# Chiral matter wavefunctions in warped compactifications

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# Outline

1. Motivation and setup
2. Unmagnetized branes
3. Magnetized branes
4. Warped kinetic terms
5. Conclusions

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# Motivation

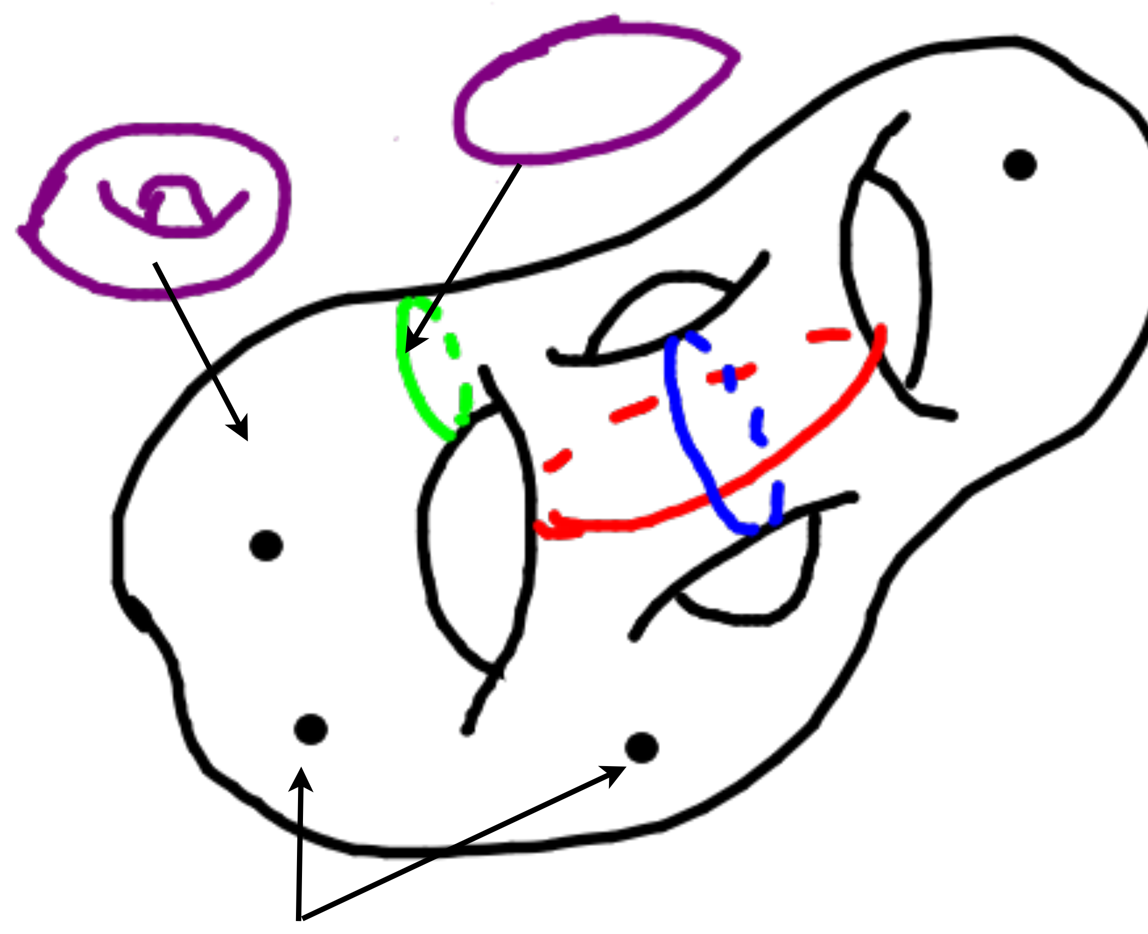
- Warping plays an important role in string models:
  - Generation of the electroweak hiearchy [RS; GKP]
  - Stabilization of moduli [KKLT;...]
  - Late-time acceleration [KKLT;...] and inflation [KKLMMT;...]
  - Gauge/gravity duality
- In type II theories, realistic models require open strings
- A 4D effective action is valuable for detailed phenomenology

