a \left( M_\chi\ \chi^a + 45\ \frac{\phi^a}{\hat{M}}\ 16_3 + 45\ \frac{S^{ab}}{\hat{M}}\ 16_b + A^{ab}\ 16_b \right), \tag{101}$$

where $\phi^a$, $S^{ab} = S^{ba}$, $A^{ab} = -A^{ba}$ are the familon fields whose VEVs break the family symmetry, $M_\chi = \hat{M}(1+\alpha X+\beta Y)$ with $X$, $Y$ charges associated with $U(1)_{X,Y}$, the orthogonal $U(1)$ subgroups of $SO(10)$, and $\{\chi^a, \bar{\chi}_a\}$ are the heavy Froggatt–Nielsen fields. After the heavy $\chi$ states are integrated out of the theory, we obtain the effective Fermion mass operators given in figure 12. These four Feynman diagrams lead to the following Yukawa matrices for quarks and charged leptons.

$$
\begin{aligned}
Y_u &= \begin{pmatrix} 0 & \epsilon'\rho & -\epsilon\xi \\ -\epsilon'\rho & \tilde{\epsilon}\rho & -\epsilon \\ \epsilon\xi & \epsilon & 1 \end{pmatrix} \lambda, \\
Y_d &= \begin{pmatrix} 0 & \epsilon' & -\epsilon\sigma\xi \\ -\epsilon' & \tilde{\epsilon} & -\epsilon\sigma \\ \epsilon\xi & \epsilon & 1 \end{pmatrix} \lambda, \\
Y_e &= \begin{pmatrix} 0 & -\epsilon' & 3\epsilon\xi \\ \epsilon' & 3\tilde{\epsilon} & 3\epsilon \\ -3\epsilon\xi & -3\epsilon & 1 \end{pmatrix} \lambda
\end{aligned}
\tag{102}
$$

with

$$
\begin{aligned}
&\xi = \frac{\langle\phi^1\rangle}{\langle\phi^2\rangle}; \qquad \tilde{\epsilon} \propto \frac{\langle S^{22}\rangle}{\hat{M}}; \\
&\epsilon \propto \frac{\langle\phi^2\rangle}{\hat{M}}; \qquad \epsilon' \sim \frac{\langle A^{12}\rangle}{\hat{M}}; \\
&\sigma = \frac{1+\alpha}{1-3\alpha}; \qquad \rho \sim \beta \ll \alpha.
\end{aligned}
\tag{103}
$$

The model has only nine arbitrary Yukawa parameters (six real parameters $\{|\lambda|, |\epsilon|, |\tilde{\epsilon}|, |\rho|, |\sigma|, |\epsilon'|\}$ and three phases $\{\Phi_\epsilon = \Phi_{\tilde{\epsilon}}, \Phi_\rho, \Phi_\sigma\}$) to fit the 13 Fermion masses and mixing angles (we have taken $\xi = 0$). The fit to the low energy data is given in table 6[11]. More details of this fit are found in Blažek *et al* (1999) (table 7), and the predictions for proton decay are found in Dermíšek *et al* (2001). Note that the model fits most of the precision electroweak data quite well. In Blažek *et al* (1999) there is also a prediction for $\sin 2\beta = 0.54$, which should be compared with the present experimental value, 0.727 (0.036). The prediction for $\sin 2\beta$ is off by $5\sigma$. In addition, the present experimental value for $V_{\mathrm{ub}}/V_{cb}$ is 0.086 (0.008), and hence this fit (table 6) is somewhat worse than before. Both these quantities are predictions due solely to the zeros in the 11, 13 and 31 elements of the Yukawa matrices (Hall and Rasin 1993, Roberts *et al* 2001, Kim *et al* 2004). These poor fits are remedied with the addition of a non-vanishing 13/31 element, i.e. $\xi \neq 0$. In this case a good fit is obtained with one additional real parameter (Kim *et al* 2004).

### 5.3. Neutrino masses

The combined data from all neutrino experiments can be fit by the hypothesis of neutrino oscillations with the neutrino masses and mixing angles given by

$$
\begin{aligned}
\Delta m^2_{\mathrm{atm}} &= |m_3^2 - m_2^2| \approx 3 \times 10^{-3}\,\mathrm{eV}^2, \\
&\sin 2\theta_{\mathrm{atm}} \approx 1, \\
\Delta m^2_{\mathrm{sol}} &= |m_2^2 - m_1^2| \approx 7 \times 10^{-5}\,\mathrm{eV}^2, \\
&0.8 < \sin 2\theta_{\mathrm{sol}} < 1.
\end{aligned}
\tag{104}
$$

[11] Note that some of the data used in this fit have improved significantly in recent years.

**Table 6.** Fit to Fermion masses and mixing angles for $SO(10)$ GUT with $SU(2) \times U(1)^n$ family symmetry (Blažek *et al* 1999).

| Observable | Data ($\sigma$) masses (GeV) | Theory |
|---|---|---|
| $M_Z$ | 91.187 (0.091) | 91.17 |
| $M_W$ | 80.388 (0.080) | 80.40 |
| $G_\mu \times 10^5$ | 1.1664 (0.0012) | 1.166 |
| $\alpha_{\mathrm{EM}}^{-1}$ | 137.04 (0.14) | 137.0 |
| $\alpha_s(M_Z)$ | 0.1190 (0.003) | 0.1174 |
| $\rho_{\mathrm{new}} \times 10^3$ | −1.20 (1.3) | +0.320 |
| $M_t$ | 173.8 (5.0) | 175.0 |
| $m_b(M_b)$ | 4.260 (0.11) | 4.328 |
| $M_b - M_c$ | 3.400 (0.2) | 3.421 |
| $m_s$ | 0.180 (0.050) | 0.148 |
| $m_d/m_s$ | 0.050 (0.015) | 0.0589 |
| $Q^{-2}$ | 0.002 03 (0.000 20) | 0.002 01 |
| $M_\tau$ | 1.777 (0.0018) | 1.776 |
| $M_\mu$ | 0.105 66 (0.000 11) | 0.1057 |
| $M_e \times 10^3$ | 0.5110 (0.000 51) | 0.5110 |
| $V_{\mathrm{us}}$ | 0.2205 (0.0026) | 0.2205 |
| $V_{cb}$ | 0.039 20 (0.00 30) | 0.0403 |
| $V_{\mathrm{ub}}/V_{cb}$ | 0.0800 (0.02) | 0.0691 |
| $\hat{B}_K$ | 0.860 (0.08) | 0.8703 |
| $B(b \to s\gamma) \times 10^4$ | 3.000 (0.47) | 2.995 |
| Total $\chi^2$ | 3.39 | |

**Table 7.** Input parameters at $M_G$ for fit in table 6 (Blažek *et al* 1999).

| | |
|---|---|
| $(1/\alpha_G, M_G, \epsilon_3)$ | $(24.52, 3.05 \times 10^{16}$ GeV, $-4.08\%)$ |
| $(\lambda, \epsilon, \sigma, \tilde{\epsilon}, \rho, \epsilon', \xi)$ | (0.79, 0.045, 0.84, 0.011, 0.043, 0.0031, 0.00) |
| $(\Phi_\sigma, \Phi_\epsilon = \Phi_{\tilde{\epsilon}}, \Phi_\rho)$ | (0.73, −1.21, 3.72) rad |
| $(m_{16}, M_{1/2}, A_0, \mu(M_Z))$ | (1000, 300, −1437, 110) GeV |
| $((m_{H_d}/m_{16})^2, (m_{H_u}/m_{16})^2, \tan\beta)$ | (2.22, 1.65, 53.7) |

For recent theoretical analyses of the data, see Barger *et al* (2003), Maltoni *et al* (2003), Gonzales–Garcia and Peña–Garay (2003). This so-called bi-large neutrino mixing is well described by the PMNS mixing matrix,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \approx \begin{pmatrix} c_{\mathrm{sol}} & s_{\mathrm{sol}} & 0 \\ -s_{\mathrm{sol}}/\sqrt{2} & c_{\mathrm{sol}}/\sqrt{2} & 1/\sqrt{2} \\ -s_{\mathrm{sol}}/\sqrt{2} & c_{\mathrm{sol}}/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix},$$

which takes mass eigenstates into flavour eigenstates. The 1–3 mixing angle satisfies $\sin\theta_{13} < 0.2$ at $3\sigma$ (Maltoni *et al* 2003).

Using the seesaw mechanism (Glashow 1979, Gell-Mann *et al* 1979, Yanagida 1979, Mohapatra and Senjanovic 1980), neutrino masses are given in terms of two completely independent $3 \times 3$ mass matrices, i.e. the Dirac mass matrix, $m_\nu$, and a Majorana mass matrix, $M_N$, via the formula $\mathcal{M}_\nu = m_\nu^T M_N^{-1} m_\nu$. The smallness of neutrino masses is explained by the large Majorana mass scale, of the order of $10^{14}$–$10^{15}$ GeV, very close to the GUT scale. In addition, the large mixing angles needed to diagonalize $\mathcal{M}_\nu$ can be related directly to large mixing in $m_\nu$, in $M_N$ or in some combination of both. Lastly, the Dirac neutrino mass matrix, $m_\nu$, is constrained by charged Fermion masses in $SO(10)$ but not in $SU(5)$, where it is completely independent.

