Three family models from the heterotic string

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Outline

- 3 family orbifold GUT on $\mathcal{M}_4 \times S_1/(Z_2 \times Z_2')$

$$\Updownarrow$$

- heterotic string compactified on $[T_2]^3/Z_6$ + Wilson lines

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- $E(6)$ example

  Orbifold breaking E(6) to PS

  PS $\longrightarrow$ SM via Higgs

  3rd family in bulk

  1st & 2nd families on SO(10) fixed pts.

  Gauge coupling unification & proton decay

  Fermion masses

- Conclusions

SUSY E(6) on $\mathcal{M}_4 \times S_1/(Z_2 \times Z_2')$

$M_c = (\pi R)^{-1} \ll M_*$

- E(6) breaks to PS by orbifold parities

$$\begin{array}{ccc} & P & & P' & \\ \mathrm{E}(6) & \rightarrow & \mathrm{SO}(10) & \rightarrow & \mathrm{SU}(4)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \ [= \mathrm{PS}] \end{array}$$

- Gauge and hypermultiplet in bulk –

$$(V,\ \Sigma)\ [\mathbf{78}]\ +\ (\mathbf{27} \oplus \overline{\mathbf{27}})$$

Consider (+ +) modes $\Longrightarrow$ 3rd family and Higgs

$$f_3^c = (\bar{4}, 1, 2), \quad f_3 = (4, 2, 1), \quad h = (1, 2, 2)$$

$V = \mathbf{78} \rightarrow \mathbf{45} \rightarrow$ adjoint of PS

$\Sigma = \mathbf{78} \rightarrow (\ \mathbf{16} \oplus \overline{\mathbf{16}}\ ) \rightarrow f_3^c + \bar{\chi}^c$

$\mathbf{27} \rightarrow \mathbf{16} \rightarrow f_3$

$\overline{\mathbf{27}} \rightarrow \mathbf{10} \rightarrow h$

3rd family Yukawa unification

$\lambda_t = \lambda_b = \lambda_\tau = g \equiv \sqrt{4\pi\alpha_G}$

$$\int_0^{\pi R} dx_5\ g_5\ (\overline{\mathbf{27}}\ \Sigma\ \mathbf{27}) \rightarrow g\ h\ f_3^c\ f_3$$

$$g = g_5\ \sqrt{M_c}$$

PS breaks to SM

- PS breaking to SM by Higgs vevs

$$\chi^c = (\bar{4}, 1, 2),\ \bar{\chi}^c = (4, 1, 2)$$

$$\langle \chi^c \rangle = \langle \bar{\chi}^c \rangle = M_b$$

- Need additional bulk states – $3\ (\ \mathbf{27} \oplus \overline{\mathbf{27}}\ )$

$3\ (\ \mathbf{27} \oplus \overline{\mathbf{27}}\ ) \rightarrow\ \ 2\ (\mathbf{16}) \oplus \overline{\mathbf{16}} \rightarrow\ \ 2\ (\chi^c) + \bar{\chi}^c + 3\ \ C$

Now total $\Longrightarrow\quad 2(\chi^c + \bar{\chi}^c) + 3\ \ C[= (6, 1, 1)]$

- Superpotential W gives mass to color triplets and breaks PS to SM along D & F flat directions

$W = M_{eff}\ C_1\ C_2 + (\chi^c\ \chi^c\ +\ \bar{\chi}^c\ \bar{\chi}^c)\ C_3$

1st & 2nd family ?