# Motivation (cont.)

- Alternatively, consider an F-theory compactification
- In such constructions must satisfy the D3-tadpole condition

$$\frac{\chi(X)}{24} = N_{\mathrm{D3}} + \int_{B_3} H^{(3)} \wedge F^{(3)}$$

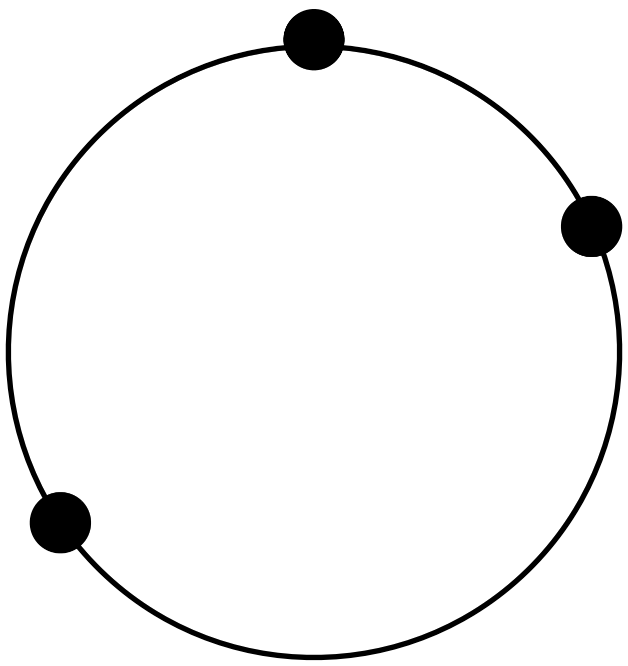
- Additional ingredients will cause warping which will modify 4D EFT



# Warped effective field theory

- In a flat background, CFT techniques can be used to determine an EFT (Kähler metrics, Yukawa couplings [Lüst et. al.; Cvetič et. al.; Ibáñez et. al;...])

- However, warping in type II usually involves Ramond-Ramond fluxes and it is difficult to make use of CFT techniques



# Warped effective field theory

- Alternative: Dimensional reduction of a higher dimensional EFT
- Requires knowledge of wavefunctions of light degrees of freedom
- Here, my focus is on open string modes (see B. Underwood's talk for closed strings)

$$-\frac{1}{2g_8^2}\int_{\mathcal{W}} \mathrm{d}^8x\,\sqrt{g}F^2$$

$$A_\mu(x,y) = A_\mu(x)\,s(y)$$

$$-\frac{1}{4g_8^2}\int_{\mathbb{R}^{1,3}} \mathrm{d}^4x F_{\mu\nu}F^{\mu\nu}\int_{\mathcal{S}_4} \mathrm{d}^4y\sqrt{g}s^2$$

$$\boxed{\frac{1}{g_4^2} = \frac{\mathcal{V}_{\mathcal{S}_4}}{g_8^2}}$$

# Setup

- For simplicity, compactify IIB on $\mathbb{T}^6 = \mathbb{T}_1^2 \times \mathbb{T}_2^2 \times \mathbb{T}_3^2$

$$\mathrm{d}s_{10}^2 = \mathrm{e}^{2\alpha}\mathrm{d}x_4^2 + \mathrm{e}^{-2\alpha}\mathrm{d}z^m\mathrm{d}\bar{z}^{\overline{m}}$$

