

Ontology via semantics?  
Introduction to the special issue of *Linguistics  
and Philosophy* on the semantics of cardinal  
numbers

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The present issue consists of a suite of papers most of which were presented in earlier form at a workshop on the semantics of cardinal numbers that we organized at The Ohio State University in March of 2012. Each paper submitted for this special issue of *Linguistics and Philosophy* was subjected to the journal’s usual, rigorous blind peer review. The reviewers were not aware that the articles were intended for a special issue, lest this lead them to lower their standards. Further, each article was reviewed by at least one linguist and at least one philosopher, in addition to the guiding sub-editors. Peter Pagin graciously took editorial responsibility for those articles contributed by authors associated with Ohio State (Barlew and Snyder), while Roberts guided the others. Not all articles originally submitted made the cut. We are very grateful to Peter and to the reviewers, as well as to the authors, for their diligence and patience.

These notes are intended to provide some background about the significance of this collection in the fields of linguistic semantics and the philosophy of language.

## 1 Frege, via Dummett, gets us started

Michael Dummett (e.g., 1973, 1981, 1991) is credited with a sea-change in how to understand the philosophy of Gottlob Frege. It is clear that Frege took numbers, say natural numbers, to exist, objectively. Dummett describes Frege as an “arch-Platonist”. And Frege held that numerals, terms like *zero*, *one* and *four thousand seven hundred* are singular terms—expressions whose linguistic role is to denote objects. In contemporary terms, Frege (or Dummett’s Frege) insisted that numerals are of type *e*, mapping onto entities in any model for the language. It is natural to think that Frege held that numerals are singular terms *because* numbers exist: semantics follows ontology.

Dummett argues that it is the other way around. Frege first noted that numerals function as singular terms. This is presumably a thesis about ordinary

German, or at least the German used by scientists and mathematicians. One of Frege's primary examples is this one:

The number of Jupiter's moons is four. (1)

Some statements in this form are true. There is presumably some true statement about the number of moons of Jupiter (vagueness aside), or the number of children had by George H. W. Bush, or the number of electrons in a given molecule of water. So, the semantic facts about sentences like (1), and the non-linguistic facts, entail that numbers exist.

In contemporary philosophy, this is sometimes called an "easy argument" for an otherwise controversial metaphysical claim. One starts with an everyday, rather innocuous statement, such as (1) above. We verify the truth of a sentence in this form by counting or otherwise observing the relevant facts about moons, children, etc. Then we draw a metaphysical conclusion, that numbers exist in this case.

Of course, it can't be *that* easy. The first thing one must do is identify what makes a linguistic item a singular term. It surely won't do to simply say that a singular term is, by definition, something that denotes an object. For then a nominalist—someone who denies that numbers exist—can simply accuse the Fregean of begging the question. Who says that *four* in (1) is a singular term? Again, Dummett has Frege arguing that numbers exist because numerals are singular terms. For this to make sense, we need some way to identify a linguistic item as a singular term independently of its role as denoting an object.

So far as we know, Frege never provided a criterion for being a singular term. On Frege's behalf, Dummett once attempted this, in terms of inference patterns licensed by a given linguistic expression (1973:54-80), but his characterization proved to be incorrect: it misclassifies a number of terms. Bob Hale (1987) then improved Dummett's characterization, by providing inferential/syntactic tests, along with some articulated "constraints", that, supposedly, singular terms, and only singular terms, pass. The details do not matter here.

Frege was aware that number expressions also appear in other grammatical categories; they are not always singular terms. He wrote:

Since what concerns us here is to define a concept of number that is useful for science, we should not be put off by the attributive form in which number also appears in our everyday use of language. This can always be avoided. For example, the proposition 'Jupiter has four moons' can be converted into 'The number of Jupiter's moons is four'. Here the *is* should not be taken as the mere copula ... Here *is* has the sense of 'is equal to', 'is the same as'. We thus have an equation that asserts that the expression *The number of Jupiter's moons* designates the same object as the word *four*. (1884, §57)

One question concerns the relationship between (1) above and the following:<sup>1</sup>

Jupiter has (exactly) four moons. (2)

The two sentences seem to say the same thing—they seem to have the same truth-conditions. Yet (1), if true, entails the existence of numbers—the heavy ontological conclusion—but (2) does not, or so it seems.