5th Dimension

SO(10) brane E(6) bulk SU(6)× $SU(2)_R$ brane

[————————————————————]

0 πR

• In bulk or on SU(6)× $SU(2)_R$ brane — $M_c < M_G$ *

On SU(6)× $SU(2)_R$ brane, one family $\in$ $[(15, 1) \oplus (\bar{6}, 2)]$

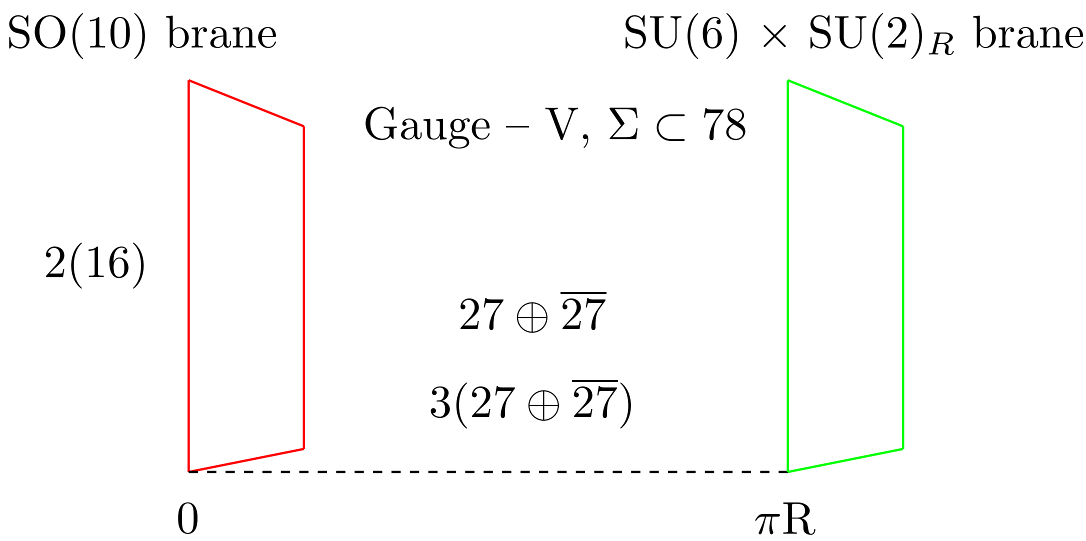
• On SO(10) brane — $M_c \geq M_G$ *

* NO problem with proton decay !

♢ **In string theory, don't get to choose** *easily*

Summary

E(6) orbifold GUT



- 3rd family and Higgs

$\Sigma \rightarrow 16 \oplus \overline{16} \rightarrow f_3^c \oplus \bar{\chi}^c$

$27 \rightarrow 16 \rightarrow f_3$

$\overline{27} \rightarrow 10 \rightarrow h$

- Yukawa unification

$\int_0^{\pi R} dx_5 \; g_5 \; (\overline{27} \; \Sigma \; 27) \quad \rightarrow \quad g \; (h \; f_3^c \; f_3)$

$\Longrightarrow \lambda_t = \lambda_b = \lambda_\tau = g = \sqrt{4\pi\alpha_G}$

- PS breaking sector

$3(27 \oplus \overline{27}) \rightarrow 2(16) \oplus \overline{16} \rightarrow$
$2(\bar{4}, 1, 2)[= 2 \; \chi^c] \oplus (4, 1, 2)[= \bar{\chi}^c] \oplus 3(6, 1, 1)[= 3 \; C]$

Heterotic string compactified on $[T_2]^3/Z_6$

$[T_2]^3 = G(2) \otimes SU(3) \otimes SO(4)$ root lattice

$Z_6 = Z_2 \otimes Z_3$

Orbifold defined by rotation on torus
$\mathbf{Z^i} \to e^{2\pi i v_i} \ \mathbf{Z^i} \ \ (i = 1, 2, 3)$

$v_3 = \frac{1}{3} \ (1, -1, 0), \quad v_2 = \frac{1}{2} \ (1, 0, -1)$

Rotations embedded in $E(8) \times E(8)$ root lattice via shift vectors + Wilson lines

Satisfy NON-TRIVIAL constraints of modular invariance !

