Modal Subordination: It would eat you first!

Abstract:
Modal subordination is a puzzle about anaphora in discourse. In *A wolf might come in. It would eat you first!* it seems to take as antecedent *a wolf* in the preceding clause, not referring to a specific wolf, despite the fact that the indefinite NP is in a scope island and could normally serve as antecedent in subsequent discourse only if it has a specific interpretation, as in *A wolf might come in. It is hungry.* Crucial to the puzzling interpretation is the presence of the modal *would*. We explore a range of related examples involving other modal elements to generalize and elucidate the puzzle, and give an overview of the relevant literature. We argue that a satisfactory account of modal subordination must involve an anaphoric understanding of domain restriction, an appreciation of the selectional restrictions on the lexical elements introducing modality, and a variety of pragmatic processes and principles.

1. Introduction to modal subordination: Illustrations and the general pattern

Suppose Hansel and Gretel are arguing over whether to lock the door to the forest hut where they’re seeking refuge for the night. Gretel tells Hansel:

(1) A wolf might come in. It would eat you first!

What concerns Gretel in (1) is the possibility of some wolf or other coming in. Compare:

(2) A wolf might come in. I saw it prowling around outside earlier.

What Gretel says in (2) can only be understood on the assumption that she has a particular wolf in mind, so that logically *a wolf* takes wide scope relative to *might*. What forces this is that the only way to find an antecedent for *it* in the second sentence is to assume that Gretel had a specific wolf in mind in uttering *a wolf*. That was not necessarily the case in (1); we can understand (1) to be about a specific wolf, but it’s not typically the first interpretation that comes to mind. Our intuitions tell us that the antecedent of *it* in the second sentence may be *a wolf* in the first while still taking the indefinite to be non-specific, evoking a merely hypothetical wolf. What licenses that interpretation in (1)?

Consider this exchange before Hansel and Gretel have entered the hut:

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1 I am grateful to the editors of SemCom, to Daniel Gutzmann, and to two anonymous reviewers for their extremely helpful comments and criticism.
(3) Hansel: I’m scared to go in! There might be a wolf in the hut, or a bear!
Gretel: Don’t be scared. I don’t see any large mammal tracks around here, and the hut
doors are securely closed.
Hansel:
(3a) It got in through the window!
(3b) It might have gotten in through the window!

(3) underlines the correlation between the scope of a wolf relative to might and the possibility of
serving as antecedent for a pronoun in a subsequent utterance. Since Hansel and Gretel are most
naturally understood to be talking about hypothetical wild animals in the first two utterances,
anaphoric it in (3a) seems quite odd, infelicitous, because it seems to presuppose some specific
animal antecedent where there is none under discussion. But it in (3b) is felicitous; as in (1), it is
the hypothetical wolf or bear that seems to serve as antecedent. Since the only difference
between (3a) and (3b) is the presence of would (and the perfect aspect required for use of that
modal auxiliary), we suspect that the difference in available readings is a function of the
presence or absence of the modal. This would then correlate with the available readings of (1)—
with a modal in the second sentence and a possible reading about a merely hypothetical wolf,
versus (2)—without a modal in the second sentence and with it only understood to be a specific
wolf.

The phenomenon illustrated in (1) and (3b) was dubbed modal subordination by Roberts
(1987, 1989). This was the first detailed consideration and general account of the phenomenon,
though related examples had been noticed and informally discussed earlier by Lakoff (1972),
Karttunen (1974), and McCawley (1981). Here is a preliminary schematic representation of
modal subordination in the examples we have seen above:

\[ [S_1 \ldots \text{ModOP}_1[\ldots X \ldots \ldots \ldots] \ldots \ [S_2 \ldots \text{ModOP}_2[\ldots Y \ldots \ldots \ldots]] \]

It involves two clauses, S₁ and S₂, syntactically independent of each other, and each containing a
modal operator. In (1), for example, ModOP₁ is might, while ModOP₂ is would. In (3)+(3b), both
operators are realized by might. Y in S₂ is an anaphoric element (it in (1) and (3)) whose apparent
antecedent is X in S₁ (a wolf). To capture this interpretation, Roberts argued, the content under
the scope of ModOP₂, its prejacent, should be understood as in some sense subordinate to the
semantic content under the scope of ModOP₁.

But this preliminary schema is insufficiently general to capture the full range of examples
involving modal subordination. For example, it may occur without overt modal auxiliaries.²
Note that Kratzer (1977, 1981) argued that indicative conditionals without overt modal
auxiliaries themselves have the implicit force of necessity:

(4) If a wolf gets in, then Hansel and Gretel are in trouble.

(4) means that in any scenario in which a wolf gets in, the kids are in trouble, with the force of
necessity. Similarly, we find:

² pace Klecha (2011), as we will discuss below.
Hansel’s question pertains to a hypothetical event in which a wolf gets in. In Gretel’s reply in (5), though it doesn’t include a modal auxiliary we understand *then* to be anaphoric to the irrealis event that’s at issue. Gretel isn’t asserting that the interlocutors are currently in trouble. Rather, the being-in-trouble is itself understood to be irrealis, a state following on the hypothetical event entertained by Hansel. Hence, Gretel’s reply is modally subordinate to Hansel’s hypothetical question, and it conveys the proposition expressed by (4).

Notice also that in (5), there is no overt anaphoric element in Gretel’s modally subordinate utterance. However, this is only a superficial difference here: With or without *then*, there is independent reason to think that there is an anaphoric relation between the *if*-clause and the main clause in (4) and between Hansel’s and Gretel’s utterances in (5): In the main clause (4) and Gretel’s utterance (5) the time at which the trouble obtains (the Event Time of the eventuality denoted) is understood to be a subinterval of the contextually given Reference Time (Reichenbach 1947; Hinrichs 1981,1986). Partee (1984) argues that the Reference Time is an anaphoric element resolved to some salient event in prior discourse. Here, the Reference Time will be the time immediately following the event of the wolf getting in, so that the interval in which Hansel and Gretel are in trouble is also understood to immediately follow after the wolf’s getting in. That is, e.g., in (5), Gretel isn’t claiming that they are already in trouble at the time of utterance, but only at such time as a wolf gets in. But since the wolf’s getting in is itself only hypothetical, the Reference Time can only be accessed via modal subordination.

Not only need the two clauses in the modal subordination schema not contain any overt modal elements, but S₁, the clause containing the apparent antecedent, needn’t have a modal or hypothetical flavor. Consider the following:

(6) John doesn’t have a car.
#It’s in the garage.
(7) John doesn’t have a car.
It would be in the garage.