Frege states that the “attributive” use of numerals can “always be avoided”, presumably by paraphrasing sentences like (2) as sentences like (1). As indicated at the start of the passage, Frege’s interest here (in 1884) is not so much in the semantics of natural languages, but in defining “a concept of number that is useful for science”. But if Dummett’s interpretation is correct, Frege’s “arch-Platonism” about numbers is driven by semantics, presumably the semantics of ordinary languages—including the languages used in science and mathematics. Of course, this is not the place to address the underlying exegetical issues. Our topic here concerns the relationship between semantics and ontology.

## 2 The metaphysical battle lines

A group of neo-logicist philosophers pursue what Dummett (1991) calls the *substantival strategy*. Crispin Wright (1983) begins with “the syntactic priority thesis”, that if a linguistic item functions as a singular term in a certain type of sentence—typically an atomic sentence of a formalized language—and if that sentence is true, then the term denotes something. And examples like (1) above are cited as the primary cases of interest. With Frege, the substantialists are content to paraphrase (or formalize, or regiment) sentences like (2) into sentences like (1). See, for example, Wright (1983), Tennant (1987, 1997), Hale (1987), and Hale & Wright (2001). The central claim is that (1) is primary, and we can draw ontological conclusions from the truth of sentences in this form. This is the easy argument.

The opposition takes the opposite tack, what Dummett calls the *adjectival strategy*. The theme here is that uses of numerical terms like (2) are primary. Uses like (1) above are to be paraphrased away. It can hardly be doubted that in the language of arithmetic itself, the field of number theory, numerals function as singular terms, and quantifiers range over natural numbers. Harold Hodes [1984] refers to a language like that as a “coding fictionalism”. Using the resources of higher-order logic, he shows how to paraphrase any statement in the language of arithmetic into one that does not entail the existence of numbers. Here the adjectival strategy is a defense of nominalism.

The theorists mentioned so far, on both sides of the divide, do occasionally cite ordinary language “data” to support their accounts, but it must be admitted that they do so selectively. More important, perhaps, there is no attempt to give semantic analyses of the locutions to be paraphrased away—sentences like

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<sup>1</sup>We inserted the paranthetical *exactly* into Frege’s example in order to avoid the question of whether *Jupiter has four moons* means “Jupiter has at least four moons” or “Jupiter has exactly four moons”. Frege clearly intends the latter, and so we make that explicit.

(2) for the substantival strategy and sentences like (1) for the adjectival. The partisans to the ontological debate are content to note the paraphrases, claiming that no expressive power is lost along the way. Perhaps the goal, all along, was the same as Frege's, to develop a language that is useful for science and mathematics. Despite the sea-change that Dummett brought to light, the primary aim seems to be ontological. Those who accept the existence of numbers adopt the substantival strategy while nominalists go the adjectival route.

### 3 Semantics of cardinal expressions

Recent years have seen a lot of work by philosophers of language, philosophers of mathematics, and linguists on the semantics of cardinal expressions. The targets include expressions like (1) and (2) above, along with many other constructions involving number terms. The goal is to provide an adequate, preferably compositional semantic account of all of the various uses of such expressions, showing how they are related. No legitimate statement is to be ignored or simply paraphrased away.

Thomas Hofweber (2005, 2007) argues that *four* serves the same function in both (1) and (2). In both sentences, it is a quantificational determiner, and not a singular term. So, against Frege's explicit claim to the contrary, the sentence (1) is not an identity statement; rather it comes from a sentence like (2) due to a shift in focus. So the result is a defense of nominalism—or at least an argument that the thesis that numbers exist gets no support from the semantics of natural languages..