Consider first $[T_2]^3/Z_3$ +
W_3 in SU(3) torus
SO(4) torus $R^{-1} \gg l_s = M_*^{-1}$

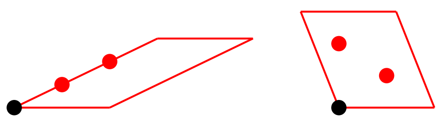
$v_3 = \frac{1}{3}\ (1, -1, 0)$

$V_3 = \frac{1}{3}\ (2, 2, 2, 0, 0, 0, 0, 0) \oplus (\cdots)$

$W_3 = \frac{1}{3}\ (1, -1, 0, 0, 0, 0, 0, 0) \oplus (\cdots)$

$\Longrightarrow \quad N = 2$ SUSY in 5D bulk

G_2 $\qquad SU_3$ $\qquad SO(4)$



$V,\ \Sigma\ \in E(6)$

$(\mathbf{27} \oplus \overline{\mathbf{27}})$

$3(\mathbf{27} \oplus \overline{\mathbf{27}})$

$\mathrm{G}_2 \oplus \mathrm{SU}_3 \oplus \mathrm{SO}_4$ lattice with $\mathbb{Z}_3$ fixed points. The fields $V,\ \Sigma,$ and $\mathbf{27}(\in U_1) + \overline{\mathbf{27}}(\in U_2)$ are bulk states from the untwisted sectors. On the other hand, $3 \times (\mathbf{27} + \overline{\mathbf{27}})$ are "bulk" states located on the $T_{(0,1)}/T_{(0,2)}$ twisted sector (G_2, SU_3) fixed points.

Applying Z_2 orbifold + Wilson line in 5th Dimension

Breaks N = 2 to N = 1 SUSY

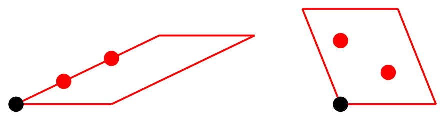
Defines parities $P, \;\; P'$

$P \Leftrightarrow (v_6; V_6; W_3)$ $\qquad$ $P' \Leftrightarrow (v_6; V_6 + W_2; W_3)$

$V_6 = V_2 - V_3$, $\qquad$ $W_2 = \frac{1}{2}\,(1,0,0,0,0,1,1,1) \oplus (\cdots)$

Consider massless states, i.e. (+ +) modes of ($P \;\; P'$) from bulk and $T_{(0,1)}/T_{(0,2)}$ twisted sector (G_2, SU(3)) fixed points.

Find



$V \in PS \;\; (f_3^c + \bar{\chi}^c) \in \Sigma$

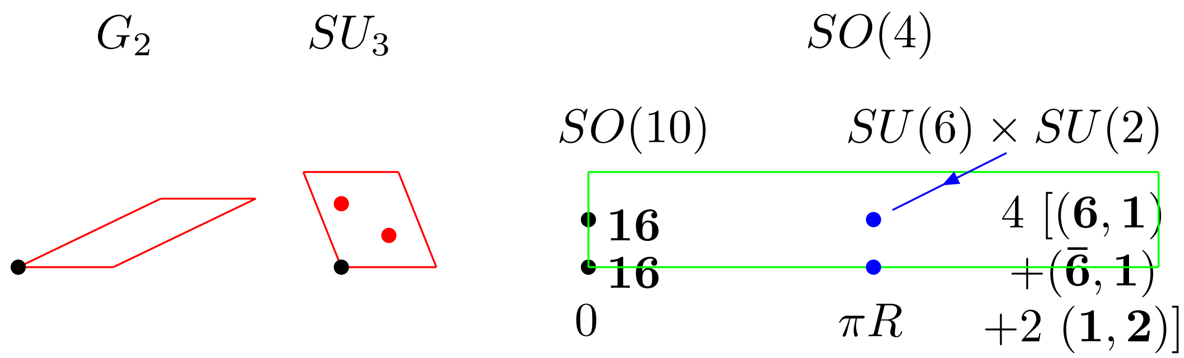
$f_3 \in \mathbf{27} + h \in \overline{\mathbf{27}}$