(6) illustrates that normally an indefinite under the scope of negation cannot serve as antecedent for a pronoun in subsequent discourse. Intuitively, *it* in (6) seems to refer to some specific entity, but *a car* under the scope of negation in the first sentence doesn’t refer, so there is no antecedent and the example is infelicitous. But in (7) *a car* in that same first sentence, still understood as having narrow scope relative to negation, does seem to license anaphoric *it* in the second utterance. The difference is that in (7) the anaphora is facilitated by *would*, with a counterfactual sense. In this context, the sentence is understood to mean ‘if John did have a car, it would be in the garage’. The subordination of the modal’s prejacent is to the content under the scope of the negation in the first sentence.

Modal subordination also occurs with use of the future auxiliary *will*:

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3 On *will*, see Ogihara (1996), Abusch (1997), and Gennari (2000), and the references cited therein.
The birds will get hungry (this winter).

If Edna forgets to fill the birdfeeder, she will feel very bad. The birds will get hungry.

(8) without prior context seems like a prediction, with the force of necessity: ‘(come what may) the birds will get hungry’. But in (9), the prediction is restricted to those future branches in which Edna forgets to fill the birdfeeder, and the speaker is not committed to that actually happening. So the truth of (8) is not compatible with the birds failing to get hungry, while that of (9) is—so long as Edna doesn’t forget.

Further, the subordinated operator OP₂ in S₂ may be introduced by some element other than a modal auxiliary:4

John may wear a disguise.

Otherwise, the paparazzi will dog him relentlessly.

With overt otherwise, the second sentence has the conditional sense ‘if he does not wear a disguise, the paparazzi will dog him’. But it seems that otherwise itself has a conditional flavor, as well as triggering anaphora; it means something like ‘if not p’, where p is some circumstance under discussion.

And we find a perfectly parallel phenomenon involving quantificational adverbials (QAdvs) like always, sometimes, usually, and their ilk:

Harvey courts a girl at every convention.

She always comes to the banquet with him.

The girl is usually very pretty. (Karttunen 1976)

In (11), a girl occurs under the scope of every convention, so that we understand that Harvey is a serial offender: the indefinite is non-specific. Then always and usually pick up on this non-specific scenario, so that we understand them to mean something like ‘on those occasions’ or ‘when he does this’.

In the literature on QAdvs (see the line of development from Lewis 1975 through Barwise & Perry 1983, Kratzer 1989, Berman 1987, and de Swart 1991), quantificational adverbs have been analyzed as involving quantification over situations, effecting a unification with the semantics of modals and propositional attitudes. de Swart (1992) argues convincingly that QAdvs are generalized quantifiers over events. Cipria & Roberts (2000) argue that events are a type of situation. If we follow Kratzer (1989) in taking situations to be partial worlds, then generalized quantifiers over events constitute a special sub-class of modal operators. Arguably, most QAdvs (always and usually, but not, e.g., normally) quantify over only event-situations in the actual world; i.e., in Kratzer’s (1981) terminology they are totally realistic modals. Then examples like (11) are straightforward cases of modal subordination. No particular girl is referred to, though the girls in question are real girls in actual situations, not merely hypothetical. Note that in (11) we have a

4 The observation about otherwise is inspired by related examples due to Corblin (1994). See also Webber et al. (2006).
bare present tense across all three sentences, which here has the sense of the habitual present. (We could have used the simple past habitual, as well.) The habitual itself has a modal aspect; see again Cipria & Roberts (2000). So the choice of tense here, as with the future in (9), arguably facilitates the modal subordination.

Compare (12) (Roberts 1996), where the present progressive second sentence fails to convey the habitual sense required for modal subordination to the generalization in the first sentence. Then because it seems to refer to a particular entity, and there is none salient in the context of utterance, the anaphora is infelicitous. But in (13), the generic bare present and always together license modal subordination, and we have felicitous anaphora.

(12) Usually Fred buys a muffin every morning and eats it at the office. #It's being baked.
(13) Usually Fred buys a muffin every morning and eats it at the office. It's always oat bran.

Finally, in the QAdv cases, modal subordination can also arise without an overt operator in the second sentence, so long as the tense and aspect are compatible with that interpretation, as in the second sentence of (14):

(14) Usually Fred buys a muffin every morning and eats it at the office. He buys [a cup of coffee], too.

In (14), the anaphoric element is too, whose felicity requires that there be some salient situation of the sort Fred buys x, where x other than the denotation of the focused constituent a cup of coffee (Rooth 1985, 1992; Heim 1992). There is no salient specific situation of that type, but the bare present buys permits the habitual interpretation, so that we understand this sentence to be modally subordinate to the first: ‘whenever Fred buys a muffin to eat in the office, he buys a cup of coffee too’, with too anaphoric to the proposition that Fred bought a muffin on any particular instantiation of that typical situation. Note the universal force of this interpretation, ‘whenever’, like that of the bare present consequent in (4) or Gretel’s response in (5).

We also find modal subordination under attitude predicates, first noticed (though not under that description) by Lakoff (1972) and McCawley (1981):

(15) John wants to catch a fish. It's a big one that lurks in the shallows of the river.
(16) John wants to catch a fish. He plans to eat it for supper.

There are two interpretations of the first sentence of (15), one in which a fish takes wide scope relative to expect, so that there’s a particular fish John wants to catch, and one in which he wants to catch some non-specific fish or other. Taking a fish as antecedent for it in (15) is only felicitous on the first, specific interpretation of the indefinite. But in (16), it in the complement of plan can be understood non-specifically, resulting in the interpretation ‘John plans to eat whatever fish he catches for supper’. A Hintikka-style (1969) interpretation of attitude predicates
like want, plan, and many others takes them to involve quantification over possible worlds, and hence to effectively have a modal interpretation. So we can paraphrase (16) as ‘in all the worlds consistent with John’s desires, he catches a fish, and in all the worlds consistent with his plans he catches a fish and eats it for supper…’.

Modal subordination can occur in cases where the operator in the first sentence and that in the second are not the same type. In (1) the two modal auxiliaries have different force: might with the force of possibility, would with that of necessity. In (7), we saw negation in the first sentence, would in the second. We can also mix modal auxiliaries and QAdvs, as in (17) below, with QAdv never in the first sentence, would in the second. (18) involves consider, one of the modal-like attitude predicates, and the modal auxiliary would. Here there is no overt anaphoric element; but regret is factive, implicating the truth of its complement. In the context following utterance of the first sentence, it’s to be presumed that Mary has not yet attended graduate school, but is only considering doing so. Nonetheless, this factive is felicitous because would facilitates modal subordination to the circumstances Mary is considering, those in which she does get her Ph.D: ‘if Mary gets her Ph.D. in linguistics, she wouldn’t regret attending graduate school’.

(17) I never eat a big breakfast.
    It would make me sleepy all morning.

(18) Mary is considering getting her Ph.D. in linguistics.
    She wouldn’t regret attending graduate school.

Finally, the two clauses involved in modal subordination needn’t be indicative, as we saw in (5) above and in (19), with the content of the question modally subordinate to that of the complement of want, and also in (20), with the prejacent of would subordinate to the content of the imperative clause:

(19) Abe: Stefan wants to buy a new car.
    Joan: Where would he keep it? on the street?