Friederike Moltmann (2013) and Katherina Felka (2014) argue that (1) is not an identity statement. Instead, (1) is what is called a specificational sentence, and the best analysis of these suggests that it expresses a question-answer pair, where *the number of Jupiter's moons* expresses a question about the cardinality of Jupiter's moons, and post-copular *four* expresses an answer to that, namely the one expressed by (2). So, again, we get no support for the existence of numbers, at least not from this area of semantics.

Moltmann's contribution to the present issue tells a different story, albeit one that does not support the easy argument. She brings to bear new linguistic evidence from German and English to argue that a crucial syntactic argument in Moltmann (2013) was flawed. However, she argues that there is still semantic evidence against the Fregean account.

In their contribution, Balcerak Jackson & Penka argue that *four* is a referring expression in the relevant examples, and neither a determiner nor an adjective. However, what it refers to is a degree. They discuss how this might be compatible with various anti-Platonist strategies for characterizing numbers in terms of degrees. But they point to linguistic evidence which could be problematic for any such strategy, concluding that more work is necessary to determine whether the semantics of the relevant expressions is compatible with nominalism.

Snyder comes to a similar conclusion to that of Balcerak Jackson & Penka about the meaning of *four* in (1), but via a very different route. He offers

evidence that *four* and its ilk are polysemous, their several senses related via regular, independently motivated type-shifting processes. He agrees with Moltmann (2013) that (1) is not equative but specificational; and that in such sentences, *four* does not refer to a number. But it *is* referential in (1): it refers to a degree—one of its independently attested senses. However, he argues that degrees are constructed semantically from numbers, which are basic semantic objects, even if they aren't the denotations of expressions like *four* in (1).

Barlew argues that both Moltmann (2013) and Snyder (this issue) err in taking the meaning of (1) in the Fregean argument to be specificational. The linguistic evidence he considers argues that such sentences are ambiguous, with an equative meaning compatible with the Fregean account. The expressions whose denotata are equated are, thus, arguably referring expressions. However, he notes, this leaves open the question of what they refer to, so that his conclusion might be consistent with that of Balcerak Jackson & Penka.

Weighing the evidence in the current issue, one might be tempted to conclude that even if something like the adjectival strategy wins the battle over the semantics of sentences like (1), so that expressions like *four* denote tropes or degrees, the nominalist loses the ontological war.

## 4 Semantics and ontology

Or does she?

It all comes down to the vexed question of what the semanticist is trying to do. The sea-change in the interpretation of Frege suggests that the goal of semantics is to provide the truth-conditions of sentences in various contexts. If, on this view, the correct semantic theory has it that an utterance of a given sentence is true only if a certain kind of object exists, and if the sentence is true in the context of utterance, then the object exists. This perspective on semantics underlies the easy argument.

Balcerak Jackson and Penka put it well:<sup>2</sup>

... we take for granted a view of ontological commitment according to which the commitments of a sentence are determined by looking at what entities must exist if the sentence is to be literally true—for example, because the sentence contains referring expressions that purport to refer to the entities, or because the truth of the sentence requires that the entities be among the values of its existentially

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<sup>2</sup>Note that Balcerak Jackson & Penka talk about a *sentence* being true, and the commitments of a *sentence*, a locution common in philosophy and logic. In fact, as Barlew (this issue) is at pains to make clear, it is *utterances of sentences*—sentences in context—which are true or false, and not sentences themselves. The vast majority of natural language sentences are context-sensitive—if only because of the context-sensitivity of tense—and hence have no truth value apart from a context of utterance. This is clear even in the earliest formal approaches to truth conditional semantics, e.g. that of Montague (1968). If a sentence carries a presupposition that a certain kind of entity exists (as is arguably the case, e.g., with sentences containing definite descriptions, like (1)), then we can say that any felicitous use of that sentence carries the commitment in question.

bound variables. This way of thinking about ontological commitment is also common ground among the participants of the debate over the puzzle of how we can get something from nothing. It is why participants in that debate tend to regard natural language semantics as pertinent to it: insofar as semantics uncovers the contributions to truth conditions made by the constituents of sentences like (1) and (2), semantics helps reveal their ontological commitments. It is also why proposed solutions to the puzzle tend to be as much semantic as ontological: if one takes the view, for example, that (1) does not in fact carry ontological commitment to numbers, one must look for an account of the semantic function of [*the number of Jupiter's moons*] and [*four*] as they occur in (1) that is compatible with this view, and provide evidence for it.