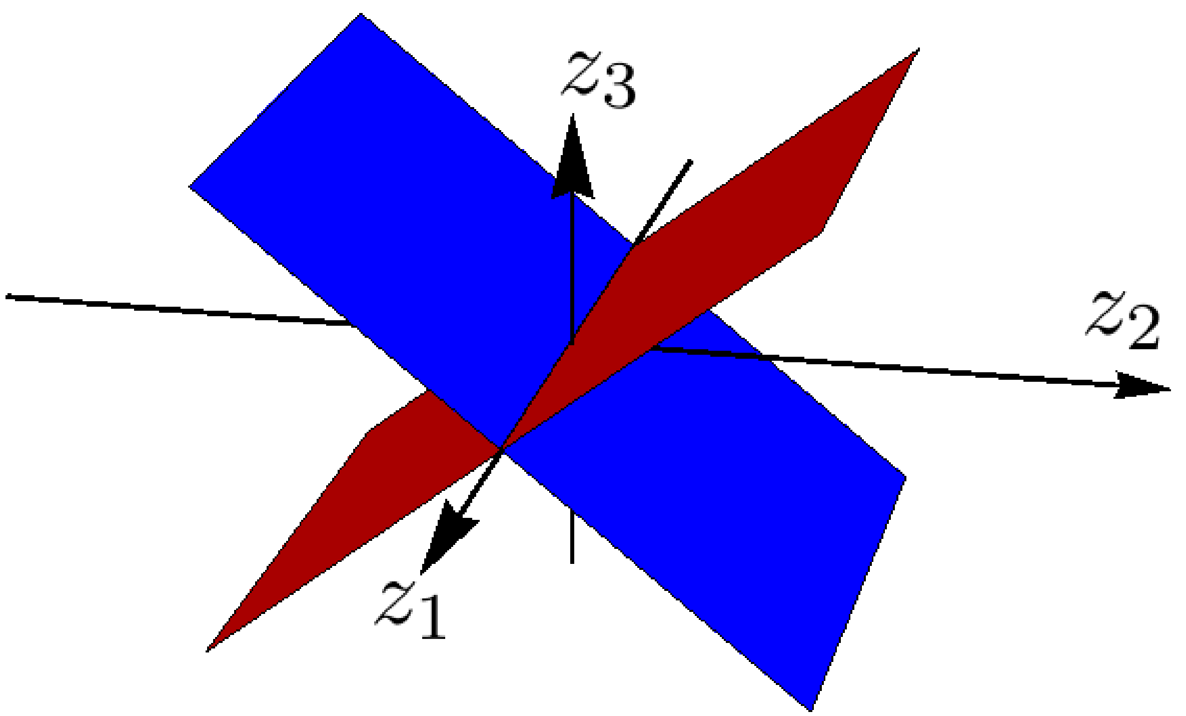
$$F^{(5)} = (1 + *_{10})\, F_{\mathrm{ext}}^{(5)} \quad F_{\mathrm{ext}}^{(5)} = \mathrm{e}^{4\alpha} \wedge \mathrm{dvol}_{\mathbb{R}^{1,3}} \quad G^{(3)} = 0$$

- Add two probe D7 branes

$$\mathrm{D7}_1 : z^3 = M_3 z^2$$

$$\mathrm{D7}_2 : z^3 = -M_3 z^2$$

- Gives a $\mathrm{U}(1) \times \mathrm{U}(1)$ gauge symmetry



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# Single D-brane

- For a single D7-brane, the 4D bosonic d.o.f. are
  - Gauge boson $A_\mu$
  - Wilson lines $A_a$
  - Worldvolume deformations $\Phi$

- Dynamics described by the DBI+CS action

$$M_{\alpha\beta} = \mathrm{P}\big[g_{\alpha\beta} + B_{\alpha\beta}\big] + \lambda F_{\alpha\beta}$$

$$\begin{aligned} S_{\mathrm{D7}} =& S_{\mathrm{D7}}^{\mathrm{DBI}} + S_{\mathrm{D7}}^{\mathrm{CS}} \\ S_{\mathrm{D7}}^{\mathrm{DBI}} =& -\tau_{\mathrm{D7}} \int_{\mathcal{W}} \mathrm{d}^8 x \big(\mathrm{Im}\,\tau\big)^{-1} \sqrt{M_{\alpha\beta}} \\ S_{\mathrm{D7}}^{\mathrm{CS}} =& \tau_{\mathrm{D7}} \int_{\mathcal{W}} \mathrm{P}\Big[\mathcal{C} \wedge \mathrm{e}^{B^{(2)}}\Big] \wedge \mathrm{e}^{\lambda F^{(2)}} \end{aligned}$$

$$\lambda = 2\pi\alpha'$$

# Intersections as Higgsing

- When the branes are coincident, the symmetry is enhanced to $\mathrm{U}(2)$. The transverse fluctuations are promoted to an adjoint-valued scalar $\Phi$

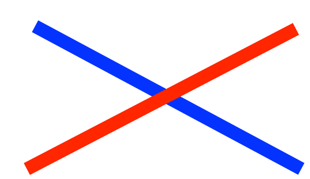
Position of $\mathrm{D7}_1$ Position of $\mathrm{D7}_2$

$$\Phi = \begin{pmatrix} \phi^a & \phi^- \\ \phi^+ & \phi^b \end{pmatrix}$$

$7_1$-$7_2$ strings (bifundamental)

- vevs for $\phi^{a,b}$ correspond to background D7 positions

$$\langle \phi^a \rangle = \lambda^{-1} M_3 z^2 \quad \langle \phi^b \rangle = -\lambda^{-1} M_3 z^2$$