$2(\chi^c) + \bar{\chi}^c + 3C \in 3(\mathbf{27} + \overline{\mathbf{27}})$

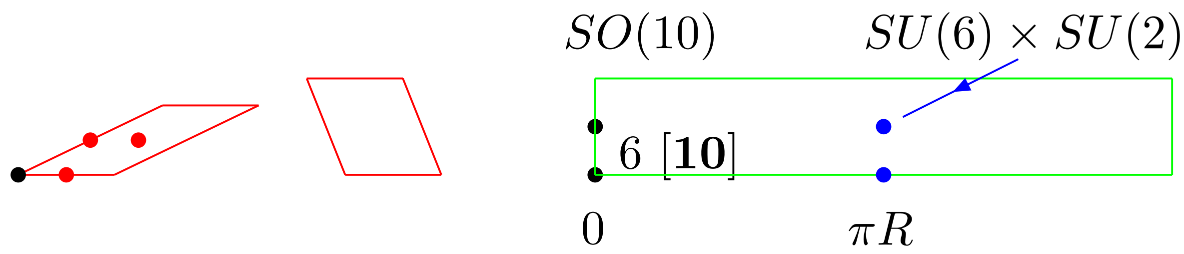
Table 1: Parities of the bulk states in model A1.

States	P	P'	States	P	P'
$V(\mathbf{15},\mathbf{1},\mathbf{1})$	$+$	$+$	$\Sigma(\mathbf{15},\mathbf{1},\mathbf{1})$	$-$	$-$
$V(\mathbf{1},\mathbf{3},\mathbf{1})$	$+$	$+$	$\Sigma(\mathbf{1},\mathbf{3},\mathbf{1})$	$-$	$-$
$V(\mathbf{1},\mathbf{1},\mathbf{3})$	$+$	$+$	$\Sigma(\mathbf{1},\mathbf{1},\mathbf{3})$	$-$	$-$
$V(\mathbf{6},\mathbf{2},\mathbf{2})$	$+$	$-$	$\Sigma(\mathbf{6},\mathbf{2},\mathbf{2})$	$-$	$+$
$V(\mathbf{4},\mathbf{2},\mathbf{1})$	$-$	$+$	$\Sigma(\mathbf{4},\mathbf{2},\mathbf{1})$	$+$	$-$
$V(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})$	$-$	$+$	$\Sigma(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})$	$+$	$-$
$V(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	$-$	$-$	U_3 $\Sigma(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	$+$	$+$
$V(\mathbf{4},\mathbf{1},\mathbf{2})$	$-$	$-$	U_3 $\Sigma(\mathbf{4},\mathbf{1},\mathbf{2})$	$+$	$+$
U_1 $H(\mathbf{4},\mathbf{2},\mathbf{1})$	$+$	$+$	$H^c(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})$	$-$	$-$
$H(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	$+$	$-$	$H^c(\mathbf{4},\mathbf{1},\mathbf{2})$	$-$	$+$
$H(\mathbf{6},\mathbf{1},\mathbf{1})$	$-$	$+$	$H^c(\mathbf{6},\mathbf{1},\mathbf{1})$	$+$	$-$
$H(\mathbf{1},\mathbf{2},\mathbf{2})$	$-$	$-$	U_2 $H^c(\mathbf{1},\mathbf{2},\mathbf{2})$	$+$	$+$
$H(\mathbf{4},\mathbf{2},\mathbf{1})_+$	$+$	$-$	$H^c(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})_+$	$-$	$+$
$H(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_+$	$+$	$+$	$H^c(\mathbf{4},\mathbf{1},\mathbf{2})_+$	$-$	$-$
$H(\mathbf{6},\mathbf{1},\mathbf{1})_+$	$-$	$-$	$H^c(\mathbf{6},\mathbf{1},\mathbf{1})_+$	$+$	$+$
$H(\mathbf{1},\mathbf{2},\mathbf{2})_+$	$-$	$+$	$H^c(\mathbf{1},\mathbf{2},\mathbf{2})_+$	$+$	$-$
$H(\mathbf{4},\mathbf{2},\mathbf{1})_-$	$-$	$+$	$H^c(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})_-$	$+$	$-$
$H(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})_-$	$-$	$-$	$H^c(\mathbf{4},\mathbf{1},\mathbf{2})_-$	$+$	$+$
$H(\mathbf{6},\mathbf{1},\mathbf{1})_-$	$+$	$+$	$H^c(\mathbf{6},\mathbf{1},\mathbf{1})_-$	$-$	$-$
$H(\mathbf{1},\mathbf{2},\mathbf{2})_-$	$+$	$-$	$H^c(\mathbf{1},\mathbf{2},\mathbf{2})_-$	$-$	$+$