Of course, it is not up to the semanticist to say whether a given statement in the form (1) or (2) *is* true. Suppose that the truth conditions for a given sentence require the existence of a certain object, or kind of object. It may be that science, or metaphysics, or common sense, tells us that the object does not exist, or that objects of that kind do not exist, for example, in a statement about the planet Vulcan, or the substances caloric or phlogiston. In such a case, the utterance in question is not true. It is either false, or suffers from presupposition failure, or . . . So the nominalist has an option, even if Frege, the neo-Fregeans, Snyder, Balcerak Jackson and Penka, . . . are right in their semantic analyses of sentences in the form (1) and/or sentences in the form (2). She can claim that all utterances of such sentences are false, or suffer from presupposition failure, or something like that, and she may go on to give a pragmatic account of how the use of such sentences can nonetheless convey true propositions. In philosophy, this is sometimes called an “error theory”.<sup>3</sup>

But this orientation toward semantics is not the only option available. Other perspectives might not sanction any direct connection between the output of a semantic theory and the existence of anything.

Consider the problem from the perspective of a different domain than the semantics of number. If you could look out this window, you would surely agree that the following is true today:

The sky is blue. (3)

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<sup>3</sup>A classic source, concerning ethics, is J. L. Mackie (1977). Here is a closely related example that arose in a seminar that we taught together a few years ago, concerning the use of mereological sums in giving the semantics for plural terms and mass nouns (e.g., Link 1983). A metaphysics student objected, arguing that, according to some accounts, unrelated things do not have a sum, or fusion. A standard example, is “Bill Clinton’s nose and the Eiffel Tower”. Merological nihilists argue that there is no such thing (van Inwagen 1990). A linguistics student asked us to consider the following example:

Bill Clinton’s nose and the Eiffel Tower touched.

This at least looks like it is in subject-predicate form, saying that something “touched”. If the sentence is to be true (and, of course that is a big ‘if’) we need a subject.

Yet one can reasonably question whether there is actually an entity referred to by *the sky*. What is it about the world, then, that makes (3) true in this context? In the usual formal semantic models for English, this requires that there be some entity in the domain of the model which is in the extension of *blue*. But in actual usage, it seems, instead, that (3) is true by virtue of the fact that some perceptual phenomenon, which we entify in referring to it as the sky, gives the appearance of being blue.

Emmon Bach (1986) has this to say about such requirements:

What exactly are we claiming when we put forward our theories about model structures for natural languages? The enterprise looks very close to metaphysics or ontology, describing what some philosophers like to call “the ultimate furniture of the world.” Do things such as properties, kinds, quantities of matter, stages, and so on really exist? I would claim that those are philosophical or scientific questions, not linguistic ones. As a linguist, I feel perfectly justified in sidestepping such questions. Consequently, I like to say that what I am doing here is not metaphysics *per se* but *natural language metaphysics*. Some philosophers claim that all metaphysical enterprise is the analysis of language (this was a prominent part of the program of logical positivists like Rudolf Carnap). But here, too, as a linguist I can be—indeed, I think I *should* be—perfectly neutral. What we are doing is simply seeking linguistic evidence for the nature of the semantic structures that we seem to need to give a good account for the meanings of natural language expressions. Of course, this evidence is relevant to nonlinguistic questions within broader scientific or philosophical contexts. One such context is that of psychology: How do the tentative answers that we find in the linguistic domain relate to questions and answers in other domains such as nonlinguistic cognition, perception, and so on? The broadest such context is philosophical: What is the world really like? How do we fit into it? How do linguistic categories relate to reality? It seems to me that the best contribution that the linguist can make to these ultimate questions seems to be to work out precise theories for linguistic systems as such.