(20) Get a cat. It would take care of the mouse problem in your basement.

Note that in the imperative in (20), a cat doesn’t refer to some specific cat—this is a suggestion for future action with respect to some non-specific animal, so that it would be infelicitous to follow up with It’s taking care of my mouse problem, with it referring to a cat. And the second clause needn’t be indicative. The addressee in (20) could respond How would you get along with it?. But the would is crucial here. How do you get along with it? would be an infelicitous response.

Taking into account all these types of examples, here is a more general schema for the type of logical form found in modal subordination, involving two independent sentences uttered sequentially:

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5 Some would say that it presupposes it, but this is controversial. For the purposes of discussion here, I will assume that this is presupposition; it makes no difference to the ultimate analysis.
where:

- $S_1$ and $S_2$ are both complete clauses, neither syntactically subordinate to the other, not necessarily in the indicative;
- $Y$ is an anaphoric element or other presupposition trigger under the scope of ModOP$_2$ in $S_2$’s logical form. $Y$ may be a pronoun or other anaphoric definite, too, a tense presupposing a Reference Time, or a factive presupposing the truth of its complement, among other presupposition triggers;
- $X$ under the scope of OP$_1$ seems to satisfy the presupposition triggered by $Y$;
- OP$_1$ may be a modal (1, 3), negation (7), the imperative (20), future tense (9, 10), a QAdv (11, 13, 14, 17), or a modal operator in the lexical semantics of an attitude predicate (16,18,19), and
- ModOP$_2$ is some type of modal operator, e.g. a modal auxiliary (either overt: 1,3b,7,17,19,20; or covert: 5), future tense (9), modal adverbial (10), QAdv (11,13), generic present (14), or the modal operator in the lexical content of an attitude predicate (16).

Summarizing, the examples above illustrate three essential features of the phenomenon of modal subordination, across a sequence of uttered sentences $S_1$ and $S_2$ in the schema above:

1. In the interpretation of $S_2$, a presupposition trigger $Y$ itself occurs under the scope of a modal operator.
2. The only content in the context of utterance which would satisfy the presupposition triggered by $Y$ occurs under the scope of OP$_1$ in $S_1$, so that the satisfying content $X$ itself is non-specific (if introduced by an indefinite, as in (1)) or irrealis (if propositional, as in (18)).
3. Even though $S_2$ may have no if-clause itself, that part of it headed by ModOP$_2$ has an interpretation wherein part of the hypothetical assumption from which the ModOP$_2$-headed constituent is understood to follow (if its modal force is that of necessity) or with which it must be compatible (if the force is possibility) is the denotation of $X$.

2. The theoretical problem

Modal subordination phenomena like those considered above appear to pose problems for otherwise robust generalizations about discourse (non-bound) anaphora, and about presupposition satisfaction more generally. To understand why, we need to briefly consider some background: the usual assumptions about presupposition projection and satisfaction (§2.1), and the phenomenon of scope islands (§2.2). Then we will consider how modal subordination challenges these otherwise well-motivated generalizations (§2.3).

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6 $S_2$ may have an if-clause. Consider:

(i) A wolf might come in. If I hid, it would eat you first!
Here the conditional is still understood as modally subordinate to the first clause, with the if-clause enriched to yield the interpretation ‘if a wolf came in and I hid, it would eat you first’.
§2.1 Presupposition projection and satisfaction

In classical accounts of presupposition projection and satisfaction, stemming from the work of Karttunen (1974) and Stalnaker (1973, 1974) and epitomized by Heim (1983), a presupposition such as that triggered by an anaphoric element (e.g. a pronoun or too) or a factive predicate (like regret) is a felicity constraint on the context in which it is interpreted: in order for the utterance containing the trigger to be felicitous, the context (roughly, the information available to the interlocutors) must satisfy that presupposition. Roughly, in the case of a factive, this means that the context has to entail the truth of the factive’s complement; for an anaphoric element, the context has to make available a unique antecedent, satisfactorily resolving the anaphora. The key to characterizing how satisfaction works across a very wide range of cases is Heim’s notion of the local context of interpretation for the expression including the trigger.

A dynamic semantic theory like that of Kamp (1981), Heim (1982, 1983), Kamp & Reyle (1993) and much subsequent work not only calculates the truth conditional content of an utterance, but also predicts how it will lead to the modification of the input context of utterance to yield a new context reflecting its content. Moreover, context is updated not only after full interpretation of a given utterance, to yield an updated context of utterance for the subsequent utterance (the resulting global context), but utterance-internally, so that parts of the sentence being interpreted may contribute in crucial ways to the interpretation of the remainder of that utterance itself. For example, this is simple and relatively obvious for conjunctions. The first conjunct of (22) clearly influences the context for interpretation of the second conjunct in just the way that the first sentence in (21) influences the context for the second. Thus, we can say that the first conjunct serves as part of the local context for the interpretation of the second conjunct:

(21) John has a dog. He walks it in the park.
(22) John has a dog, and he walks it in the park.

In this case, the content of the first conjunct would ultimately make a contribution to the resulting global context, just like in the first sentence in (21). But update also happens under the scope of operators, and in this case, the contribution to local context differs from that to the resulting global context. In a complex utterance like the famous “donkey sentences” of Geach (1967), illustrated by (23) below, the antecedent of a conditional (23) contributes a temporary update to the context of utterance in such a way as to make available an antecedent for a pronoun or to otherwise satisfy some presupposition in the consequent.

(23) If Hansel let a wolfi get in, iti bit him.

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7 Space restrictions preclude giving a satisfactorily introduction to presupposition projection and satisfaction, and the notion of local context. For those unfamiliar with these notions and the basic literature, see Beaver & Geurts (2011) for a brief overview of the literature on presupposition. For dynamic semantics and the notion of local context, see the Chapter on “Donkey anaphora: Farmers and bishops” in this volume, and the useful overviews in Geurts, Beaver & Maier (2015), and Nouwen, Brasoveanu, van Eijk & Visser (2016). More recent work on dynamic semantics includes Barker & Shan (2014); Martin (2013, 2016); and AnderBois et al. (2015).

8 Some of the update rules on local context found in dynamic semantic theories, like that for conjunctions and conditionals, are actually foreshadowed in the rules for presupposition projection due to Karttunen (1974), but in a less general way.
The basic idea is simple. (23) is not about a specific wolf. But neither does the underlined indefinite c-command the coindexed pronoun; hence the anaphoric relationship between them is not one of syntactic binding. Instead, just as in (22), where after utterance of the first conjunct the context is updated to reflect the existence of a salient dog, in interpreting the if-clause in (23), the input context of utterance is temporarily updated to reflect the fact that in the hypothetical situation offered for consideration there was a wolf who got in. This updated context then serves as the local context of interpretation for the main clause, and accordingly, the wolf in that hypothetical context can serve as antecedent for it. After the conditional has been fully interpreted, the temporary modification of the context is dropped—we’re no longer considering the arbitrary wolf; and the resulting truth conditional content is added to the input context to yield the context for the subsequent utterance. Since there was no specific wolf referred to, the proposition included in the final update doesn’t contain information about any particular wolf, and follow-ups to (23) like (24) are therefore infelicitous:

(24) #It$_i$ was very hungry.