# Myers action

- Bosonic fluctuations governed by the Myers action

$$S_{\mathrm{D7}} = S_{\mathrm{D7}}^{\mathrm{DBI}} + S_{\mathrm{D7}}^{\mathrm{CS}}$$

symmetrization

$$S_{\mathrm{D7}}^{\mathrm{DBI}} = -\tau_{\mathrm{D7}} \int_{\mathcal{W}} \mathrm{d}^8 x \, \mathrm{Str}\left\{ \left(\mathrm{Im}\,\tau\right)^{-1} \sqrt{\det M_{\alpha\beta} \det Q^i{}_j} \right\}$$

interior product

$$S_{\mathrm{D7}}^{\mathrm{CS}} = \tau_{\mathrm{D7}} \int_{\mathcal{W}} \mathrm{Str}\left\{ \mathrm{P}\left[ \mathrm{e}^{\mathrm{i}\lambda \iota_\Phi \iota_\Phi} \mathcal{C} \wedge \mathrm{e}^{B^{(2)}} \right] \wedge \mathrm{e}^{\lambda F^{(2)}} \right\}$$

non-Abelian pullback: $\mathrm{P}\left[v_\alpha\right] = v_\alpha + \lambda v_i D_\alpha \Phi^i$

where:

$$M_{\alpha\beta} = \mathrm{P}\left[E_{\alpha\beta} + \left(\mathrm{Im}\,\tau\right)^{-1/2} E_{\alpha i}\left(Q^{-1} - \delta\right)^{ij} E_{j\beta}\right] + \lambda\left(\mathrm{Im}\,\tau\right)^{1/2} F_{\alpha\beta}$$

$$E_{MN} = g_{MN} + \left(\mathrm{Im}\,\tau\right)^{1/2} B_{MN} \qquad Q^i_j = \delta^i_j - \mathrm{i}\lambda\left[\Phi^i, \Phi^k\right]\left(\mathrm{Im}\,\tau\right)^{-1/2} E_{kj}$$

# Myers action (cont.)

- Bulk fields given as a non-Abelian Taylor expansion

adjoint valued

$$\Psi\left[\Phi\right] = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \Phi^{i_1} \cdots \Phi^{i_n} \partial_{i_1} \cdots \partial_{i_n} \Psi_0$$

neutral

need small angle

$$= \Psi_0 + \mathcal{O}\left(\lambda\right)$$

- Leading order in $\alpha'$, action is Higgsed warped Yang-Mills

- Equations of motion are second order and hard to solve in general

# Fermionic action

- To get first order equations, can use fermionic action
- d.o.f. encoded in two 10D M-W spinors $\Theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$
- Abelian case: [Martucci, Rosseel, Van denBleeken, Van Proeyen;...]

$$S^{\mathrm{F}}_{\mathrm{D7}} = \frac{1}{g_8^2} \int \mathrm{d}^8 x \, (\mathrm{Im}\,\tau)^{-1} \sqrt{\det M_{\alpha\beta}} \bar{\Theta} P_-^{\mathrm{D7}} \big[ (\mathcal{M}^{-1})^{\alpha\beta} \Gamma_\beta \mathcal{D}_\alpha - \mathcal{O} \big] \Theta$$

Involves $\Gamma_{\mathrm{D7}}$

$$\delta_\epsilon \Psi_M = \mathcal{D}_M \epsilon$$
$$\delta_\epsilon \lambda = \mathcal{O} \epsilon$$

# Fermionic action (cont.)

- In a warped geometry ($G^{(3)} = 0$),

$$\mathcal{D}_M = \nabla_M + \frac{1}{4}\not{F}^{(5)}\Gamma_M \qquad \mathcal{O} = 0$$

- After $\kappa$-fixing

$$S^{\mathrm{F}}_{\mathrm{D}7} = \frac{1}{g_8^2}\int_{\mathcal{W}} \mathrm{d}^8x\,\bar{\theta}\Big\{\mathrm{e}^{-\alpha}\not{\partial}_{\mathbb{R}^{1,3}} + \mathrm{e}^{\alpha}\not{\partial}_{\mathbb{T}^4} + \mathrm{e}^{\alpha}\frac{1}{2}\not{\partial}_{\mathbb{T}^4}\alpha\big(1 + 2\Gamma_{\mathcal{S}^4}\big)\Big\}\theta$$