The major challenge for theories of neutrino masses is to obtain two large mixing angles, as compared with charged Fermions, where we only have small mixing angles in $V_{\rm CKM}$. There are several interesting suggestions for obtaining large mixing angles in the literature (for recent reviews of models of neutrino masses, see Altarelli and Feruglio (2003), Altarelli *et al* (2003), King (2003)):

- *Degenerate neutrinos and RG running (Mohapatra et al 2003, Casas et al 2003).* It was shown that starting with three degenerate Majorana neutrinos and small mixing angles at a GUT scale, RG running can lead to bi-large neutrino mixing at low energies.
- *Minimal renormalizable $SO(10)$ (Goh et al 2003, Bajc et al 2003).* $SO(10)$ with Higgs in the 10 and $\overline{126}$ representations can give predictable theories of Fermion masses with naturally large neutrino mixing angles.
- *Dominant Majorana neutrinos (King 1998, 2000).* It was shown that large neutrino mixing can be obtained via coupling to a single dominant right-handed neutrino.
- *Minimal Majorana sector (Frampton et al 2002, Raby 2003, Raidal and Strumia 2003).* It was shown that a simple model with two right-handed neutrinos can accommodate bi-large neutrino mixing with only one CP violating phase. In such a theory, CP violating neutrino oscillations measured in low energy accelerator experiments are correlated with the matter—anti-matter asymmetry obtained via leptogenesis.
- *Lopsided charged lepton and down quark matrices (Lola and Ross 1999, Nomura and Yanagida 1999, Albright and Barr 2000a, b, 2001, Altarelli et al 2000, Barr and Dorsner 2003).* In $SU(5)$ (or even in some $SO(10)$ models) the down quark mass matrix is related to the transpose of the charged lepton mass matrix. A large left-handed $\mu$–$\tau$ mixing angle is thus related directly to a large right-handed $s$–$b$ mixing angle. Whereas right-handed quark mixing angles are not relevant for CKM mixing, the large left-handed charged lepton mixing angle can give large $\nu_\mu$–$\nu_\tau$ mixing.

Let us now consider the last two mechanisms in more detail.

*5.3.1. SU(5) × U(1) flavour symmetry.* One popular possibility has the large $\nu_\mu$–$\nu_\tau$ mixing in the Dirac charged Yukawa matrix with

$$Y_e = \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \lambda^4 = Y_d^T. \tag{105}$$

The neutrino Dirac Yukawa matrix and the Majorana matrix are given by

$$Y_\nu = \begin{pmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & 0 & 1 \\ \lambda & 0 & 1 \end{pmatrix}, \tag{106}$$

$$M_N = \begin{pmatrix} \lambda^2 & 1 & \lambda \\ 1 & 0 & 0 \\ \lambda & 0 & 1 \end{pmatrix}, \tag{107}$$

where we use the results of Altarelli *et al* (2000). The light neutrino mass matrix is given by the standard seesaw formula. We obtain

$$\mathcal{M}_\nu = U_e^{\rm tr} \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} U_e v_u^2/M, \tag{108}$$

where $v_u$ is the VEV of the Higgs doublet giving mass to the up quarks, $M$ is the heavy Majorana mass scale and *all entries in each matrix are specified up to order 1 coefficients*. $U_e$ is the mixing matrix taking left-handed charged leptons into the mass eigenstate basis. It is given by

$$\begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix} = U_e^{\mathrm{tr}}(Y_e Y_e^\dagger)U_e^* \langle H_d \rangle^2, \tag{109}$$

where

$$Y_e Y_e^\dagger = \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} \tag{110}$$

*up to order 1 coefficients*. It is clear that without order 1 coefficients, the neutrino mixing matrix is the identity matrix. Hence, the arbitrary order 1 coefficients are absolutely necessary for obtaining bi-large neutrino mixing[12]. Hierarchical neutrino masses with $m_3 \gg m_2 \gg m_1$ and bi-large neutrino mixing can be obtained naturally (Altarelli and Feruglio 2003).

Of course, different $U(1)$ charge assignments for all the fields can lead to other experimentally acceptable solutions to the solar neutrino problem. For a review and further references, (see Altarelli and Feruglio (2003)).

*5.3.2. $SO_{10} \times [SU_2 \times U_1^n]_{\mathrm{FS}}$ model.* Within the context of the $SO_{10} \times [SU_2 \times U_1^n]_{\mathrm{FS}}$ model, bi-large neutrino mixing is obtained naturally using the mechanism of the minimal two Majorana neutrino sector. The Dirac neutrino mass is fixed once charged Fermion masses and mixing angles are fit. It is given by the formula

$$Y_\nu = \begin{pmatrix} 0 & -\epsilon'\omega & \frac{3}{2}\epsilon\xi\omega \\ \epsilon'\omega & 3\tilde{\epsilon}\omega & \frac{3}{2}\epsilon\omega \\ -3\epsilon\xi\sigma & -3\epsilon\sigma & 1 \end{pmatrix} \lambda$$

with $\omega = 2\sigma/(2\sigma - 1)$ and the Dirac neutrino mass matrix given by $m_\nu \equiv Y_\nu(v/\sqrt{2}) \sin\beta$. Of course, all the freedom is in the Majorana neutrino sector. The FGY ansatz (Frampton *et al* 2002) is obtained with the following Majorana neutrino sector (Raby 2003):

$$W_{\mathrm{neutrino}} = \frac{\overline{16}}{\hat{M}}(N_1\, \tilde{\phi}^a\, 16_a \;+\; N_2\, \phi^a\, 16_a \;+\; N_3\, \theta\, 16_3) + \frac{1}{2}(S_1\, N_1^2 \;+\; S_2\, N_2^2),$$

where $\{N_i, i = 1, 2, 3\}$ are $SO(10)$- and $SU(2)$ - flavour singlets. In this version of the theory, the symmetric two index tensor flavon field $S^{ab}$ is replaced by an $SU(2)$ doublet $\tilde{\phi}^a$ such that $S^{ab} \equiv \tilde{\phi}^a\tilde{\phi}^b/\hat{M}$. Note that since the singlet $N_3$ has no large Majorana mass, it gets a large Dirac mass by mixing directly with $\bar{\nu}_3$ at the GUT scale. Thus $\bar{\nu}_3$ is removed from the seesaw mechanism, and we effectively have only two right-handed neutrinos taking part.

Integrating out the heavy neutrinos, we obtain the light neutrino mass matrix given by

$$\mathcal{M}_\nu = U_e^{\mathrm{tr}}[D^{\mathrm{tr}}\hat{M}_N^{-1}D]U_e, \tag{111}$$

where $U_e$ is the unitary matrix diagonalizing the charge lepton mass matrix and

$$D^{\mathrm{tr}} \equiv \begin{pmatrix} a & 0 \\ a' & b \\ 0 & b' \end{pmatrix}, \qquad \hat{M}_N \equiv \begin{pmatrix} \langle S_1 \rangle & 0 \\ 0 & \langle S_2 \rangle \end{pmatrix} \tag{112}$$

[12] I thank Altarelli (private communication) for emphasizing this point.

with

$$b \equiv \epsilon' \omega \lambda \left( \frac{M_2}{\langle \phi^1 \rangle} \right) \frac{\hat{M}}{v_{16}} \frac{v \sin \beta}{\sqrt{2}}, \tag{113}$$

$$b' \equiv -3\epsilon \xi \sigma \lambda \left( \frac{M_2}{\langle \phi^1 \rangle} \right) \frac{\hat{M}}{v_{16}} \frac{v \sin \beta}{\sqrt{2}},$$

$$a \equiv -\epsilon' \omega \lambda \left( \frac{M_1}{\langle \tilde{\phi}^2 \rangle} \right) \frac{\hat{M}}{v_{16}} \frac{v \sin \beta}{\sqrt{2}},$$

$$a' \equiv (-\epsilon' \xi^{-1} + 3\tilde{\epsilon}) \omega \lambda \left( \frac{M_1}{\langle \tilde{\phi}^2 \rangle} \right) \frac{\hat{M}}{v_{16}} \frac{v \sin \beta}{\sqrt{2}}.$$

We obtain

$$b \sim b' \tag{114}$$

naturally since $\epsilon' \sim \epsilon \xi$. In addition, we can accommodate

$$a \sim a' \tag{115}$$

with minor fine-tuning of O(1/10) since $\epsilon' \xi^{-1} \sim \tilde{\epsilon}$. Note that this is the Frampton–Glashow–Yanagida ansatz (Frampton *et al* 2002) with a bi-large neutrino mixing matrix obtained naturally in a SUSY GUT.

### 5.4. *Flavour violation*

Quarks and leptons come in different flavours: up, down, charm, strange, top, bottom, electron, muon, tau. We observe processes where bottom quarks can decay into charm quarks or up quarks. Hence quark flavours (for quarks with the same electric charge) are interchangeable. In the SM, this is parametrized by the CKM mixing matrix. We now have direct evidence from neutrino oscillation experiments showing that tau, muon and electron numbers are not conserved separately. Yet, we have never observed muons changing into electrons. In supersymmetric theories there are many more possible ways in which both lepton and quark flavours can change. This is because scalar quarks and leptons carry the flavour quantum numbers of their SUSY partners. Thus flavour violation in the scalar sector can lead to flavour violation in the observed fermionic sector of the theory. This gives rise to the SUSY flavour problem. We consider two examples here: $\mu \to e\gamma$ and $B_s \to \mu^+\mu^-$. We show why SUSY GUTs and/or neutrino masses can cause enhanced flavour violation beyond that of the SM. In this section we consider several ways of solving the SUSY flavour problem.