The same kind of context-update explains how factive and other non-anaphoric presuppositions can also be merely locally satisfied, as we see in (25) with factive regret:

(25) [context: Sven has a terrible disease which is usually transmitted to the sufferer’s offspring. We don’t know whether Sven has kids, but we take him to be a caring guy.] If Sven is a father, he regrets that he has children.

The real key to the local update in such cases is that the logical form of a conditional like (25), according to Kratzer, has a tripartite structure (see Heim 1982; von Fintel 1994; Bach et al. 1995):

\[
\text{Operator} \ [ \text{Restriction} , \text{Nuclear Scope} ]
\]

Following Kratzer (1981), Heim (1982), all conditionals contain a modal operator, either overt or, in a superficially bare conditional, an implicit modal with universal force. The if-clause serves to make explicit part of the domain restriction on the modal; the main clause is its nuclear scope, and the whole conditional expresses a relation between the restriction and scope, just as with generalized quantifiers generally (Barwise & Cooper 1981). Utterances with QAdvs have the same tripartite structure; again, the restriction may be completely or partly implicit: As Lewis (1975) noted, if- or when-clauses may serve to restrict QAdvs, the main clause then serving as nuclear scope:

(26) Usually, if a boy lets a wolf get in, it bites him.

In a dynamic semantics, contextual update in all these cases operates in the same fashion. In interpreting the tripartite logical form, we first temporarily update the global context of utterance with the content of the restriction, using the result as the local context for interpretation of the nuclear scope. But that temporary update expires once the nuclear scope has been interpreted.

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9 The best real explanation of why local update works in these tripartite structures is due to Chierchia (1995). He notes that this structure is the tripartite schema of the logical form of generalized quantifiers, and that, as observed...
Then the resulting truth conditional content of the whole tripartite structure, as given by the semantics of the operator itself, is added to the input global context to yield the output global context. Normally only indefinites which have a specific interpretation, escaping the scope of the operator in logical form, bring about the addition of a possible antecedent for anaphoric triggers in subsequent discourse. So in (2) from above, if *a wolf* has wider scope than *might*, it is a possible antecedent for *it* in the subsequent sentence:

(2)  **A wolf** *might* come in. I saw it prowling around outside earlier.

But as we saw in (23), non-specific, narrow scope indefinites do not result in an update which licenses anaphora in the subsequent global context, explaining the infelicity of (24).

Felicitous presupposition only requires satisfaction in the local context of interpretation of the presupposition trigger (the anaphoric pronoun, *too*, factive verb, etc.). The same kind of temporary, merely local update facilitates the local satisfaction of the factive presupposition triggered by *regret* in (25): the *if*-clause restriction of the conditional sets up a local context which entails that Sven has children, and this local context is available for the interpretation of the nuclear scope in the main clause; but the interpretation of the whole conditional does *not* entail that Sven has children. Thus, in this example the factive presupposition doesn’t project (impose a constraint on the global context)—it does not imply that Sven has children. This differs from (27), where the presupposition that Sven has kids does project—i.e., it implies that Sven has kids; or in (28), which seems to presuppose that all X-sufferers have children:

(27)  If Sven suffers from X, he regrets that he has children.
(28)  Every X-sufferer regrets that he has children.

Anaphoric presuppositions may be similarly satisfied in the merely local context, as in (29) where *children* in the local context provided by the *if*-clause serves as antecedent for *them*. If not, as in (30), the presupposition of an antecedent projects, so that the example is only felicitous in a global context which itself provides an antecedent for *them*:

(29)  If Sven has children, he loves them.
(30)  If Sven loves them, he’ll tell them the truth.

The same kind of local context update licenses the understood relationship in (4) between the time of the event denoted by the *if*-clause and that of the main clause: in this case the understood Reference Time is temporarily updated to the time of the hypothetical event of the wolf’s getting in, and this is the time when Hansel and Gretel are understood to get in trouble.

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by Barwise & Cooper (1981), generalized quantifiers are conservative. To be conservative means that for quantifier $Q$, restriction $A$ and nuclear scope $B$, $Q \{A, B\}$ is true iff $Q \{A \cap B\}$ is true as well. Thus, the local context update proposed by Heim and others is not just a stipulation, but reflects this deeper semantic principle.
§2.2 Anaphora and scope islands

As we saw above, the hallmark of modal subordination is that the interpretation of the second sentence in a sequence S₁ . . . S₂ seems to depend on content that is borrowed, in some sense, from the utterance of S₁, even though the borrowed content is itself under the scope of OP₁—a modal, QAdv (at every convention in (11)), or negation (in (6)). In simple cases of modal subordination like (3) involving the same modal auxiliary as operator in both S₁ and S₂, one might entertain the hypothesis that the OP₁ in S₁ (might) takes scope over the content of the entire second sentence S₂. Then all the content introduced under the scope of OP₁ would still be local in S₂, just as it is in the main clause of a conditional. But there are both theoretical and empirical problems with this hypothesis.

First, there is good reason to think that tensed clauses, including tensed main clauses S₁ and S₂ in our examples, are scope islands, that is that the scope of any operator in such a clause is restricted to that clause itself. Rodman (1976) pointed out that relative clauses are islands to quantifier scope, so that the quantifier can’t “get off the island”. He offered examples like these:

(31) Guinevere has a bone in every corner of the room.
(32) Guinevere has a bone that’s in every corner of the room.
(33) Guinevere met a representative from every city in Ohio. (inverse linking)

Unlike (31), (32) gives rise only to a very odd interpretation in which the same bone is in every corner of the room: every corner of the room cannot take wide scope out of the relative clause in which it occurs, to permit it to scope over the existential head NP, a bone... This contrasts both with the locative adjunct in every corner of the room in (31), and the NP-internal complement from every city in Ohio in (33), in both of which the universal can take wide scope over a bone: (31) permits different bones in different corners, and (33) may be true in a case where Guinevere met different representatives from different cities. So the scope-islandhood of the relative clause in (32) is specific to that construction.