4-cycle chirality

- Non-Abelian modification (leading $\alpha'$) [Wynants]:

$$\not{\partial} \to \not{D} \qquad \delta\mathcal{L} = -\mathrm{i}\bar{\theta}\mathrm{e}^{-\alpha}\Gamma_i\left[\Phi^i, \theta\right]$$

and take trace

# Adjoint fields

- Warping effect on adjoint zero-mode wavefunctions are mild

Vector multiplet: $A_\mu \sim \mathrm{const}$ $\qquad \psi_0 \sim \mathrm{e}^{-3\alpha/2}$

Wilson line multiplet: $A_m \sim \mathrm{const}$ $\qquad \psi_{1,2} \sim \mathrm{e}^{\alpha/2}$

Deformation multiplet: $\phi \sim \mathrm{const}$ $\qquad \psi_3 \sim \mathrm{e}^{-3\alpha/2}$

Consistent with supersymmetry

# Equations of motion

- For the bifundamental modes, take the ansatz

$$\psi^{\mp}_{0,3} = \mathrm{e}^{-3\alpha/2}\chi^{\mp}_{0,3} \qquad \psi^{\mp}_{1,2} = \mathrm{e}^{\alpha/2}\chi^{\mp}_{1,2}$$

gaugino, modulino

wilsonini

- Equations of motion:

$$\begin{aligned} 0 =& \partial_1\chi_1^{\mp} + \partial_2\chi_2^{\mp} + \mathrm{e}^{-4\alpha} D_3^{\mp}\chi_3^{\mp} \\ 0 =& \partial_1\chi_0^{\mp} + \partial_2^*\chi_3^{\mp} - D_3^{\pm *}\chi_2^{\mp} \\ 0 =& \partial_1^*\chi_3^{\mp} - \partial_2\chi_0^{\mp} - D_3^{\pm *}\chi_1^{\mp} \\ 0 =& \partial_1^*\chi_2^{\mp} - \partial_2^*\chi_1^{\mp} + \mathrm{e}^{-4\alpha}\hat{D}_3^{\mp}\chi_0^{\mp} \end{aligned}$$

$$D_3^{\mp} = \mp\mathrm{i}M_3\bar{z}^{\bar{2}}$$

# BPS conditions

- For a single D7 brane, the equations of motion follow from F- and D-flatness conditions: [Jockers, Louis; Martucci]

$$W = \int_{\mathcal{S}_4} \mathrm{P}\big[\gamma\big] \wedge \mathrm{e}^{\lambda F^{(2)}} \qquad D = \int_{\mathcal{S}_4} \mathrm{P}\big[\mathrm{Im}\,\eta\big] \wedge \mathrm{e}^{\lambda F^{(2)}}$$

fundamental 3-form

$$\mathrm{d}\gamma = \Omega \wedge \mathrm{e}^{B^{(2)}} \qquad \eta = \mathrm{e}^{2\alpha}\mathrm{Im}\,\tau\,\mathrm{e}^{\mathrm{i}J} \wedge \mathrm{e}^{B^{(2)}}$$

warped Kähler form

- Comparing to the CS-action, the non-Abelian version should be (see also [Butti et. al.])

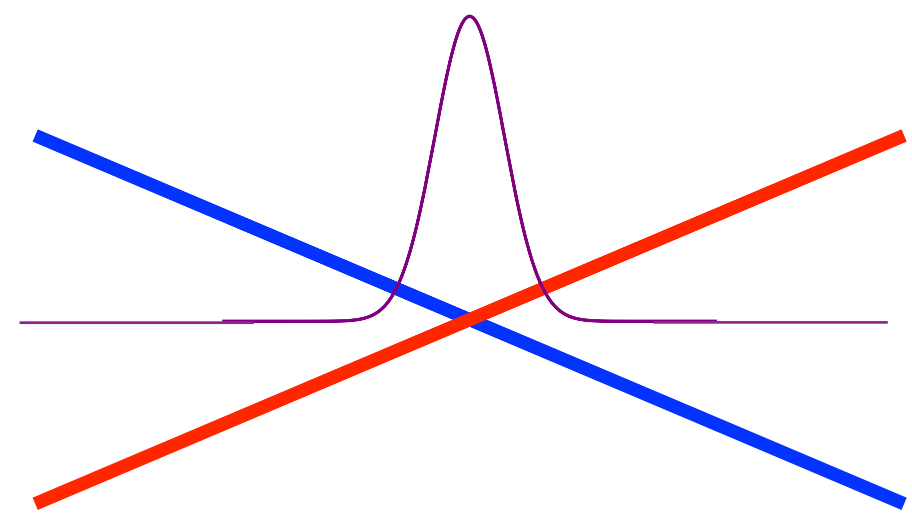
$$W = \int_{\mathcal{S}_4} \mathrm{Str}\Big\{\mathrm{P}\big[\mathrm{e}^{\mathrm{i}\lambda \iota_\Phi \iota_\Phi}\gamma\big] \wedge \mathrm{e}^{\lambda F^{(2)}}\Big\} \quad D = \int_{\mathcal{S}_4} \mathrm{S}\Big\{\mathrm{P}\big[\mathrm{e}^{\mathrm{i}\lambda \iota_\Phi \iota_\Phi}\mathrm{Im}\,\eta\big] \wedge \mathrm{e}^{\lambda F^{(2)}}\Big\}$$