$T_{(1,2)}$ and $T_{(1,0)}$ twisted sectors



$G_2 \oplus SU(3) \oplus SO(4)$ lattice with Z_6 fixed points. The $T_{(1,2)}$ twisted sector states sit at these fixed points.



$G_2 \oplus SU(3) \oplus SO(4)$ lattice with Z_2 fixed points. The $T_{(1,0)}$ twisted sector states sit at these fixed points.

D_4 family symmetry !

Note *unrequested*

$[(6,1,1) \oplus (1,2,2)] \oplus [(4,1,1) \oplus (1,2,1)] \oplus (1,1,2) \oplus (1,1,1)$

Gauge coupling unification & Proton decay

- 5D RG equations

M_s = string scale, M_{PS} = PS breaking scale,
M_c = 5D compactification scale

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha_s} + b_i^{MSSM} \ln \frac{M_{PS}}{\mu} + (b_{++}^{PS} + b_{brane}^{PS})_i \ln \frac{M_s}{M_{PS}} - \frac{1}{2}(b_{++}^{PS} + b_{--}^{PS})_i \ln \frac{M_s}{M_c} + b^G \left(\frac{M_s}{M_c} - 1 \right)$$

- 4D RG equations

$M_G \approx 3 \times 10^{16}$ GeV, $\alpha_G^{-1} \approx 24$
and included threshold correction at M_G.

$$\frac{2\pi}{\alpha_i(\mu)} = \frac{2\pi}{\alpha_G} + b_i^{MSSM} \ln \frac{M_G}{\mu} + 6\, \delta_{i3}$$

- $2\pi/\alpha_s = \pi/4\ (M_{Planck}/M_s)^2$ **

Find —

$M_{PS} = e^{-3/2}\ M_G \sim 7 \times 10^{15}$ GeV,
$M_s(MAX) = e^2\ M_G \sim 2 \times 10^{17}$ GeV
** $\Longrightarrow \alpha_s^{-1} \sim 450$, α_s too small — PROBLEM !

NO solution !!

Give up perturbative heterotic string boundary conditions

Eleven-dimensional Hořava-Witten

$$\frac{2\pi}{\alpha_s} = \frac{1}{2(4\pi)^{5/3} M\rho} \left(\frac{M_{\rm Pl}}{M}\right)^2$$

M — eleven-dimensional Newton's constant by $\kappa^{2/3} = M^{-3}$

ρ — size of the eleventh dimension

Now find solution

$M_s \simeq M = 2M_G,$
$M_c \simeq M_{PS} = e^{-3/2} M_G$
$M\rho \simeq 2 \Longrightarrow \rho \sim M_G^{-1}$

♢ Enhanced proton decay rate — dimension-six operators

$$\tau(p \to e^+\pi^0) = \ 3 \times 10^{33} \left(\frac{0.015\ \text{GeV}^3}{\beta_{\text{lattice}}}\right)^2 \ \text{yrs}$$