Szabolcsi (2010:18) argues that the underlying generalization is broader, and that the scope of a quantificational NP is limited to the tensed clause in which it occurs. Generally, you can bind into a tensed clause from a c-commanding quantificational operator, just not out of it. Some of her examples:

(34) That every boy was hungry surprised his mother.
not a possible reading: ‘for every boy, that he was hungry surprised his own mother’
(35) Some judge saw that every contestant was cheating.
not a possible reading: ‘every contestant was such that some judge or other saw that s/he was cheating’

10 For background to this discussion, see Rodman (1976), May (1988), Reinhart (1991) and Fox & Sauerland (1996), as well as Szabolcsi (2010) and the references therein.
11 The phenomenon of specific indefinites has been taken by some to argue that indefinite NPs can escape scope islands. Whether or not this is the case, the resulting specific interpretation of the indefinite is not the interpretation of interest in modal subordination involving an indefinite NP as X in our schema, as we saw in the initial discussion of examples (1) and (2).
In accord with this generalization, we see that the antecedent of a conditional is a scope island:

(36)  #If every farmer planted his fields last spring, he must now be preparing to harvest.

Here the quantificational DP *every farmer* cannot scope over the main clause to bind *he*.

And quite generally, matrix sentences are scope islands. Thus, the second sentence in (37) is infelicitous due to anaphora failure, since the only potential antecedent for *he* is bound by *every* in the preceding sentence, which serves as a scope island. This is despite the fact that the intended interpretation is perfectly felicitous when the content is in a relative clause that itself is under the scope of *every*, in (38):\(^{12}\)

(37)  Every farmer planted his fields last spring.
        #I heard that he’s now preparing to harvest.

(38)  Every farmer planted last spring the fields that I heard that he’s now preparing to harvest.

Hence, theoretically it seems quite implausible to account for anaphora resolution (and presupposition satisfaction more generally) in modal subordination by taking the operator in the first sentence S\(_1\) to take scope over the entire second sentence S\(_2\), since this would constitute an isolated counter-example to the otherwise very robust generalization about scope islands.

There are other, empirical problems with attempting to account for modal subordination with scope extension. For one thing, as we saw in (1), even when S\(_1\) and S\(_2\) both contain overt modals, those models may have distinct force: *might* has the force of possibility in S\(_1\), while *would* has the force of necessity in S\(_2\). We get the wrong truth conditions if we simply extend the scope of *might* over the entire second sentence, resulting in the too-weak interpretation ‘it might be that it would eat you first’. The attested interpretation is stronger—the speaker is committed to it being necessarily the case, should the wolf come in, that it eats the addressee first.

The existence of cases involving mixed attitude predicates also argues against scope extension. For example, the second sentence of (18) cannot mean that Mary is considering that she wouldn’t regret attending graduate school. Instead, it is an assertion by the speaker that Mary wouldn’t regret a particular decision, if she makes it:

(18)  Mary is *considering* getting her Ph.D. in linguistics.
        She *wouldn’t* regret *attending* graduate school.

And scope extension certainly would not apply in cases involving negation in the first sentence, as in (17).

\(^{12}\) There are *prima facie* counterexamples to this claim. For example, some speakers find the following fine:

(i)  Every farmer received a federal subsidy.
        He changed his crop rotation schedule in order to qualify, and his income was significantly enhanced.

This is an example of a phenomenon Roberts (1989) dubbed *telescoping*. She argues that telescoping does *not* involve binding. See §4 below.
(17)  I never eat a big breakfast.  
It would make me sleepy all morning.

That would yield the unattested interpretation ‘it’s not ever the case that there’s a big breakfast that I eat and that it necessarily makes me sleepy all morning’.

Scope extension also cannot account for cases where the interpretation of $S_2$ clearly draws on content given in multiple previous irrealis clauses, each with its own wide-scope modal auxiliary, QAdv, or attitude predicate. Consider the following, modified from Roberts (1995):

(39)  If Audrey met a sorcerer, she’d be delighted.  
Sorcerers often have leprechaun companions.  
Leprechauns sometimes have a pot of gold.  
If Audrey was really lucky, she might get the sorcerer to get his leprechaun to let her have some of his gold.

We understand the last sentence in (39) to mean ‘if Audrey was really lucky and met a sorcerer who had a leprechaun companion that had a pot of gold, she might get the sorcerer to get the leprechaun to let her have some of the leprechaun’s gold’. That is, the implicit enrichment of the overt if-clause restriction on might is taken from the restrictions (a sorcerer) and nuclear scopes (leprechaun companions, a pot of gold) of multiple operators in distinct preceding sentences: would and the mixed-force QAdvs often and sometimes. None of those operators is understood to extend its scope over subsequent discourse.

Finally, extension of scope would not account for cases where the subordinate modal itself is in the complement of an attitude predicate.

(40)  John wants to give the boss a piece of his mind.\(^{13}\)  
Alexa thinks he would get fired.

The second sentence is understood to mean ‘Alexa thinks that if John gave his boss a piece of his mind, John would get fired’, so that the borrowed content and conditional force have no effect on the embedding Alexa thinks.

§2.3  The challenge of modal subordination

Now we can appreciate the problem posed by modal subordination for the classical theories of presupposition projection. Consider again the general logical form instantiated by the varied examples we have considered:

\[
[S_1 \ldots OP_1[\ldots X \ldots]] \ldots [S_2 \ldots ModOP_2[\ldots Y \ldots]] \ldots
\]

Modal subordination shows that intuitively $X$ can serve as antecedent for $Y$, and the goal is to explain how this is possible. $X$ occurs within the restriction or nuclear scope of OP$_1$. Any

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\(^{13}\) To give someone a piece of one’s mind is to tell them one’s harshly negative assessment of their behavior.
contextual update under the scope of OP₁ making available X as potential antecedent, expires after the logical form headed by OP₁ has been fully interpreted, in keeping with the general context-update procedure for such logical forms (§2.1). But independently supported constraints on scope argue that the tensed sentence S₁ is a scope island (§2.2), so that OP₁ cannot take scope over S₂. Therefore, none of the content in the scope of OP₁ is in the global context of interpretation for S₂, and in particular X is not available as antecedent for Y.

If a presupposition fails to be satisfied in the context of utterance, it is said to project. That is to say, if a presupposition is triggered by Y in S₂ and the global context of utterance resulting from interpretation of S₁ fails to satisfy it—to entail the factive complement or provide an antecedent for the anaphoric trigger—then assuming there is no antecedent for Y in S₂ itself, we would expect S₂ to have an unsatisfied presupposition. The verb regret is factive, its use presupposing the truth of its complement. So in (18), repeated below, the classical characterization of presupposition projection would predict that in the absence of prior context not given here, the second sentence will presuppose that Mary has attended or will attend graduate school.

(18) Mary is considering getting her Ph.D. in linguistics. She wouldn’t regret attending graduate school.

Yet that is clearly not our impression. Similarly, the classical story about presuppositions incorrectly predicts that the second sentence in (1) should be infelicitous in the context given because that global context fails to yield an antecedent for it.

(1) A wolf might come in. It would eat you first!

Of course, Karttunen, Stalnaker and Heim all admitted of the possibility that other contextual factors might play a role in satisfying presuppositions. But in their accounts, these factors were all constrained by the kinds of scope factors considered in §§2.1 and 2.2.