This principled ontological agnosticism was first discussed in Bach (1986), where he concludes:

Do the fundamental distinctions that are reflected in the overt and covert categories of natural language correspond in any way to the structure of the world? How could they not? But this is where linguistics stops.

Bach’s natural language metaphysics is supported by detailed cross-linguistic analysis in a different realm—the discussion of mass and count entities—by Jeffrey Pelletier, who concludes (2011:41):

[T]he +MASS/+COUNT distinction in language should not be used to make claims about either the physical or the conceptual reality/importance of any possibly related distinction [between “things” and “stuff”].

Instead, such distinctions reflect what “people talk as if there is.” If one language makes the things/stuff distinction in a different way from another (as Pelletier argues is common), that needn’t reflect a deep difference in world view. It’s just that once a language has certain a grammatical distinction, like the mass/count distinction or a distinction in grammatical gender, this gets systemized; so speakers have to make arbitrary decisions about how to apply it in a particular case where it has no natural bearing: maidens in German are neuter and English boats are *she*. As Moltmann explains in her article in this volume, there is a difference between German and English in the way that a distinction is drawn between sortal and non-sortal NPs; but this needn’t have any bearing on the nature of the world so-described

One might take the same tack toward the linguistic evidence presented by Lewis (1986) to argue that there are multiple actual possible worlds. Lewis concludes (p.3):

Why believe in a plurality of worlds? — Because the hypothesis is serviceable. . .

and offers the following comparison (p.4):

Set theory offers the mathematician great economy of primitives and premises, in return for accepting rather a lot of entities unknown to Homo javanensis. It offers an improvement in what Quine calls ideology, paid for in the coin of ontology. It’s an offer you can’t refuse.

As a semanticist, making a model for natural language that includes modals, one needs something like possible worlds. But as a semanticist one can remain agnostic about their ontological status: unlike Lewis, we can use them in our models but take them only to be rather abstract “ways things might be”.

Similarly for the sky. Just because a term is “serviceable” doesn’t mean that it is singular in the sense of having a referent that can be individuated in any metaphysically interesting sense. We know that there is no dome over our heads, but talk about it because it permits us to convey a certain kind of information about the environment, without associated ontological commitments.

More generally, why should we assume that because we have an expression of type *e*, it is a singular term, denoting some actual entity in the world? A predicate needs arguments. But do all arguments thereby refer, in any ontologically interesting sense?

If one promotes a view like Bach’s, what does it mean that the meaning of a sentence is its truth conditions—what the world would have to be like in order

for a particular utterance of that sentence to be true? How can we say that (1) and (2) are true in the actual world of utterance without committing ourselves to the existence in that world of entities that make them true?

We suspect that Bach would say that this is not his business: “this is where linguistics stops”. Or one might say that such truth conditions correctly reflect how speakers conceive of or effectively talk about the world as being, on the basis of their collective experience. If (3) is true in a context of utterance, we might say that this is so by virtue of some abstract object, like ‘the apparent dome over the earth’, giving the appearance to the human eye of having a certain property. Then if (1) and (2) refer to numbers, or degrees, or tropes, perhaps such things ‘exist’ only in the sense that we all talk as if they do: they appear again and again to serve a useful function in characterizing other entities, like the class of Jupiter’s moons; hence, they seem to exist. Philosophers might draw ontological conclusions on the basis of the utility of such statements, but this depends upon an indirect relationship between the relevant number expressions, their truth conditional contributions, and the world so-described.

Such a view isn’t ontological skepticism so much as conservatism about what language itself reveals about the world it is used to accurately describe. Nothing in the present collection of papers adjudicates between this type of view and that reflected in the quote above from Balcerak Jackson & Penka. Instead, the authors offer careful semantic analysis about what the truth conditions of the relevant expressions should be, and about what the contribution of expressions like *four* is to those truth conditions. None of this can be ignored in future discussions.

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