But first let us illustrate the problem with a few of the dominant examples (Gabbiani *et al* 1996). In the first column of table 8, we present four flavour violating observables with their experimental bounds. In the second and third columns we present the bounds on flavour violating scalar mass corrections,

$$\delta_{ij}^{f} \equiv \Delta m_{ij}^{2\,f} / \bar{m}^2, \tag{116}$$

where $i, j = 1, 2, 3$ are family indices, $\Delta m_{ij}^{2\,f}$ is an off-diagonal scalar mass insertion for $f = \{\text{quark, lepton}\}$ flavour (treated to lowest non-trivial order in perturbation theory) in the flavour diagonal basis for quarks and leptons and $\bar{m}$ is the average squark or slepton mass squared. The subscripts LL refer to left-handed squark or slepton mass insertions. There are separate limits on RR and LR mass insertions (Gabbiani *et al* 1996) that are not presented here. The only difference between the second and third columns is the fiducial value of $\bar{m}^2$. Clearly, as the mean squark or slepton mass increases, the fine-tuning necessary for avoiding

**Table 8.** Some constraints from the non-observation of flavour violation on squark, slepton and gaugino masses (Gabbiani *et al* 1996). For the electron electric dipole moment, we use the relation $d_N^e \sim 2(100/m_{\tilde{l}}(\mathrm{GeV}))^2 \sin\Phi_{A,B} \times 10^{-23}$ e cm.

| Observable | Experimental bound (1) | Experimental bound (2) |
|---|---|---|
| $B(\mu \to e\gamma) < 1.2 \times 10^{-11}$ | $\lvert(\delta^l_{12})_{\mathrm{LL}}\rvert < 2.1 \times 10^{-3} \left(\frac{m_{\tilde{l}}(\mathrm{GeV})}{100}\right)^2$ | $\lvert(\delta^l_{12})_{\mathrm{LL}}\rvert < 0.8 \left(\frac{m_{\tilde{l}}(\mathrm{TeV})}{2}\right)^2$ |
| $\Delta m_K < 3.5 \times 10^{-12}$ MeV | $\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LL}}\rvert} < 1.9 \times 10^{-2} \left(\frac{m_{\tilde{q}}(\mathrm{GeV})}{500}\right)$ | $\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LL}}\rvert} < 7.6 \times 10^{-2} \left(\frac{m_{\tilde{q}}(\mathrm{TeV})}{2}\right)$ |
| $\epsilon_K < 2.28 \times 10^{-3}$ | $\sqrt{\lvert\mathrm{Im}(\delta^d_{12})^2_{\mathrm{LL}}\rvert} < 1.5 \times 10^{-3} \left(\frac{m_{\tilde{q}}(\mathrm{GeV})}{500}\right)$ | $\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{\mathrm{LL}}\rvert} < 6.0 \times 10^{-3} \left(\frac{m_{\tilde{q}}(\mathrm{TeV})}{2}\right)$ |
| $d_N^e < 4.3 \times 10^{-27}$ e cm | $\sin\Phi_{A,B} < 4 \times 10^{-4} \times \left(\frac{m_{\tilde{l}}(\mathrm{GeV})}{100}\right)^2$ | $\sin\Phi_{A,B} < 0.16 \times \left(\frac{m_{\tilde{l}}(\mathrm{TeV})}{2}\right)^2$ |

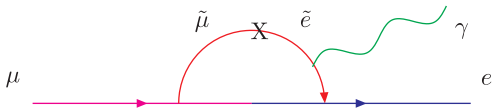


**Figure 13.** The one-loop contribution to the process $\mu \to e\gamma$ proportional to an off-diagonal scalar muon—electron mass term in the charged lepton mass eigenstate basis.

significant flavour violation is dramatically reduced. As seen in figure 13, the amplitude for $\mu \to e\gamma$ is proportional to $\delta^e_{12}$ and is suppressed by $1/\bar{m}^2$ since it is an effective dimension 5 operator.

*5.4.1. The origin of flavour violation in SUSY theories.* There are three possible ways of avoiding large flavour violation.

(i) Having squarks and sleptons, with the same SM gauge charges, being degenerate and, in addition, having the cubic scalar interactions proportional to the Yukawa matrices.
(a) Alignment of squark and slepton masses with quark and lepton masses.
(ii) The Fermion and scalar mass matrices are 'aligned' when, on the basis of where Fermion masses are diagonal, the scalar mass matrices and cubic scalar interactions are approximately diagonal as well.
(iii) Heavy first and second generation squarks and sleptons.

The CMSSM (or mSUGRA) is an example of the first case. It has a universal scalar mass $m_0$ and tri-linear scalar interactions proportional to Yukawa matrices. These initial conditions correspond to a symmetry limit (Hall *et al* 1986)—dubbed minimal flavour violation (Ciuchini *et al* 1998)—where the only flavour violation occurs in the CKM matrix at the messenger scale for SUSY breaking or, in this case, the Planck scale. GMSB, where squarks and sleptons obtain soft SUSY breaking masses via SM gauge interactions, is another example of the first case (for a review, see Giudice and Rattazzi (1999)). In this case the messenger mass is arbitrary. Finally, within the context of perturbative heterotic string theory, dilaton SUSY breaking gives universal scalar masses at the string scale. For moduli SUSY breaking, on the other hand, scalar masses depend on modular weights, whose values are very model dependent.

Abelian flavour symmetries can be used to align quark (lepton) and the corresponding squark (slepton) mass matrices, but they still require one of the above mechanisms for obtaining degenerate scalar masses at zeroth order in symmetry breaking. Non-abelian symmetries, on the other hand, can both align Fermion and scalar mass matrices and guarantee the degeneracy of the scalar masses at zeroth order.

Finally, since the most stringent limits from flavour violating processes come from the lightest two families, if the associated squarks and sleptons are heavy, these processes are suppressed (Dimopoulos and Giudice 1995). A natural mechanism for obtaining this inverted scalar mass hierarchy with the first and second generation scalars heavier than the third was discussed by Bagger *et al* (1999, 2000).

If the messenger scale for SUSY breaking is above the GUT scale or even above the seesaw scale for neutrino masses, then squark and slepton masses can receive significant flavour violating radiative corrections due to this beyond the SM physics (Borzumati and Masiero 1986, Georgi 1986, Hall *et al* 1986, Leontaris *et al* 1986, Barbieri *et al* 1995a, b, Hisano *et al* 1995, 1996). In addition, if the Fermion mass hierarchy is due to flavour symmetry breaking using a Froggatt–Nielsen mechanism, then upon integrating out the heavy Froggatt–Nielsen sector, new flavour violating soft SUSY breaking terms may be induced (Dimopoulos and Pomarol 1995, Pomarol and Dimopoulos 1995). Hence, the low energy MSSM is sensitive to physics at short distances. This is both a problem requiring natural solutions and a virtue leading to new experimentally testable manifestations of SUSY theories.

For example, in SUSY GUTs, colour triplet Higgs fields couple quarks to leptons. As a consequence, flavour mixing in the quark sector can, via loops, cause flavour mixing, proportional to up quark Yukawa couplings, in the lepton sector. This comes via off-diagonal scalar lepton masses in the basis where charged lepton masses are diagonal. While the one-loop contribution of charm quarks give a branching ratio $Br(\mu \to e\gamma) \sim 10^{-15}$ (Hall *et al* 1986), the top quark contribution leads to very observable rates (Barbieri *et al* 1995a) near the experimental bounds. Moreover, new experiments will soon test these results. In addition, experimental evidence for neutrino oscillations makes it clear that the lepton sector has its own intrinsic flavour violation. In the SM, these effects are suppressed by extremely small ($<1$ eV) neutrino masses. In SUSY, however, flavour violation in the (s)neutrino sector leads, again via loops, to eminently observable mixing in the charged (s)lepton sector (Hisano *et al* 1995, 1996). There are a large number of papers in the literature that try to use low energy neutrino oscillation data in an attempt to predict rates for lepton flavour violation. However, a bottom-up approach is fraught with the problem that low energy oscillation data cannot constrain completely the neutrino sector (Casas and Ibarra 2001, Lavignac *et al* 2001, 2002). It has been shown that neutrino oscillation data and $l_i \to l_j\gamma$ measurements can nevertheless provide complementary information on the seesaw parameter space (Davidson and Ibarra 2001, Ellis *et al* 2002). In a recent analysis, it was shown that lepton flavour violation can constrain typical SUSY $SO(10)$ theories (Masiero *et al* 2003). Finally, it is important to note that the same physics can lead to enhanced contributions to flavour conserving amplitudes such as the anomalous magnetic moment of the neutrino ($a_\mu$) (Chattopadhyay and Nath 1996, Moroi 1996) and the electric dipole moments of the electron ($d_e^e$) and neutron ($d_e^n$) (Dimopoulos and Hall 1995, Hisano and Tobe 2001, Demir *et al* 2003). Moreover, the rates for these flavour violating processes increase with $\tan\beta$.

Let us now consider flavour violating hadronic interactions at large $\tan\beta$. We focus on a few important examples, in particular the processes $B \to X_s\gamma$, $B \to X_s l^+ l^-$ forward–backward asymmetry and $B_s \to \mu^+\mu^-$. For a more comprehensive study, see Carena *et al* (1994), Hall *et al* (1994), Hempfling (1994), Blažek *et al* (1995), Chankowski and Pokorski (1997), Misiak *et al* (1998), Huang and Yan (1998), Hamzaoui *et al* (1999), Choudhury and Gaur (1999), Babu and Kolda (2000), Bobeth *et al* (2001), Carena *et al* (2001), Chankowski and Slawianowska (2001), Dedes *et al* (2001), Huang *et al* (2001), Isidori and Retico (2001), Buras *et al* (2002, 2003), Dedes and Pilaftsis (2003). The SM contribution to $B \to X_s\gamma$

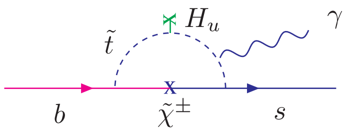


**Figure 14.** The one-loop chargino contribution to the process $B \to X_s\gamma$ proportional to $\lambda_t$ and $\tan\beta$.

has significant uncertainties, but the calculated branching ratio is consistent with the latest experimental data. SUSY contributions are divided typically into two categories, i.e. the contributions contained in a two Higgs doublet model and then the rest of the SUSY spectrum. The charged Higgs contribution has the same sign as the SM contribution and thus increases the predicted value for the branching ratio. This spoils the agreement with the data, and thus a lower limit on the charged Higgs mass is obtained. In the minimal flavour SUSY scenario, the additional SUSY contribution is dominated by the chargino loop (figure 14). The sign of this term depends on the sign of $\mu$. For $\mu > 0$ (this defines my conventions), the chargino contribution is the opposite sign of the SM contribution to the coefficient, $C_7$, of the magnetic moment operator $O_7 \sim \bar{s}_L \Sigma_{\mu\nu} b_R F^{\mu\nu}$. Moreover, this contribution is proportional to $\tan\beta$. For small or moderate values of $\tan\beta$ the SUSY correction to $C_7$ is small, and for $\mu > 0$ it tends to cancel the charged Higgs and SM contributions. This is in the right direction, giving good agreement with the data. For $\mu < 0$ the agreement with the data gets worse and can only work for large Higgs and squark masses, so that the overall SUSY contribution is small. On the other hand, for $\mu > 0$ and large $\tan\beta \sim 50$, there is another possible solution, with the total SUSY contribution to $C_7$ equal to twice the SM contribution but with opposite sign. In this case $C_7^{\text{total}} = C_7^{\text{SM}} + C_7^{\text{SUSY}} \approx -C_7^{\text{SM}}$, and good fits to the data are obtained (Blažek and Raby 1999). Although the sign of $C_7$ is not observable in $B_s \to X_s\gamma$, it can be observed by measuring the forward–backward asymmetry in the process $B \to X_s\, l^+\, l^-$ (Huang and Yan 1998, Huang *et al* 1999, Lunghi *et al* 2000, Ali *et al* 2002, Bobeth *et al* 2003), where forward (backward) refers to the positive lepton direction with respect to the B flight direction in the rest frame of the di-lepton system[13].