Thus, we need an account of how in modal subordination, content under the scope of OP₁ apparently manages to satisfy presuppositions triggered under ModOP₂.

3. The general form of an explanation of modal subordination:

Recall the summary of features of modal subordination at the end of §1, now restated in the terms introduced in §2. We have a logical form:

\[ [S₁ . . .OP₁[. . .X . . .] . . .] . . . [S₂ . . .ModOP₂[. . .Y . . .]] . . . \]

where crucially:
1. Y in S₂ is a presupposition trigger; and the only content X which would satisfy the presupposition it triggers occurs under the scope of OP₁ in the scope island S₁.
2. ModOP₂ in S₂ is a modal operator which has scope over Y.
3. The ModOP₂-headed constituent in S₂ has an interpretation wherein part of its restriction consists of X.
Underlying the explanation for features 2 and 3 is the now widely accepted Kratzerian semantics of the tripartite structure of a modal operator-headed constituent like ModOP₂[...Y...] in S₂. According to Kratzer (1981, 2012), use of a modal operator in natural language assumes two functions, a Modal Base \( f \) and Ordering Source \( g \), which together give the contextually intended restriction for the interpretation of the modal. Kratzer (1977) uses in view of paraphrases to give the intuitive content of such restrictions, so that one and the same modal, \textit{must} in (41) and (42), may have very different flavors—epistemic in (41), deontic in (42), as a function of the contextually given \( f \) and \( g \):

(41) In view of what we’ve just learned from the weather satellite, it must be raining in Seattle.
(42) In view of the laws in the state of Missouri, you must signal before turning right.

Kratzer argues that when there is an overt \textit{if}-clause modifying a modally-headed clause, the \textit{if}-clause enriches the otherwise-contextually given Modal Base \( f \); hence, contextual factors are always part of the determination of the restriction. Given this independently motivated semantics for modals, we now expect that ModOP₂ in the schema above will itself have some restriction, which may in part be given by an overt \textit{if}-clause but need not be overt at all. This predicts that typically the ModOP₂-headed constituent in S₂ will have a conditional-like interpretation, with its restriction at least partly given contextually.

In theories of dynamic interpretation, as we saw above, it is assumed that in the interpretation of an operator-headed tripartite structure, the local context with respect to which the operator’s scope is interpreted has been temporarily enriched with the content of its restriction. So despite the fact that due to X’s occurrence in a scope island it is inaccessible for presupposition satisfaction after S₁, if we can explain how the content denoted by X in can serve as part of the contextually-given restriction on ModOP₂, we can explain all the features of modal subordination. Put another way, if content in the scope of OP₁ can somehow serve to restrict ModOP₂, that explains how the X so “borrowed” comes to locally—in the scope of ModOP₂—satisfy the presupposition triggered by Y.

I’ll exemplify this explanation informally with (12) and (13), involving QAdv. (12) is unacceptable because the only potential antecedent for \textit{it} in the second sentence, a \textit{muffin}, is under the scope of \textit{usually} in the first sentence; in the scope of \textit{usually}, a \textit{muffin} is temporarily available to serve as antecedent, as we see in the felicitous \textit{eats it at the office}. But that availability “expires” after interpretation of the first sentence, a scope island, leaving \textit{it} in S₂ with no antecedent and resulting in infelicity. But in (13), though we have the same S₁ and restricted availability of a \textit{muffin} to serve as antecedent in subsequent sentences, we can contextually restrict the modal-like QAdv \textit{always} in the second sentence to only range over situations in which Fred has purchased a morning muffin, “borrowing” the content under the scope of \textit{usually} in S₁ to limit that domain. Since this restriction involves a muffin, it makes available an antecedent for \textit{it} in the operator’s nuclear scope \textit{it’s oat bran}.

(12) \textbf{Usually} Fred buys a \textit{muffin} every morning and \textit{eats it} at the office.
\#It’s being baked.
(13) **Usually** Fred buys a **muffin** every morning and eats **it** at the office.
    **It's always** oat bran.

Thus, in the summary characterization of modal subordination above, feature 1 captures the way in which the puzzle of modal subordination is one about the apparent inaccessibility of \( X \), illustrated in examples like (12). The presence of the modal-like operator in \( S_2 \) (feature 2) facilitates the conditional-like interpretation (feature 3), “borrowing” content from under the scope of some operator(s) in preceding discourse to serve as the intuitive conditional restriction. This domain restriction of ModOP\(_2\) licenses the attested presupposition satisfaction in the modally subordinate cases like (13), unlike cases like (12) without a modal.

As we will review in the next section, a number of particular proposals have been made regarding why and how \( X \) can be used to restrict ModOP\(_2\). But most of these essentially assume this type of explanation of modal subordination, wherein under the schema above \( X \) is understood to be part of the domain restriction of ModOP\(_2\).

§4. Accounts of Modal Subordination

We have seen that modal subordination occurs in an utterance involving a modal operator when the interpretation of some expression under the scope of the modal depends on information that seems not to be available at the point where the expression occurs, yet the utterance is nonetheless felicitous. In such cases, the felicity is due to restriction of the modal operator’s domain using content under the scope of an operator in a previous utterance, the domain restriction making the relevant information locally available.\(^{14}\)

Kratzer’s general approach to modal semantics, and work over the past twenty years on the relationships between different flavors of modality in the Kratzerian account offer a strong, intuitively appealing foundation for an account of how and when modal subordination is possible. The tradition coming out of Hintikka (1969) that analyzes attitude predicates as involving a modal element, and the kinds of detailed work on the lexical semantics of attitude predicates found in Asher (1987), Heim (1992), and Maier (2015) allow us to extend that general conception of modality to the attitude predicates, explaining their relationship to modal auxiliaries and other modal elements. Add to this the work by de Swart (1992) and others on QAdvs, cited above, and Frank’s (1997) characterization of modals as generalized quantifiers over worlds, and we have the independently motivated tools to permit us to generalize over the different types of operators instantiating ModOP\(_2\) in the schema for modal subordination: all involve modal operators, their restriction at least in part contextually retrieved.

The large body of work on dynamic interpretation that has grown out of Kamp (1981, Kamp & Reyle 1993) and Heim (1992)—most recently Barker & Shan (2014); Martin (2013, 2016), and AnderBois et al. (2015)—all agree that content in an operator’s restriction is available to help satisfy presuppositions triggered in its nuclear scope, as we sketched in §2.1. In general, it is understood that indefinites and referential expressions like proper names and indexicals serve as triggers to introduce **discourse referents** into the context of utterance, which can serve as

\(^{14}\) Thanks to an anonymous reviewer for suggestions about how to best informally characterize modal subordination.
antecedents across discourse. Think of a discourse referent as a *file of information* (Heim 1982) about a particular entity under discussion in the discourse. These files are part of the information contained in a context of interpretation. But unlike the discourse referents for names and indexicals, discourse referents corresponding to indefinites only persist throughout the scope of any wider scope operators that dominate the indefinite, as illustrated above. Karttunen characterizes this as the limited “lifespan” of the discourse referent introduced by an indefinite. Then it is the discourse referent which is the true antecedent for anaphora in discourse, not the NP that leads to its introduction; and this permits us to capture constraints on the potential of indefinites to serve as apparent antecedents for discourse (as opposed to bound) anaphora.