These yield the previous equations of motion with $\psi_0 = 0$

# Unwarped zero mode

- In the absence of warping, the zero modes are exponentially localized on the intersection

$$
\begin{aligned}
\psi^{\mp}_{0,1} &= 0 \\
\psi^{\mp}_{2} &= \sigma^{\mp}(x^{\mu})\mathrm{e}^{-M_3\left|z^2\right|^2} \\
\psi^{\mp}_{3} &= \pm\,\mathrm{i}\sigma^{\mp}(x^{\mu})\mathrm{e}^{-M_3\left|z^2\right|^2}
\end{aligned}
$$



4D field

- Mixture of deformation modulus and Wilson line of the un-Higgsed theory

# Warped zero mode

- For arbitrary warping, no simple analytic solution
- In the weak warping case, can treat the warping as a perturbation

$$\mathrm{e}^{-4\alpha} = 1 + \epsilon\beta \qquad\qquad \epsilon \ll 1$$

- Can then expand the warped zero mode in terms of the unwarped massive modes

# Unwarped spectrum

- The equation of motion for the massive modes is

$$\mathbf{D}^{\mp}\mathbf{X}_{\lambda}^{\mp} = m_{\lambda}\mathbf{X}_{\lambda}^{\pm *} \qquad \mathbf{D}^{\mp} = \begin{pmatrix} 0 & \partial_1 & \partial_2 & D_3^{\mp} \\ -\partial_1 & 0 & D_3^{\pm *} & -\partial_2^* \\ -\partial_2 & -D_3^{\pm *} & 0 & \partial_1^* \\ -D_3^{\mp} & \partial_2^* & -\partial_1^* & 0 \end{pmatrix} \quad \mathbf{X}_{\lambda}^{\mp} = \begin{pmatrix} \chi_0^{\mp} \\ \chi_1^{\mp} \\ \chi_2^{\mp} \\ \chi_3^{\mp} \end{pmatrix}$$

- Easiest to work in a rotated basis $\mathbf{X}'^{\mp} = \mathbf{J}^{-1}\mathbf{X}^{\mp}$

$$\mathbf{J} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1/\sqrt{2} & \mathrm{i}/\sqrt{2} \\ & & \mathrm{i}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$\begin{aligned} \partial_1 &\to \partial_1 \\ \partial_2 &\to \hat{D}_2'^{\mp} = \frac{1}{\sqrt{2}}\left(\partial_2 \pm M_3 \bar{z}^{\bar{2}}\right) \\ \partial_3 &\to \hat{D}_3'^{\mp} = \frac{\mathrm{i}}{\sqrt{2}}\left(\partial_2 \mp M_3 \bar{z}^{\bar{2}}\right) \end{aligned}$$

# Unwarped spectrum (cont.)

- Boundary conditions:
  - Periodicity along $\mathbb{T}_1^2$
  - Localized on intersection

- $-$ - sector modes built from ladder operators (giving two simple harmonic oscillator algebras) and Fourier modes

lowering: $\mathrm{i}\hat{D}_2'^{-}$, $\mathrm{i}\left(\hat{D}_3'^{+}\right)^*$

raising: $\mathrm{i}\left(\hat{D}_2'^{+}\right)^*$, $\mathrm{i}\hat{D}_3'^{-}$

$$\varphi^-_{mnpl} = \underbrace{h_{mn}}_{\text{Fourier mode}}\left(z^1, \bar{z}^{\bar{1}}\right)\left[\mathrm{i}\left(\hat{D}_2'^{+}\right)\right]^l\left(\mathrm{i}\hat{D}_3'^{-}\right)^p \mathrm{e}^{-\kappa\left|z^2\right|^2}$$