*5.4.2. Flavor violating Higgs couplings at large* $\tan\beta$. The MSSM has two Higgs doublets that at tree level satisfy the Glashow–Weinberg condition for natural flavour conservation. Up quarks get mass from $H_u$ and down quarks and charged leptons get mass from $H_d$. Thus when the Fermion mass matrices are diagonalized (and neglecting small neutrino masses) the Higgs couplings to quarks and leptons are also diagonal. However, this is no longer true once SUSY is broken and radiative corrections are considered. In particular, for large values of $\tan\beta$ the coupling of $H_u$ to down quarks, via one-loop corrections, results in significant flavour violating vertices for neutral and charged Higgs. These one-loop corrections to the Higgs couplings contribute to an effective Lagrangian (equation (117)) (Blažek *et al* 1995, Chankowski and Pokorski 1997). The chargino contribution (figure 7) is proportional to the square of the up quark Yukawa matrix, which is not diagonal in the diagonal down quark mass basis. As a result, at one-loop order, the down quark mass matrix is no longer diagonal (equation (118)). This leads to $\tan\beta$ enhanced corrections to down quark masses and to CKM matrix elements (Blažek *et al* 1995). Upon re-diagonalizing the down quark mass matrix, we obtain the effective flavour violating Higgs–down quark Yukawa couplings given in equations (119) and (120) (Chankowski and Pokorski 1997, Babu and Kolda 2000, Bobeth *et al* 2001, Chankowski and Slawianowska 2001, Dedes *et al* 2001, Huang *et al* 2001, Isidori

[13] I thank Tobe for pointing out this possibility to me.

and Retico 2001, Buras *et al* 2002, 2003, Dedes and Pilaftsis 2003).

$$\mathcal{L}_{\rm eff}^{\rm ddH} = -\bar{d}_{Li}\lambda_{di}^{\rm diag} d_{Ri} H_d^{0*} - \bar{d}_{Li}\Delta\lambda_d^{ij} d_{Rj} H_d^{0*} - \bar{d}_{Li}\delta\lambda_d^{ij} d_{Rj} H_u^0 + \text{h.c.} \tag{117}$$

$$m_d^{\rm Diagonal} = V_d^L\big[\lambda_d^{\rm diag} + \Delta\lambda_d + \delta\lambda_d \tan\beta\big] V_d^{R\dagger} \frac{v\cos\beta}{\sqrt{2}} \tag{118}$$

$$\begin{aligned}\mathcal{L}_{\rm FV}^{i\neq j} = &-\frac{1}{\sqrt{2}}\bar{d}'_i\big[F_{ij}^h P_R + F_{ji}^{h*} P_L\big] d'_j h - \frac{1}{\sqrt{2}}\bar{d}'_i\big[F_{ij}^H P_R + F_{ji}^{H*} P_L\big] d'_j H \\ &-\frac{\mathrm{i}}{\sqrt{2}}\bar{d}'_i\big[F_{ij}^A P_R + F_{ji}^{A*} P_L\big] d'_j A,\end{aligned} \tag{119}$$

where

$$\begin{aligned} F_{ij}^h &\simeq \delta\lambda_d^{ij}(1+\tan^2\beta)\cos\beta\cos(\alpha-\beta), \\ F_{ij}^H &\simeq \delta\lambda_d^{ij}(1+\tan^2\beta)\cos\beta\sin(\alpha-\beta), \\ F_{ij}^A &\simeq \delta\lambda_d^{ij}(1+\tan^2\beta)\cos\beta. \end{aligned} \tag{120}$$

This leads to $\tan\beta$ enhanced flavour violating couplings for the neutral Higgs Bosons. For example, the branching ratio $B(B_s \rightarrow \mu^+\mu^-)$ is proportional to $\tan\beta^4$ and inversely proportional to the fourth power of the CP odd Higgs mass $m_A$ (Choudhury and Gaur 1999, Babu and Kolda 2000, Bobeth *et al* 2001, Chankowski and Slawianowska 2001, Dedes *et al* 2001, Huang *et al* 2001, Isidori and Retico 2001, Buras *et al* 2002, 2003, Dedes and Pilaftsis 2003), since the contributions of the two CP even Higgs Bosons cancel approximately (Babu and Kolda 2000). The present D0 and CDF bounds constrain $m_A \geqslant 250$ GeV for $\tan\beta \sim 50$, although this result is somewhat model dependent (Dermíšek *et al* 2003). Note that the D0 and CDF bounds from Tevatron Run 2 now give $B(B_s \rightarrow \mu^+\mu^-) < 1.2 \times 10^{-6}$ (CDF) (Lin 2003) at 95% CL and $< 1.6 \times 10^{-6}$ (D0) (Kehoe 2003). An order of magnitude improvement will test large $\tan\beta$ SUSY for $m_A$ up to $\sim$500 GeV (Dermíšek *et al* 2003) (see figure 15).

## 6. SUSY dark matter

The two most popular dark matter candidates are axions or the LSP of SUSY. Both are well motivated cold dark matter candidates. There have been several recent studies of SUSY dark matter in light of the recent WMAP data. For these analyses and references to earlier works, see Chattopadhyay *et al* (2003), Ellis *et al* (2003c, d, e, f), Roszkowski *et al* (2003). These calculations have been performed with different assumptions about soft SUSY breaking parameters, assuming the CMSSM boundary conditions at $M_G$ (Chattopadhyay *et al* 2003, Ellis *et al* 2003b, c, d, f) or arbitrary low energy scalar masses (Ellis *et al* 2003d). Soft SUSY breaking outside the realm of the CMSSM has also been considered. For example, soft breaking with non-universal Higgs masses has been analysed recently by Ellis *et al* (2003a, b), Roszkowski *et al* (2003). In the latter case, the soft SUSY breaking parameters consistent with $SO(10)$ Yukawa unification were studied.

In the limit of large squark and slepton masses, it is important to have efficient mechanisms for dark matter annihilation. Most recent studies have focused on the neutralino LSP in the limit of large $\tan\beta$ and/or the FP limit. In both cases there are new mechanisms for efficient neutralino annihilation. For large $\tan\beta \geqslant 40$, neutralino annihilation via direct s-channel neutral Higgs Boson exchange dominates (Roszkowski *et al* 2001, Ellis *et al* 2001a, b). In this limit the CP even and odd Higgs Bosons have large widths due to their larger coupling to bottom quarks and $\tau$ leptons. In the FP limit, on the other hand, the neutralino LSP is a mixed

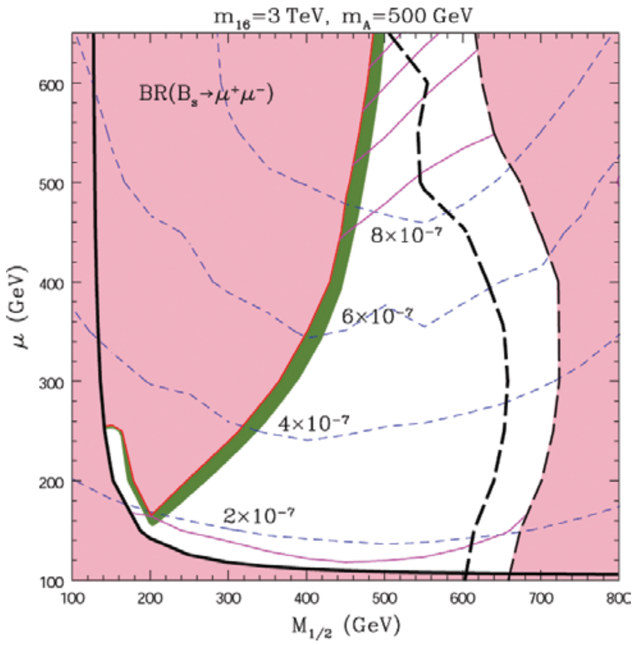


**Figure 15.** The dashed (blue) lines are contours for a constant branching ratio $B(B_s \to \mu^+\mu^-)$ as a function of $\mu$, $M_{1/2}$ for $m_{16} = 3$ TeV. The green (dark grey) shaded region is consistent with the recent WMAP data for dark matter abundance of the neutralino LSP. The light shaded region in the lower left-hand corner (separated by the solid line) is excluded by chargino mass limits, while the light shaded region in the upper left (right-side) is excluded by a cosmological dark matter abundance that is too large (Higgs mass that is too light).

Higgsino–gaugino state. Thus it has more annihilation channels than the pure bino LSP case, valid for values of the universal scalar mass $m_0 < 1$ TeV (Feng *et al* 2000c, 2001).

Direct detection (Ellis *et al* 2003b, e, Roszkowski *et al* 2003, Munoz 2003) and/or indirect detection (Baer and Farrill 2003, de Boer *et al* 2003) of neutralino dark matter has also been considered. In fact, de Boer *et al* (2003) suggest that some indirect evidence for SUSY dark matter already exists.

## 7. Open questions

It is beyond the scope of this review to comment on many other interesting topics affected by supersymmetric theories. Several effective mechanisms for generating the matter–anti-matter asymmetry of the universe have been suggested, including the Affleck–Dine mechanism (Affleck and Dine 1985), which is a purely supersymmetric solution, and leptogenesis (Fukugita and Yanagida 1986), which is not necessarily supersymmetric. There have also been many interesting studies of inflation in a SUSY context. Finally, we have only made passing references to superstring theories and SUSY breaking mechanisms or Fermion masses there.