Super-Kamiokande bound 5.7×10^{33} years @ 90% CL

Yukawa couplings

$D_4 = \{\pm 1,\ \pm\sigma_1,\ \pm\sigma_3,\ \mp i\sigma_2\}$ family symmetry

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}: \qquad f_1 \leftrightarrow f_2$$
$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}: \quad f_2 \rightarrow -f_2$$

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \ \text{doublet;} \quad f_3 \quad \text{singlet}$$

PS breaking VEVs

$$O_i = \langle \chi^c_\alpha \ \bar{\chi}^c_i \rangle, \quad i = 1, 2$$

- Fermion mass matrix [simple form]

$$(f_1 \ f_2 \ f_3)\ h\ \mathcal{M} \begin{pmatrix} f_1^c \\ f_2^c \\ f_3^c \end{pmatrix}$$

$$\mathcal{M} = \begin{pmatrix} (O_2\ \tilde{S}_e + S_e) & (O_2\ \tilde{S}_o + S_o) & (O_1\ O_2\ \phi_e + \tilde{\phi}_e) \\ (O_2\ \tilde{S}_o + S_o) & (O_2\ \tilde{S}_e + S_e) & (O_1\ O_2\ \phi_o + \tilde{\phi}_o) \\ \phi'_e & \phi'_o & 1 \end{pmatrix}$$

Problems and Virtues

- # Virtues
  - $E(6) \rightarrow SO(10) \rightarrow PS$
  - Three families (**16** of SO(10)) + Higgs (**10**) + PS breaking sector
  - D_4 Family symmetry $\Longrightarrow$ hierarchy of masses and mixing
  - Baryon and Lepton # violation in effective low energy field theory

    $f\ f\ f^c\ \langle\chi^c\ S^n\rangle \Longrightarrow Q\ L\ \bar{D} + L\ L\ \bar{E}$

    $f^c\ f^c\ f^c\ \langle\chi^c\ S^n\rangle \Longrightarrow \bar{U}\ \bar{D}\ \bar{D}$

    $f\ f\ f\ f\ \langle S^n\rangle \Longrightarrow Q\ Q\ Q\ L$?

    NOT found to order S^8 – inconsistent with string selection rules !

- # Problem
  - Effective dimension 4 R parity violating operators via color triplet mixing

    $\langle\chi^c\ S^n\rangle\ C\ f^c\ +\ \langle S^m\rangle\ C\ C$

    $\Longrightarrow \hat{M}_{PS}\ T\ \bar{D}\ +\ \hat{M}_s\ T\ \bar{T}$ and $C = T + \bar{T}$

    Implies – massless $\bar{D}^0 \sim \bar{D} + \frac{\hat{M}_{PS}}{\hat{M}_s}\ \bar{T}$

    $C\ f^c\ f^c \Longrightarrow \bar{D}^0 \bar{D} \bar{U}$

    $C\ f\ f \Longrightarrow \bar{D}^0\ Q\ L$

    B and L – R parity violating terms O$(\hat{M}_{PS}/\hat{M}_s)$ — too large ??
  - Vector-like exotics w/fractional charge need large mass O(M_G)

Conclusions

- Obtained UV completion of E(6) orbifold GUT in 5D

- Obtained cubic and higher order *effective Yukawa couplings*

- Need more study of baryon and lepton number violation

- We have two other three family models — one with SO(10) in bulk

- Just the beginning

Expand search to $Z_N \otimes Z_2$ orbifolds +
1 (or 2) Wilson lines in SO(4) direction
$\Longrightarrow$ Effective 5 or 6 D orbifold GUTs

- Promising new directions for 3 family models !

- w/ D_4 family symmetry for fermion mass hierarchy and suppress flavor violation !

- and observable proton decay rate w/ $p \rightarrow \pi^0 \; e^+$!!