# Unwarped spectrum (cont.)

- Unwarped spectrum:

$$m_\lambda^2 = m^2 + n^2 + M_3\big(l + p + 1\big) \qquad \Phi_\lambda'^- = \big(\varphi_{mnlp}^-, 0, 0, 0\big)$$

$$m_\lambda^2 = m^2 + n^2 + M_3\big(l + p + 1\big) \qquad \Phi_\lambda'^- = \big(0, \varphi_{mnlp}^-, 0, 0\big)$$

$$m_\lambda^2 = m^2 + n^2 + M_3\big(l + p\big) \qquad \Phi_\lambda'^- = \big(0, 0, \varphi_{mnlp}^-, 0\big)$$

$$m_\lambda^2 = m^2 + n^2 + M_3\big(l + p + 2\big) \qquad \Phi_\lambda'^- = \big(0, 0, 0, \varphi_{mnlp}^-\big)$$

# Expanding the warped zero mode

- Write the warped zero mode as

$$\mathbf{X}^- = \mathbf{\Phi}_0^- + \sum_\lambda c_\lambda \mathbf{\Phi}_\lambda^-$$

unwarped modes

- To leading order

$$c_\lambda = \frac{1}{m_\lambda^2} \int_{\mathcal{S}_4} \mathrm{d}^4 y \left(\mathbf{\Phi}_\lambda^-\right)^* \cdot \left(\mathbf{D}_0^+\right)^* \beta \mathbf{K}^- \mathbf{\Phi}_0^-$$

where

$$\mathbf{D}^- = \mathbf{D}_0^- + \epsilon\beta\mathbf{K}^- + \mathcal{O}\left(\epsilon^2\right)$$

- Examples given in [1011.xxxx]

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# Chirality

- Without magnetic flux, the spectrum is vector-like
- In order to have a chiral theory, the intersection must be magnetized

$$\frac{1}{2\pi}\int_{\mathbb{T}^2} F^{(2)} = M_1\sigma_3$$

- SUSY requires [Marino, Minasian, Moore, Strominger; ...]

$$F^{(2)} = - *_4 F^{(2)}$$

Hodge-$*$ on $\mathcal{S}_4$

# Unwarped zero modes

- For example, if $M_1 > 0$, only the $-$-sector has zero modes

- Due to magnetic flux, wavefunction are quasi-periodic
[Cremades, Ibáñez, Marchesano;...]

$$\varphi_0^{j,-} = \mathrm{e}^{-\kappa \left|z^2\right|^2} \mathrm{e}^{2\pi \mathrm{i} M_1 z^1 \mathrm{Im}\, z^1} \vartheta \begin{bmatrix} j/2M_1 \\ 0 \end{bmatrix} \left(2M_1 z^1, \mathrm{i}2M_1\right)$$

$$\kappa = \sqrt{\left(\tfrac{M_1}{2}\right)^2 + M_3^2} \qquad\qquad j = 0, \ldots, 2M_1 - 1$$

families orthogonal: $\int_{\mathcal{S}_4} \mathrm{d}^4 y \left(\varphi_0^{j,-}\right)^* \varphi_0^{k,-} = \delta^{kj}$

# Warped zero modes

- As in unmagnetized case, warped zero mode has no general simple analytic solution

- Again, expand in unwarped massive modes

- Spectrum built from three QSHO algebras

$$\varphi^{j,-}_{nlp} = \left(\mathrm{i}D'^{-}_1\right)^n \left[\mathrm{i}\left(D'^{+}_2\right)\right]^l \left(\mathrm{i}\hat{D}'^{-}_3\right)^p \varphi^{j,-}_0$$

$$\begin{aligned} D'^{\mp}_1 &= \partial_1 \mp M_1 \bar{z}^{\bar{1}} \\ D'^{\mp}_2 &\propto \partial_2 \pm \kappa \bar{z}^{\bar{2}} \\ D'^{\mp}_3 &\propto \mathrm{i}\left(\partial_2 \mp \kappa \bar{z}^{\bar{2}}\right) \end{aligned}$$

# Warped zero modes (cont.)

- Expand warped zero mode in terms of unwarped massive modes

$$\mathbf{X}^{j,-} = \mathbf{\Phi}_0^{j,-} + \sum_{k,\lambda} c_\lambda^k \mathbf{\Phi}_\lambda^{k,-}$$

unwarped modes

then

$$c_\lambda^k = \frac{1}{m_\lambda^2} \int_{\mathcal{S}_4} \mathrm{d}^4 y \left(\mathbf{\Phi}_\lambda^{k,-}\right)^* \cdot \left(\mathbf{D}_0^+\right)^* \beta \mathbf{K}^- \mathbf{\Phi}_0^{j,-}$$

family mixing is generic

- Examples given in [1011.xxxx]