Simple 'naturalness' arguments would lead one to believe that SUSY should have been observed already. On the other hand, 'FP' or 'minimal $SO(10)$ SUSY' regions of soft SUSY breaking parameter space extend to significantly heavier squark and slepton masses *without giving up on 'naturalness'*. In both the 'FP' and mSO$_{10}$SM regions of parameter space, we expect a light Higgs with mass of the order of 114–120 GeV. Both ameliorate the SUSY flavour problem with heavy squark and slepton masses. They are nevertheless both surprisingly consistent with cosmological dark matter abundances. In addition, we have shown that the $\mathrm{mSO_{10}SM}$ satisfies Yukawa coupling unification with an inverted scalar mass hierarchy. Thus one finds first and second generation squarks and sleptons with mass of the order of several teraelectronvolts, while gauginos and third generation squarks and sleptons are much lighter. In addition, it requires large values of $\tan\beta \sim 50$, resulting in enhanced flavour violation.

SUSY GUTs are the most natural extensions of the SM, and thus they are the new 'SM' of particle physics. The mSUGRA (or CMSSM) boundary conditions at the GUT scale provide excellent fits to precision low energy electroweak data. SUSY GUTs, besides predicting gauge coupling unification, also provide a framework for resolving the gauge hierarchy problem and understanding Fermion masses and mixing angles, including neutrinos. They also give a natural dark matter candidate and a framework for leptogenesis and inflation.

BUT there are two major challenges with any supersymmetric theory. We do not know how SUSY is spontaneously broken or the origin of the $\mu$ term. We are thus unable to predict the SUSY particle spectrum, which makes SUSY searches very difficult. Nevertheless, 'naturalness' arguments always lead to some light SUSY sector, observable at the LHC, a light Higgs, with mass less than O(135 GeV), or observable flavour violating rates beyond that of the SM. Assuming SUSY particles are observed at the LHC, then the fun has just begun. It will take us many years to prove that it is really SUSY. Assuming SUSY is established, a SUSY desert from $M_Z$ to $M_G$ (or $M_N$) becomes highly likely. Thus precision measurements at the LHC or a linear collider will probe the boundary conditions at the very largest and fundamental scales of nature. With the additional observation of proton decay and/or precise GUT relations for sparticle masses, SUSY GUTs can be confirmed. Hence, with experiments at teraelectronvolts scale accelerators or in underground detectors for proton decay, neutrino oscillations or dark matter, the fundamental superstring physics can be probed. Perhaps then we will understand finally who ordered three families. It is thus no wonder why the elementary particle physics community is *desperately seeking SUSY*.

## References


Affleck I and Dine M 1985 *Nucl. Phys.* B **249** 361
Albright C H and Barr S M 2000a *Phys. Rev. Lett.* **85** 244 (*Preprint* hep-ph/0002155)
Albright C H and Barr S M 2000b Phys. Rev. D **62** 093008 (*Preprint* hep-ph/0003251)
Albright C H and Barr S M 2001 *Phys. Rev.* D **64** 073010 (*Preprint* hep-ph/0104294)
Ali A, Lunghi E, Greub C and Hiller G 2002 *Phys. Rev.* D **66** 034002 (*Preprint* hep-ph/0112300)
Allanach B C, King S F, Leontaris G K and Lola S 1997 *Phys. Lett.* B **407** 275 (*Preprint* hep-ph/9703361)
Altarelli G, Feruglio F and Masina I 2000 *J. High Energy Phys.* JHEP 11 (2000) 040 (*Preprint* hep-ph/0007254)
Altarelli G and Feruglio F 2001 *Phys. Lett.* B **511** 257 (*Preprint* hep-ph/0102301)
Altarelli G and Feruglio F 2003 (*Preprint* hep-ph/0306265)
Altarelli G, Feruglio F and Masina I 2003 *J. High Energy Phys.* JHEP 01 (2003) 035 (*Preprint* hep-ph/0210342)
Alvarez-Gaume L, Claudson M and Wise M B 1982 *Nucl. Phys.* B **207** 96
Alvarez-Gaume L, Polchinski J and Wise M B 1983 *Nucl. Phys.* B **221** 495
Amaldi U, de Boer W and Fürstenau H 1991 *Phys. Lett.* **B260** 447
Ananthanarayan B, Lazarides G and Shafi Q 1991 *Phys. Rev.* D **44** 1613
Ananthanarayan B, Lazarides G and Shafi Q 1993 *Phys. Lett.* B **300** 245
Ananthanarayan B, Shafi Q and Wang X M 1994 *Phys. Rev.* D **50** 5980
Anderson G W and Castaño D J 1995 *Phys. Lett.* B **347** 300
Anderson G W, Raby S, Dimopoulos S and Hall L J 1993 *Phys. Rev.* D **47** 3702 (*Preprint* hep-ph/9209250)
Anderson G, Raby S, Dimopoulos S, Hall L J and Starkman G D 1994 *Phys. Rev.* D **49** 3660 (*Preprint* hep-ph/9308333)
Aoki S *et al* (JLQCD) 2000 *Phys. Rev.* D **62** 014506
Aoki Y *et al* (RBC) 2002 (*Preprint* hep-lat/0210008)

Appelquist T, Dobrescu B A, Ponton E and Yee H U 2001 *Phys. Rev. Lett.* **87** 181802 (*Preprint* hep-ph/0107056)
Aranda A, Carone C D and Lebed R F 2000 *Phys. Lett.* B **474** 170 (*Preprint* hep-ph/9910392)
Arkani-Hamed N, Cheng H C and Hall L J 1996 *Phys. Rev.* D **54** 2242 (*Preprint* hep-ph/9601262)
Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 *Phys. Lett.* B **429** 263 (*Preprint* hep-ph/9803315)
Aulakh C S, Bajc B, Melfo A, Senjanovic G and Vissani F 2003 (*Preprint* hep-ph/0306242)
Auto D, Baer H, Balazs C, Belyaev A, Ferrandis J and Tata X 2003 *J. High Energy Phys.* JHEP 06 (2003) 023 (*Preprint* hep-ph/0302155)
Babu K S and Barr S M 1993 *Phys. Rev.* D **48** 5354 (*Preprint* hep-ph/9306242)
Babu K S and Barr S M 2002 *Phys. Rev.* D **65** 095009 (*Preprint* hep-ph/0201130)
Babu K S and Kolda C 2000 *Phys. Rev. Lett.* **84** 228
Babu K S and Mohapatra R N 1995 *Phys. Rev. Lett.* **74** 2418
Babu K S, Pati J C and Wilczek F 2000 *Nucl. Phys.* B **566** 33
Babu K S and Strassler M J 1998 (*Preprint* hep-ph/9808447)
Baer H, Balazs C, Belyaev A and O'Farrill J 2003 *J. Cosmol. Astropartile Phys.* JCAP 09 (2003) 007 (*Preprint* hep-ph/0305191)
Baer H, Balazs C, Mercadante P, Tata X and Wang Y 2001a *Phys. Rev.* D **63** 015011 (*Preprint* hep-ph/0008061)
Baer H, Balazs C, Brhlik M, Mercadante P, Tata X and Wang Y 2001b *Phys. Rev.* D **64** 015002 (*Preprint* hep-ph/0102156)
Baer H, Mercadante P and Tata X 2000 *Phys. Lett.* B **475** 289 (*Preprint* hep-ph/9912494)
Baer H and Ferrandis J 2001 *Phys. Rev. Lett.* **87** 211803
Baer H and Farrill J O 2003 (*Preprint* hep-ph/0312350)
Bagger J, Dimopoulos S, Georgi H and Raby S 1984 *Proc. 5th Workshop on Grand Unification (Providence, RI, 12–14 April 1984)*
Bagger J A, Feng J L and Polonsky N 1999 *Nucl. Phys.* B **563** 3 (*Preprint* hep-ph/9905292)
Bagger J A, Feng J L, Polonsky N and Zhang R-J 2000 *Phys. Lett.* B **473** 264 (*Preprint* hep-ph/9911255)
Bajc B, Perez P F and Senjanovic G 2002 *Phys. Rev.* D **66** 075005 (*Preprint* hep-ph/0204311)
Bajc B, Senjanovic G and Vissani F 2003 *Phys. Rev. Lett.* **90** 051802 (*Preprint* hep-ph/0210207)
Banks T 1988 *Nucl. Phys.* B **303** 172
Banks T and Kaplunovsky V 1983 *Nucl. Phys.* B **211** 529
Barbieri R, Dvali G R and Hall L J 1996 *Phys. Lett.* B **377** 76 (*Preprint* hep-ph/9512388)
Barbieri R, Ferrara S and Savoy C A 1982 *Phys. Lett.* B **119** 343
Barbieri R and Giudice G F 1988 *Nucl. Phys.* B **306** 63
Barbieri R and Hall L J 1997 *Nuovo Cimento.* A **110** 1 (*Preprint* hep-ph/9605224)
Barbieri R, Hall L J, Raby S and Romanino A 1997a *Nucl. Phys.* B **493** 3
Barbieri R, Hall L J and Romanino A 1997b *Phys. Lett.* B **401** 47 (*Preprint* hep-ph/9702315)
Barbieri R, Hall L J and Romanino A 1999 *Nucl. Phys.* B **551** 93 (*Preprint* hep-ph/9812384)
Barbieri R, Hall L J and Strumia A 1995a *Nucl. Phys.* B **445** 219 (*Preprint* hep-ph/9501334)
Barbieri R, Hall L J and Strumia A 1995b *Nucl. Phys.* B **449** 437 (*Preprint* hep-ph/9504373)
Barbieri R and Strumia A 1998 *Phys. Lett.* B **433** 63 (*Preprint* hep-ph/9801353)
Barger V D, Berger M S and Ohmann P 1993 *Phys. Rev.* D **47** 1093 (*Preprint* hep-ph/9209232)
Barger V, Marfatia D and Whisnant K 2003 *Int. J. Mod. Phys.* E **12** 569 (*Preprint* hep-ph/0308123)
Barr S M and Dorsner I 2003 *Phys. Lett.* B **556** 185 (*Preprint* hep-ph/0211346)
Berezhiani Z G 1983 *Phys. Lett.* B **129** 99
Berezhiani Z G 1985 *Phys. Lett.* B **150** 177
Berezhiani Z 1998 *Phys. Lett.* B **417** 287 (*Preprint* hep-ph/9609342)
Berezhiani Z and Rossi A 2001 *Nucl. Phys.* B **594** 113
Berezinsky V, Bottino A, Ellis J R, Fornengo N, Mignola G and Scopel S 1996 *Astropart. Phys.* **5** 333 (*Preprint* hep-ph/9603342)
Binetruy P, Lavignac S and Ramond P 1996 *Nucl. Phys.* B **477** 353 (*Preprint* hep-ph/9601243)
Blažek T, Carena M, Raby S and Wagner C 1997a *Nucl. Phys. Proc.* **52** (Suppl. A) 133 (*Preprint* hep-ph/9608273)
Blažek T, Carena M, Raby S and Wagner C E M 1997b *Phys. Rev.* D **56** 6919 (*Preprint* hep-ph/9611217)
Blažek T, Dermíšek R and Raby S 2002a *Phys. Rev. Lett.* **88** 111804
Blažek T, Dermíšek R and Raby S 2002b *Phys. Rev.* D **65** 115004 (*Preprint* hep-ph/0201081)
Blažek T, Pokorski S and Raby S 1995 *Phys. Rev.* D **52** 4151
Blažek T and Raby S 1999 *Phys. Rev.* D **59** 095002 (*Preprint* hep-ph/9712257)
Blažek T, Raby S and Tobe K 1999 *Phys. Rev.* D **60** 113001 (*Preprint* hep-ph/9903340)
Blažek T, Raby S and Tobe K 2000 *Phys. Rev.* D **62** 055001 (*Preprint* hep-ph/9912482)
Bobeth C, Ewerth T, Kruger F and Urban J 2001 *Phys. Rev.* D **64** 074014 (*Preprint* hep-ph/0104284)