The accounts of modal subordination in the literature almost all take it to involve restriction of the domain of a modal, and use a Kratzerian semantics for modals and something like the notion of a discourse referent in dynamic semantics to explain when modal subordination is possible. These accounts fall into three general types: those based on accommodation and other pragmatic factors, those based on the lexical semantics of the relevant operators, and those that are anaphoric. We’ll very briefly sketch an overview of these accounts, their differences, and their strengths and weaknesses.

Roberts (1987,1989) focused on cases of modal subordination involving modal auxiliaries and QAdv, and on how Kratzer’s semantics and the tripartite structure of modal logical forms offers an avenue of explanation for those cases. She framed her account formally in Kamp’s (1981) Discourse Representation Theory, and (referring to our schema from §1) characterized the “borrowing” of content from under the scope of OP1 to restrict ModOP2 in terms of accommodation, a notion defined by Lewis (1979:340):

**Rule of accommodation for presupposition:** If at time *t* something is said that requires presupposition *P* to be acceptable, and if *P* is not presupposed just before *t*, then—ceteris parabus and within certain limits—presupposition *P* comes into existence at *t*.

Since in the local context under the scope of ModOP2 there is no explicit content that would satisfy the presupposition triggered by *Y*—an antecedent for a pronoun or *too*, a true proposition to satisfy the factive presupposition of *regret*, etc.—then the presupposition is accommodated (“comes into existence”) by introducing the “borrowed” content from the scope of OP1 into the restriction on ModOP2, including any discourse referents for indefinites *X* under OP1. Roberts took this “borrowing” to be motivated and constrained by the relevance of the content of S1 to that of S2, drawing on general Gricean principles for interpretation in context. She offered a similar explanation for the way that in disjunction examples like (43) the negation of the first disjunct serves to enrich the content of the second disjunct, on the assumption that disjunction offers alternative ways of resolving some question under discussion.

(43) Either there’s no bathroom in this house or it’s in a funny place.

A disjunction may be understood to offer alternative answers to some question under discussion. One is first led to entertain one possible answer and then, in the second disjunct, a second answer which may be true *in case the first disjunct is not*. Hence, pragmatically, it is natural to interpret the second disjunct as an indicative conditional whose restriction includes the negation of the
first: ‘if there is a bathroom in this house, it’s in a funny place’. In this way, the disjunction acts pragmatically like a species of modal statement—two related possibilities are entertained.

Asher (1987) and Heim (1992) focused on a wide variety of cases involving pairs of attitude predicates, and showed how the lexical semantics of some of these predicates both facilitates and constrains what we here call modal subordination. As Heim points out, though we take belief to be closed under entailment, so that (44a) entails (44b), there is evidence that wanting shouldn’t be closed under entailment, so that (45a) does not entail (45b):

(44) a) Nicholas believes he’s due a free trip on the Concorde.
    b) Nicholas believes he’s due a trip on the Concorde. ✓

(45) a) Nicholas wants a free trip on the Concorde.
    b) Nicholas wants a trip on the Concorde. X

Then effectively, in our schema for modal subordination, if ModOP2 is part of the lexical semantics of an attitude predicate whose meaning is based on the agent’s beliefs and OP1 is part of the lexical semantics of an attitude predicate which entails something about those beliefs, then the fact that the content of the complement of OP1, including X, is available as part of the domain restriction on ModOP2 simply follows from this entailment. Thus, we find (46) acceptable, with it taken to be Vijay’s job, even though the first conjunct fails to entail that he actually has one:

(46) Jayanti believes that Vijay has found a job. She’s glad it will permit him to repay his student loans.

The first conjunct entails that Jayanti believes that he Vijay a job. Since an agent’s gladness is based on what she believes to be true, the first conjunct interacts with the lexical semantics for glad to guarantee that Vijay has a job in all the worlds that reflect what makes Jayanti glad, providing a salient, relevant antecedent for it.

Drawing on this work, Roberts (1996) discusses how require and permit are directly related to deontic must and may, resulting in parallel constraints on modal subordination: As we see in (47) and (48), require and permit cannot readily be inverted in such pairs and still yield felicity, which is intuitively parallel to the constraints on deontic must and may in (49) and (50):

(47) You are required to find a bear and permitted to take its picture.
(48) #You are permitted to find a bear and required to take its picture.

(49) You must find a bear. [Then] you may take its picture.
(50) #You may find a bear. [Then] you must take its picture.

If we take the agent granting permissions and imposing requirements to be the same across the two conjuncts in (47) and (48), and similarly for the deontic modals in (49) and (50), we can understand the difference in acceptability: It is quite odd to require someone to do something which has action δ as a precondition but merely permit action δ, permission conversationally implicating that that person has the option not to undertake δ. In (48) and (50): if you don’t find
the bear (δ), you can’t take its picture; the taking of the picture is required; ergo so is finding the bear. But permit and may implicate that finding the bear not required. This yields a pragmatic contradiction in these examples, explaining their infelicity.

Extending Heim’s general approach, Maier (2015) looks at a variety of attitude predicates which involve as part of their lexical semantics a dependence on the presumed beliefs of the agent. He enriches a dynamic semantics using D(iscourse)R(epresentation)T(heory) (Kamp 1981) with representations of mental states which are related so as to reflect such “parasitic” dependencies of an agent’s desires and other non-doxastic attitudes on their beliefs. Then he shows how we can thereby derive many of the attested examples of modal subordination across attitude predicates, without accommodation. In developing a common ground that reflects the interlocutors’ joint beliefs about some agent’s mental states, the partial attitude representations resulting from the interpretation of particular assertions are “merged”. For example, he considers (51) (Maier 2015:207; the example due to Karttunen 1973):

(51) Bill believed that Fred had been beating his wife and he hoped that Fred would stop beating her.

The first conjunct of (2) tells us that Bill has a particular belief about Fred’s behavior. The second conjunct tells us that Bill has a particular hope about Fred’s future behavior. But since (following Heim 1992), hopes are based on beliefs—in Maier’s terms, they are parasitic on belief—we merge the content of these two clauses into a general representation of Bill’s mental state in which his hopes are subordinate to (dependent upon) his beliefs. In so doing in a Discourse Representation, the content of the particular hope asserted in the second conjunct Fred would stop beating her becomes logically and formally subordinate to the content of Fred had been beating his wife, the belief asserted in the first. Moreover, both are under the scope of a logical BELIEF operator in the merged representation. Through this independently motivated subordination in the merged content, her acquires a local antecedent, the discourse referent corresponding to his wife, and the anaphoric presupposition triggered by his is satisfied. Note that in this case, the resulting interpretation of the second conjunct fails to have a conditional sense like ‘if Fred had been beating his wife, Bill hoped that he would stop beating her’ or ‘Bill hoped that if Fred had been beating his wife, he would stop beating her’. That is, given the first conjunct and the relationship between the relevant attitudes reported, the domain restriction of ModOP2 is straightforwardly entailed: ‘Bill hoped that, given that Fred had been beating his wife, he would stop beating her’.