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# Adjoint fields

- Recall the adjoint field wavefunctions

$$\psi_0 \sim \mathrm{e}^{-3\alpha/2} \qquad \psi_{1,2} \sim \mathrm{e}^{\alpha/2} \qquad \psi_3 \sim \mathrm{e}^{-3\alpha/2}$$

- The resulting 4D kinetic terms are

$$\mathcal{L} \ni \mathcal{K}_{i\bar{i}} \partial_\mu \zeta^i \partial^\mu \bar{\zeta}^{\bar{i}} + \frac{1}{4g_4^2} F_{\mu\nu} F^{\mu\nu}$$

$$\boxed{\frac{1}{g_4^2} = \frac{1}{g_8^2} \int_{\mathcal{S}_4} \mathrm{d}^4 y \, \mathrm{e}^{-4\alpha}}$$

$$\boxed{\mathcal{K}_{1\bar{1}} = \mathcal{K}_{2\bar{2}} = \frac{1}{g_8^2 \mathcal{V}_\mathrm{w}} \int_{\mathcal{S}_4} \mathrm{d}^4 y}$$

$$\boxed{\mathcal{K}_{3\bar{3}} = \frac{1}{g_8^2 \mathcal{V}_\mathrm{w}} \int_{\mathcal{S}_4} \mathrm{d}^4 y \, (\mathrm{Im}\,\tau)^{-1} \mathrm{e}^{-4\alpha}}$$

$$\mathcal{V}_\mathrm{w} = \int_{X_6} \mathrm{d}^6 y \, \mathrm{e}^{-4\alpha}$$

# Chiral matter fields

- Warping modifications for chiral matter more complex

$$S = -\frac{1}{g_8^2}\int_{\mathcal{W}} \mathrm{d}^8x\,\sqrt{\tilde{g}}\,\mathrm{tr}\left\{\frac{1}{2}\eta^{\mu\nu}\tilde{g}^{ab}F_{\mu a}F_{\nu b} + \mathrm{e}^{-4\alpha}\eta^{\mu\nu}\tilde{g}_{ij}\partial_\mu\Phi^i\partial_\nu\Phi^j\right\}$$

$$\boxed{\mathcal{K}^{\mp}_{j\bar{k}} = \frac{1}{g_8^2\mathcal{V}_{\mathrm{w}}}\int_{\mathcal{S}_4}\mathrm{d}^4y\,(\mathrm{Im}\,\tau)^{-1}\left(\mathbf{X}^{k,\mp}\right)^*\cdot\mathrm{e}^{\#\alpha}\,\mathbf{X}^{j,\mp}}$$

$$\mathrm{e}^{\#\alpha} = \mathrm{diag}\left(\mathrm{e}^{-4\alpha},1,1,\mathrm{e}^{-4\alpha}\right)$$

- $\mathbf{X}^{j,\mp}$ is the warped zero mode, not simply related to unwarped zero mode

# Chiral matter fields (cont.)

- In the weak warping limit, the first order correction is

$$\boxed{\delta\mathcal{K}^{\mp}_{j\bar{k}} = \frac{1}{\mathcal{V}}\int_{\mathcal{S}_4} \mathrm{d}^4y\,(\mathrm{Im}\,\tau)^{-1}(\chi_3^{k,\mp})^*\beta\chi_3^{j,\mp} - \frac{\delta\mathcal{V}}{\mathcal{V}}(\mathcal{K}^{\mp}_{j\bar{k}})_0}$$

unwarped

- Generic warp factors will introduce off-diagonal terms in the Kähler metric

- Second order corrections make use of warped wavefunctions (work in progress...)

# Summary and future directions

- Studied the wavefunctions for bifundamental matter in warped compactifications in both the chiral and non-chiral cases
- Needed to develop warped effective field theory and detailed phenomenology (though still work to be done!)
- Warping effects are more intricate than for adjoint matter; require a series expansion in unwarped massive modes
- General warp factors induce off-diagonal terms in the Kähler metric
- Extension to Calabi-Yau case is likely tricky...
- With a non-SUSY source (such as $\overline{\text{D3}}$-branes), wavefunctions can be used to study soft terms (work in progress)