Bobeth C, Gambino P, Gorbahn M and Haisch U 2003 (*Preprint* hep-ph/0312090)
Borzumati F and Masiero A 1986 *Phys. Rev. Lett.* **57** 961
Brodsky S J, Ellis J, Hagelin J S and Sachradja C 1984 *Nucl. Phys.* B **238** 561
Buchmuller W 2001 (*Preprint* hep-ph/0107153)
Buchmuller W, Di Bari P and Plumacher M 2003 *Nucl. Phys.* B **665** 445 (*Preprint* hep-ph/0302092)
Buras A J, Chankowski P H, Rosiek J and Slawianowska L 2002 *Phys. Lett.* B **546** 96 (*Preprint* hep-ph/0207241)
Buras A J, Chankowski P H, Rosiek J and Slawianowska L 2003 *Nucl. Phys.* B **659** 3 (*Preprint* hep-ph/0210145)
Carena M *et al* 1994 *Nucl. Phys.* B **426** 269
Carena M *et al* 1995 *Phys. Lett.* B **355** 209
Carena M, Garcia D, Nierste U and Wagner C E M 2001 *Phys. Lett.* B **499** 141 (*Preprint* hep-ph/0010003)
Carena M, Quiros M and Wagner C E M 1996 *Nucl. Phys.* B **461** 407
Carone C D, Hall L J and Murayama H 1996 *Phys. Rev.* D **53** 6282 (*Preprint* hep-ph/9512399)
Carone C D and Lebed R F 1999 *Phys. Rev.* D **60** 096002
Casas J A, Espinosa J R and Navarro I 2003 *J. High Energy Phys.* JHEP 09 (2003) 048 (*Preprint* hep-ph/0306243)
Casas J A, Espinosa J R, Quiros M and Riotto A 1995 *Nucl. Phys.* B **436** 3
Casas J A, Espinosa J R, Quiros M and Riotto A 1995 *Nucl. Phys.* B **439** 466 (erratum)
Casas J A and Ibarra A 2001 *Nucl. Phys.* B **618** 171 (*Preprint* hep-ph/0103065)
Chamseddine A H, Arnowitt R and Nath P 1982 *Phys. Rev. Lett.* **49** 970
Chankowski P H, Ellis J R and Pokorski S 1997 *Phys. Lett.* B **423** 327 (*Preprint* hep-ph/9712234)
Chankowski P H, Ellis J R, Olechowski M and Pokorski S 1999 *Nucl. Phys.* B **544** 39 (*Preprint* hep-ph/9808275)
Chankowski P H and Pokorski S 1997 *Perspectives on Supersymmetry* ed G L Kane (Singapore: World Scientific) (*Preprint* hep-ph/9707497)
Chankowski P H and Slawianowska L 2001 *Phys. Rev.* D **63** 054012 (*Preprint* hep-ph/0008046)
Chattopadhyay U, Corsetti A and Nath P 2003 *Phys. Rev.* D **68** 035005 (*Preprint* hep-ph/0303201)
Chattopadhyay U and Nath P 1996 *Phys. Rev.* D **53** 1648 (*Preprint* hep-ph/9507386)
Chen M C and Mahanthappa K T 2003 *Phys. Rev.* D **68** 017301
Chen M C and Mahanthappa K T 2003 *Int. J. Mod. Phys.* A **18** 5819 (*Preprint* hep-ph/0305088)
Choudhury S R and Gaur N 1999 *Phys. Lett.* B **451** 86 (*Preprint* hep-ph/9810307)
Claudson M, Wise M and Hall L J 1982 *Nucl. Phys.* B **195** 297
Chadha S and Daniel M 1983 *Nucl. Phys.* B **229** 105
Ciuchini M, Degrassi G, Gambino P and Giudice G F 1998 *Nucl. Phys.* B **534** 3 (*Preprint* hep-ph/9806308)
Contino R, Pilo L, Rattazzi R and Trincherini E 2002 *Nucl. Phys.* B **622** 227 (*Preprint* hep-ph/0108102)
Davidson S and Ibarra A 2001 *J. High Energy Phys.* JHEP 03 (2001) 013 (*Preprint* hep-ph/0104076)
Davier M, Eidelman S, Hocker A and Zhang Z 2003 *Eur. Phys. J.* C **31** 503 (*Preprint* hep-ph/0308213)
de Boer W, Herold M, Sander C and Zhukov V 2003 (*Preprint* hep-ph/0309029)
de Boer W and Sanders C 2003 (*Preprint* hep-ph/0307049)
de Carlos B and Casas J A 1993 *Phys. Lett.* B **309** 320
Dedes A, Dreiner H K and Nierste U 2001 (*Preprint* hep-ph/0108037)
Dedes A and Pilaftsis A 2003 *Phys. Rev.* D **67** 015012 (*Preprint* hep-ph/0209306)
Degrassi G, Heinemeyer S, Hollik W, Slavich P and Weiglein G 2003 *Eur. Phys. J.* C **28** 133 (*Preprint* hep-ph/0212020)
Demir D, Lebedev O, Olive K A, Pospelov M and Ritz A 2003 (*Preprint* hep-ph/0311314)
Dermíšek R 2001 *Dubna 2001, Supersymmetry and Unification of Fundamental Interactions* pp 284–6 (*Preprint* hep-ph/0108249)
Dermíšek R, Mafi A and Raby S 2001 *Phys. Rev.* D **63** 035001 (*Preprint* hep-ph/0007213)
Dermíšek R and Raby S 2000 *Phys. Rev.* D **62** 015007 (*Preprint* hep-ph/9911275)
Dermíšek R, Raby S, Roszkowski L and Ruiz De Austri R 2003 *J. High Energy Phys.* JHEP 04 (2003) 037 (*Preprint* hep-ph/0304101)
Dimopoulos S 1983 *Phys. Lett.* B **129** 417
Dimopoulos S, Dine M, Raby S and Thomas S 1996 *Phys. Rev. Lett.* **76** 3494 (*Preprint* hep-ph/9601367)
Dimopoulos S and Georgi H 1981 *Nucl. Phys.* B **193** 150
Dimopoulos S and Giudice G F 1995 *Phys. Lett.* B **357** 573 (*Preprint* hep-ph/9507282)
Dimopoulos S and Hall L J 1995 *Phys. Lett.* B **344** 185 (*Preprint* hep-ph/9411273)
Dimopoulos S, Hall L J and Raby S 1992 *Phys. Rev. Lett.* **68** 1984
Dimopoulos S, Hall L J and Raby S 1992 *Phys. Rev.* D **45** 4192
Dimopoulos S and Pomarol A 1995 *Phys. Lett.* B **353** 222 (*Preprint* hep-ph/9502397)
Dimopoulos S and Raby S 1981 *Nucl. Phys.* B **192** 353
Dimopoulos S and Raby S 1983 *Nucl. Phys.* B **219** 479
Dimopoulos S, Raby S and Wilczek F 1981 *Phys. Rev.* D **24** 1681

Dimopoulos S, Raby S and Wilczek F 1982 *Phys. Lett.* **112B** 133
Dimopoulos S, Raby S and Wilczek F 1991 *Phys. Today* **44N10** 25
Dimopoulos S and Sutter D W 1995 *Nucl. Phys.* B **452** 496 (*Preprint* hep-ph/9504415)
Dine M and Fischler W 1982 *Phys. Lett.* B **110** 227
Dine M, Fischler W and Srednicki M 1981 *Nucl. Phys.* B **189** 575
Dine M, Leigh R and Kagan A 1993 *Phys. Rev.* D **48** 4269
Dine M and Nelson A E 1993 *Phys. Rev.* D **48** 1277 (*Preprint* hep-ph/9303230)
Dine M, Nelson A E and Shirman Y 1995 *Phys. Rev.* D **51** 1362 (*Preprint* hep-ph/9408384)
Dine M, Nelson A E, Nir Y and Shirman Y 1996 *Phys. Rev.* D **53** 2658 (*Preprint* hep-ph/9507378)
Dine M, Nir Y and Shadmi Y 2002 *Phys. Rev.* D **66** 115001 (*Preprint* hep-ph/0206268)
Dixon L J, Harvey J A, Vafa C and Witten E 1985 *Nucl. Phys.* B **261** 678
Dixon L J, Harvey J A, Vafa C and Witten E 1986 *Nucl. Phys.* B **274** 285 (1986)
Dreiner H K, Murayama H and Thormeier M 2003 (*Preprint* hep-ph/0312012)
Dudas E, Pokorski S and Savoy C A 1995 *Phys. Lett.* B **356** 45
Dudas E, Pokorski S and Savoy C A 1996 *Phys. Lett.* B **369** 255
Einhorn M B and Jones D R 1982 *Nucl. Phys.* B **196** 475
Elwood J K, Irges N and Ramond P 1997 *Phys. Lett.* B **413** 322 (*Preprint* hep-ph/9705270)
Elwood J K, Irges N and Ramond P 1998 *Phys. Rev. Lett.* **81** 5064 (*Preprint* hep-ph/9807228)
Ellis J, Falk T, Ganis G, Olive K A and Srednicki M 2001a *Phys. Lett.* B **510** 236
Ellis J R, Hisano J, Raidal M and Shimizu Y 2002 *Phys. Rev.* D **66** 115013 (*Preprint* hep-ph/0206110)
Ellis J, Kelly S and Nanopoulos D V 1991 *Phys. Lett.* B **260** 131
Ellis J, Nanopoulos D V and Rudaz S 1982 *Nucl. Phys.* B **202** 43
Ellis J, Nanopoulos D V and Olive K A 2001b *Phys. Lett.* B **508** 65
Ellis J R, Enqvist K, Nanopoulos D V and Zwirner F 1986 *Nucl. Phys.* B **276** 14
Ellis J R, Falk T, Olive K A and Santoso Y 2003a Nucl. Phys. B **652** 259 (*Preprint* hep-ph/0210205)
Ellis J R, Ferstl A, Olive K A and Santoso Y 2003b *Phys. Rev.* D **67** 123502 (*Preprint* hep-ph/0302032)
Ellis J R, Olive K A, Santoso Y and Spanos V C 2003c *Phys. Lett.* B **565** 176 (*Preprint* hep-ph/0303043)
Ellis J R, Olive K A, Santoso Y and Spanos V C 2003d Phys. Lett. B **573** 162 (*Preprint* hep-ph/0305212)
Ellis J R, Olive K A, Santoso Y and Spanos V C 2003e (*Preprint* hep-ph/0308075)
Ellis J R, Olive K A, Santoso Y and Spanos V C 2003f (*Preprint* hep-ph/0310356)
Ellis J R, Ridolfi G and Zwirner F 1991 *Phys. Lett.* B **257** 83
Espinosa J R and Zhang R J 2000a *J. High Energy Phys.* JHEP 03 (2000) 026
Espinosa J R and Zhang R J 2000b *Nucl. Phys.* B **586** 3
Eyal G 1998 *Phys. Lett.* B **441** 191 (*Preprint* hep-ph/9807308)
Faraggi A E and Pati J C 1998 *Nucl. Phys.* B **526** 21
Farrar G and Fayet P 1978 *Phys. Lett.* **B76** 575
Fayet P and Ferrara S 1977 *Phys. Rep.* **32** 249
Feng J L and Moroi T 2000 *Phys. Rev.* D **61** 095004 (*Preprint* hep-ph/9907319)
Feng J L, Matchev K T and Moroi T 2000a *Phys. Rev. Lett.* **84** 2322 (*Preprint* hep-ph/9908309)
Feng J L, Matchev K T and Moroi T 2000b *Phys. Rev.* D **61** 075005 (*Preprint* hep-ph/9909334)
Feng J L, Matchev K T and Wilczek F 2000c *Phys. Lett.* B **482** 388 (*Preprint* hep-ph/0004043)
Feng J L, Matchev K T and Wilczek F 2001 *Phys. Rev.* D **63** (045024)
Feng J L, Matchev K T 2001 *Phys. Rev.* D **63** 095003 (*Preprint* hep-ph/0011356)
Frampton P H, Glashow S L and Yanagida T 2002 *Phys. Lett.* B **548** 119 (*Preprint* hep-ph/0208157)
Frampton P H and Kephart T W 1995a *Phys. Rev.* D **51** 1
Frampton P H and Kephart T W 1995b *Int. J. Mod. Phys.* A **10** 4689
Frampton P H and Kong O C W 1996 *Phys. Rev. Lett.* **77** 1699 (*Preprint* hep-ph/9603372)
Frampton P H and Rasin A 2000 *Phys. Lett.* B **478** 424 (*Preprint* hep-ph/9910522)
Fritzsch H and Minkowski P 1975 *Ann. Phys.* **93** 193
Froggatt C D and Nielsen H B 1979 *Nucl. Phys.* B **147** 277
Fukugita M and Yanagida T 1986 *Phys. Lett.* B **174** 45
Gabbiani F, Gabrieli E, Masiero A and Silvestrini L 1996 *Nucl. Phys.* B **477** 321 (*Preprint* hep-ph/9604387)
Gavela M B, King S F, Sachrajda C T, Martinelli G, Paciello M L and Taglienti B 1989 *Nucl. Phys.* B **312** 269
Gell-Mann M, Ramond P and Slansky R 1979 *Supergravity (27–29 September 1979)* ed D Freedman *et al* (Amsterdam: North-Holland) 1980
Georgi H 1975 Particles and fields *Proc. APS Div. Particles and Fields* ed C Carlson, p 575
Georgi G 1986 *Phys. Lett.* B **169** 231
Georgi H and Glashow S L 1974 *Phys. Rev. Lett.* **32** 438

Georgi H and Jarlskog C 1979 *Phys. Lett.* B **86** 297
Georgi H, Quinn H R and Weinberg S 1974 *Phys. Rev. Lett.* **33** 451
Georgi H and Nanopoulos D V 1979 *Nucl. Phys.* B **159** 16
Girardello L and Grisaru M T 1982 *Nucl. Phys.* B **194** 65
Giudice G F, Notari A, Raidal M, Riotto A and Strumia A 2003 (*Preprint* hep-ph/0310123)
Giudice G F and Rattazzi R 1999 *Phys. Rep.* **322** 419 (*Preprint* hep-ph/9801271)
Glashow S L 1979 *Quarks and Leptons, Carg´ese (9–29 July 1979)* ed M L´evy *et al* (New York: Plenum) p 707
Goh H S, Mohapatra R N and Ng S P 2003 *Phys. Lett.* B **570** 215 (*Preprint* hep-ph/0303055)
Gonzalez-Garcia M C and Pena-Garay C 2003 *Phys. Rev.* D **68** 093003 (*Preprint* hep-ph/0306001)
Goto T and Nihei T 1999 *Phys. Rev.* D **59** 115009 (*Preprint* hep-ph/9808255)
Grinstein B 1982 *Nucl. Phys.* B **206** 387
Haber H E and Hempfling R 1993 *Phys. Rev.* D **48** 4280
Haber H E, Hempfling R and Hoang A H 1997 *Z. Phys. C* **75** 539 (*Preprint* hep-ph/9609331)
Hagiwara K, Martin A D, Nomura D and Teubner T 2003 (*Preprint* hep-ph/0312250)
Hall L J, Kostelecky V A and Raby S 1986 *Nucl. Phys.* B **267** 415
Hall L J, Lykken J and Weinberg S 1983 *Phys. Rev.* D **27** 2359
Hall L J and Murayama H 1995 *Phys. Rev. Lett.***75** 3985
Hall L J and Nomura Y 2001 *Phys. Rev.* D **64** 055003 (*Preprint* hep-ph/0103125)
Hall L J and Nomura Y 2002a *Phys. Rev.* D **65** 125012 (*Preprint* hep-ph/0111068)
Hall L J and Nomura Y 2002b *Phys. Rev.* D **66** 075004 (*Preprint* hep-ph/0205067)
Hall L J and Randall L 1990 *Phys. Rev. Lett.* **65** 2939
Hall L J and Rasin A 1993 *Phys. Lett.* B **315** 164 (*Preprint* hep-ph/9303303)
Hall L J, Rattazzi R and Sarid U 1994 *Phys. Rev.* D **50** 7048
Hamzaoui C, Pospelov M and Toharia M 1999 *Phys. Rev.* D **59** 095005
Hara Y, Itoh S, Iwasaki Y and Yoshie T 1986 *Phys. Rev.* D **34** 3399
Harvey J, Ramond P and Reiss D B 1980 *Phys. Lett.* B **92** 309
Harvey J, Ramond P and Reiss D B 1982 *Nucl. Phys.* B **199** 223
Hebecker A and March-Russell J 2002 *Phys. Lett.* B **539** 119 (*Preprint* hep-ph/0204037)
Hempfling R 1994 *Phys. Rev.* D **49** 6168
Hisano J, Moroi T, Tobe K, Yamaguchi M and Yanagida T 1995 *Phys. Lett.* B **357** 579 (*Preprint* hep-ph/9501407)
Hisano J, Moroi T, Tobe K and Yamaguchi M 1996 *Phys. Rev.* D **53** 2442 (*Preprint* hep-ph/9510309)
Hisano J and Tobe K 2001 *Phys. Lett.* B **510** 197 (*Preprint* hep-ph/0102315)
Huang C S and Yan Q S 1998 *Phys. Lett.* B **442** 209 (*Preprint* hep-ph/9803366)
Huang C S, Liao W and Yan Q S 1999 *Phys. Rev.* D **59** 011701 (*Preprint* hep-ph/9803460)
Huang C S, Liao W, Yan Q S and Zhu S H 2001 *Phys. Rev.* D **63** 114021
Huang C S, Liao W, Yan Q S and Zhu S H 2001 *Phys. Rev.* D **64** 059902 (erratum)
Ibañez L E 1982 *Phys. Lett.* B **118** 73
Ibañez L E and Lopez C 1983 *Phys. Lett.* B **126** 54
Ibañez L E and Lopez C 1984 *Nucl. Phys.* B **233** 511
Ibañez L and Ross G G 1981 *Phys. Lett.* B **105** 439
Ibañez L and Ross G G 1982 *Phys. Lett.* B **110** 215
Ibañez L and Ross G G 1992 *Perspectives on Higgs Physics* ed G L Kane, pp 229–75 (*Preprint* hep-ph/9204201)
Inoue K, Kakuto A, Komatsu H and Takeshita S 1982 *Prog. Theor. Phys.* **67** 1889
Irges N, Lavignac S and Ramond P 1998 *Phys. Rev.* D **58** 035003 (*Preprint* hep-ph/9802334)
Isidori G and Retico A 2001 (*Preprint* hep-ph/0110121)
Jeannerot R, Khalil S and Lazarides G 2001 *Phys. Lett.* B **506** 344 (*Preprint* hep-ph/0103229)
Jung C K 2002 *Brookhaven National Laboratory* (*Preprint* UNO 02-BNL)
Kakizaki M and Yamaguchi M 2002 *J. High Energy Phys.* JHEP 06 (2002) 032 (*Preprint* hep-ph/0203192)
Kane G L, Kolda C F, Roszkowski L and Wells J D 1994 *Phys. Rev.* D **49** 6173 (*Preprint* hep-ph/9312272)
Kane G L, Nelson B D, Wang T T and Wang L T (2003) (*Preprint* hep-ph/0304134)
Kaplan D B and Schmaltz M 1994 *Phys. Rev.* D **49** 3741 (*Preprint* hep-ph/9311281)
Kawamura Y 2001a *Prog. Theor. Phys.* **105** 999 (*Preprint* hep-ph/0012125)
Kawamura Y 2001b *Prog. Theor. Phys.* **105** 691 (*Preprint* hep-ph/0012352)
Kehoe R 2003, http://www-clued0.fnal.gov/kehoe/physics/fermiwc.pdf
Kim H D and Raby S 2003 *J. High Energy Phys.* JHEP 01 (2003) 056 (*Preprint* hep-ph/0212348)
Kim H D, Raby S and Schradin L 2004 (*Preprint* hep-ph/0401169)
King S F 1998 *Phys. Lett.* B **439** 350 (*Preprint* hep-ph/9806440)
King S F 2000 *Nucl. Phys.* B **576** 85 (*Preprint* hep-ph/9912492)

King S F 2003 *Preprint* hep-ph/0310204
King S F and Ross G G 2003 *Phys. Lett.* B **574** 239 (*Preprint* hep-ph/0307190)
Kitano R and Mimura Y 2001 *Phys. Rev.* D **63** 016008
Klebanov I R and Witten E 2003 *Nucl. Phys.* B **664** 3 (*Preprint* hep-th/0304079)
Langacker P and Luo M 1991 *Phys. Rev.* D **44** 817
Lavignac S, Masina I and Savoy C A 2001 *Phys. Lett.* B **520** 269 (*Preprint* hep-ph/0106245)
Lavignac S, Masina I and Savoy C A 2002 *Nucl. Phys.* B **633** 139 (*Preprint* hep-ph/0202086)
Leontaris G K, Tamvakis K and Vergados J D 1986 *Phys. Lett.* B **171** 412
LEP2 working group Barate *et al* (Aleph Collaboration) 2003 *Phys. Lett.* B **565** 61 (*Preprint* hep-ex/0306033)
Leurer M, Nir Y and Seiberg N 1993 *Nucl. Phys.* B **398** 319
Leurer M, Nir Y and Seiberg N 1994 *Nucl. Phys.* B **420** 319468
Lin C-J S (CDF Collaboration) 2003 *Beauty 2003 Conference (Pittsburgh, PA)*
Lola S and Ross G G 1999 *Nucl. Phys.* B **553** 81 (*Preprint* hep-ph/9902283)
Lucas V and Raby S 1996 *Phys. Rev.* D **54** 2261 (*Preprint* hep-ph/9601303)
Lucas V and Raby S 1997 *Phys. Rev.* D **55** 6986 (*Preprint* hep-ph/9610293)
Lunghi E, Masiero A, Scimemi I and Silvestrini L 2000 *Nucl. Phys.* B **568** 120 (*Preprint* hep-ph/9906286)
Maekawa N 2001 *Prog. Theor. Phys.* **106** 401
Maltoni M, Schwetz T, Tortola M A and Valle J W F 2003 *Phys. Rev.* D **68** 113010 (*Preprint* hep-ph/0309130)
Marciano W J and Senjanovic G 1982 *Phys. Rev.* D **25** 3092
Masiero A, Nanopoulos D V, Tamvakis K and Yanagida T 1982 *Phys. Lett.* B **115** 380
Masiero A, Vempati S K and Vives O 2003 *Nucl. Phys.* B **649** 189 (*Preprint* hep-ph/0209303)
Matalliotakis D and Nilles H P 1995 *Nucl. Phys.* B **435** 115
Melnikov K and Vainshtein A 2003 (*Preprint* hep-ph/0312226)
Misiak M, Pokorski S and Rosiek J 1998 *Adv. Ser. Direct. High Energy Phys.* **15** 795 (*Preprint* hep-ph/9703442)
Mohapatra R N, Parida M K and Rajasekaran G 2003 (*Preprint* hep-ph/0301234)
Mohapatra R N and Senjanovic G 1980 *Phys. Rev. Lett.***44** 912
Moroi T 1996 *Phys. Rev.* D **53** 6565
Moroi T 1997 *Phys. Rev.* D **56** 4424 (*Preprint* hep-ph/9512396)
Muon (g - 2) Collaboration, Bennett G W *et al* 2002 *Phys. Rev. Lett.* **89** 101804
Muon (g - 2) Collaboration, Bennett G W *et al* 2002 *Phys. Rev. Lett.* **89** 129903
Munoz C 2003 *Preprint* hep-ph/0312321
Murayama H, Olechowski M and Pokorski S 1996 *Phys. Lett.* B **371** 57
Murayama H and Pierce A 2002 *Phys. Rev.* D **65** 055009 (*Preprint* hep-ph/0108104)
Nath P and Arnowitt R 1997 *Phys. Rev.* D **56** 2820 (*Preprint* hep-ph/9701301)
Nilles H P, Srednicki M and Wyler D 1983 *Phys. Lett.* B **120** 346
Nir Y and Seiberg N 1993 *Phys. Lett.* B **309** 337
Nomura Y 2002 *Phys. Rev.* D **65** 085036 (*Preprint* hep-ph/0108170)
Nomura Y, Smith D R and Weiner N 2001 *Nucl. Phys.* B **613** 147 (*Preprint* hep-ph/0104041)
Nomura Y and Yanagida T 1999 *Phys. Rev.* D **59** 017303 (*Preprint* hep-ph/9807325)
Okada Y, Yamaguchi M and Yanagida T 1991 *Prog. Theor. Phys.* **85** 1
Olechowski M and Pokorski S 1988 *Phys. Lett.* B **214** 393
Olechowski M and Pokorski S 1995 *Phys. Lett.* B **344** 201
Ovrut B A and Wess J 1982 *Phys. Lett.* B **112** 347
Paschos E A 2003 *Preprint* hep-ph/0308261
Pati J C and Salam A 1973a *Phys. Rev.* D **8** 1240
Pati J C and Salam A 1973b *Phys. Rev. Lett.* **31** 661
Pati J C and Salam A 1974 *Phys. Rev.* D **10** 275
Pierce D *et al* 1997 *Nucl. Phys.* B **491** 3
Pokorski S 1990 *Nucl. Phys.* B **13** (Proc. Suppl.) 606
Polchinski J and Susskind L 1982 *Phys. Rev.* D **26** 3661
Polonsky N and Pomarol A 1995 *Phys. Rev.* D **51** 6532
Polony J 1977 *Budapest Preprint* KFKI-1977-93 unpublished
Pomarol A and Dimopoulos S 1995 *Nucl. Phys.* B **453** 83 (*Preprint* hep-ph/9505302)
Pomarol A and Tommasini D 1996 *Nucl. Phys.* B **466** 3 (*Preprint* hep-ph/9507462)
Pouliot P and Seiberg N 1993 *Phys. Lett.* B **318** 169
Raby S 1995 *Trieste HEP Cosmology* 1994:0126-180 (*Preprint* hep-ph/9501349)
Raby S 2001 *Proc. Higgs & SUSY (Orsay, France, March 2001)* http://www.lal.in2p3.fr/actualite/conferences/higgs2001/slides/ *Dubna 2001, Supersymmetry and Unification of Fundamental Interactions* pp 168–76 (*Preprint* hep-ph/0110203)

Raby S (Hagiwara K *et al* (Particle Data Group Collaboration)) 2002a *Phys. Rev.* D **66** 010001

Raby S 2002b *Proc. SUSY'02* (*Preprint* hep-ph/0211024)

Raby S 2003 *Phys. Lett.* B **561** 119

Raidal M and Strumia A 2003 *Phys. Lett.* B **553** 72 (*Preprint* hep-ph/0210021)

Rattazzi R and Sarid U 1996 *Phys. Rev.* D **53** 1553

Roberts R G, Romanino A, Ross G G and Velasco-Sevilla L 2001 *Nucl. Phys.* B **615** 358 (*Preprint* hep-ph/0104088)

Romanino A and Strumia A 2000 *Phys. Lett.* B **487** 165 (*Preprint* hep-ph/9912301)

Ross G G and Velasco-Sevilla L 2003 *Nucl. Phys.* B **653** 3

Roszkowski L, Ruiz de Austri, R and Nihei T 2001 *J. High Energy Phys.* JHEP 08 (2001) 024 (*Preprint* hep-ph/0106334)

Sakai N 1981 *Z. Phys.* C **11** 153

Sakai N and Yanagida T 1982 *Nucl. Phys.* B **197** 533

Shafi Q and Ananthanarayan B 1991 *ICTP Summer School Lectures*

Shafi Q and Tavartkiladze Z 2000 *Phys. Lett.* B **487** 145

Strumia A 1999 *Preprint* hep-ph/9904247

Tobe K and Wells J D 2003 *Nucl. Phys.* B **663** 123 (*Preprint* hep-ph/0301015)

Weinberg S 1982 *Phys. Rev.* D **26** 287

Weisskopf V 1939 *Phys. Rev.* **56** 72

Witten E 1981 *Nucl. Phys.* B **188** 513

Witten E 2002 *Hamburg 2002, Supersymmetry and Unification of Fundamental Interactions* vol 1, pp 472–91 (*Preprint* hep-ph/0201018)

Yanagida T 1979 *Proc. Workshop on the Unified Theory and the Baryon Number of the Universe (Tsukuba, 13–14 February 1979)* ed O Sawada and A Sugamoto, p 95

Zhang R J 1999 *Phys. Lett.* B **447** 89