Maier shows that his account offers superior empirical predictions to that of Heim, because he can correctly handle all sequences of attitude assertions of the same type (whereas Heim has difficulties with some, like hope….hope), and also because his account can explain examples which are not compatible with Heim’s crucial use of ordering semantics, like those involving dream and pretend (see Maier, p.215f). Since his DRT representations are based on independently motivated characterizations of the relevant agent’s mental states, this is a simple, elegant approach to the relevant types of examples. Note that though Maier couches the solution in DRT, there is nothing inherently representational about it; the point is that the lexical semantics for these predicates, along with the way that interlocutors track the overall information they have about agents’ mental states, entails the asymmetrical subordination.
However, as both Heim and Maier discuss, the entailment-based approach to modal subordination in attitude predicates cannot account for the full range of cases involving such predicates. Consider:\(^{15}\)

(52) Susan wants a pet. She believes she will look after it. [Cresswell 1990, cited by Heim]

Maier (pp.215-216) takes (52) to pose a problem for both his own account and Heim’s. “Heim observes that, somehow, the second ascription in such examples gets a conditional interpretation: she believes that if she gets a pet, she will look after it.” In this connection, compare (51) above with (53) (an example Maier doesn’t consider), where the order of the two attitude predicates has been changed.

(53) Bill hoped that Fred was buying his wife flowers and he believed that Fred would give them to her that evening.

Since on Maier’s account the parasitic relationship between hope and belief is asymmetric, the subordination required to make (53) felicitous—wherein the complement of hope would be part of the restriction on believe—is not automatically given by his attitude merging. If one does accept the example, with the irrealis implication of hope, one can only give it a conditional interpretation: ‘he believed that if Fred bought his wife flowers, he would give them to her that evening’, parallel to the conditional interpretation attested for (52) by Heim (1992).

Similarly, in (16) the desirable state denoted by the infinitival complement of wants is not necessarily something that John believes will come about—wanting something doesn’t entail that the agent believes that it will become true. So entailment alone cannot account for the felicity of it under plans, with the non-specific caught fish as its apparent antecedent:

(16) John \textbf{wants} to catch a fish.
    He \textbf{plans} to eat it for supper.

\(^{15}\) Maier also considers (i) to be a challenge for his theory and Heim’s:
(i) Alice fears that there is a squirrel in her kitchen cabinets. She hopes to trap it alive. [Maier ascribes the example to Schoubye (2013), but it is due to Roberts (1996)]
(ii) Alice denies that there’s a squirrel in her kitchen cabinets. #She hopes to trap it alive…

But Roberts (1996) offers an explanation for the contrast between (i) and (ii) which could arguably be captured in an extension of the lexical-entailment-based accounts:

We understand that one’s hopes are based on what one takes to be plausible; i.e. the modal base for hope yields the set of propositions whose truth the agent takes to be plausible. In (47), the first sentence tells us that Alice suspects that there’s a squirrel in the cabinets; hence, assuming that one takes one’s suspicions to be plausible, the second sentence can draw on that suspicion to provide an antecedent for it. But in (48), doubt tells us that Alice does not find it plausible that there’s a squirrel; hence, there’s no salient plausible existent appropriate as antecedent for it under the scope of subsequent hope.

If we take the set of propositions that an agent takes to be plausibly true to be a superset of those she believes to be true, then beliefs are subordinate to that set. Roberts is arguing that hopes are also, which could be represented in Maier’s structured DRT representation of an agent’s mental state. Since one presumably doesn’t find plausible those propositions whose truth one denies, the contrast with (ii) is explained on such an account.
Instead, the second sentence of (16) most naturally means something like ‘if he catches a fish, he plans to eat it for supper’, with a conditional restriction on the modality associated with plans.

Roberts (1996) discusses a number of other types of cases of modal subordination that the lexical entailment-based accounts cannot account for, including examples involving mixed operator types. Among these are examples above where ModOP<sub>2</sub> is would: (17) where OP<sub>1</sub> is a QAdv, (18) where it is consider, and (7) with negation. It also includes (10) with may (OP<sub>1</sub>) and otherwise (ModOP<sub>2</sub>). Note that both would and otherwise have a counterfactual aspect to their semantics, and this is no doubt one reason why they yield felicitous modal subordination in such cases. Compare (7) with (54):

(7)  John doesn’t have a car.
  It would be in the garage.

(54)  John doesn’t have a car.
  #It might be in the garage.

This strongly suggests that further investigation of the lexical semantics of particular modal auxiliaries would shed more light on when modal subordination is felicitous, as we’ll discuss further below. But, again, this still would fail to predict or explain the conditional interpretations of examples like (52), (53) and (16).

Nor can the lexical entailment-based approach account for Geach’s (1967) famous Hob-Nob sentences (55), which seem to require subordination across sequences of attitude reports with multiple agents:16

(55)  Hob thinks a witch has blighted Bob’s mare, and Nob wonders whether she (the witch) killed Cob’s sow.

Roberts (1996) notes that “In none of these is there evidence that an entailment of the first clause interacts with the modal base for the second intensional operator to satisfy the presupposition in the second, modally subordinate clause. All appear to be amenable to a treatment involving local accommodation. As Heim notes, "Once this mechanism is invoked, of course, the question arises to what extent it could also have been employed to yield some of the predictions that I took pains to make follow directly from the CCP [Context Change Potential] definitions," i.e., from facts about the lexical semantics of the propositional attitude predicates under investigation.”

Kibble (1995), Geurts (1995, 1998), Frank (1997), and Frank & Kamp (1997) have criticized Roberts’ (1989) use of accommodation because it is presumed to be too powerful, failing to predict when modal subordination is not felicitous. For example, Geurts (2012) complains that “…there is nothing in Roberts’s theory that rules out sequences like the following”:

[(56)] Wilma may have bought a car.
  #Fred can drive it, too

This problem is partially addressed by the accounts of modal subordination which take the domain restriction involved to be anaphoric. von Fintel (1994) and Roberts (1995) argued that domain restriction is a presuppositional phenomenon. von Fintel’s anaphoric account has been particularly influential, and most subsequent work on modal subordination adopts his view and aims to account for modal subordination by using different formal realizations of dynamic context update to modal how ModOP2 gets its domain restricted anaphorically.

Kibble (1995), Geurts (1995, 1998), Frank (1997), and Frank & Kamp (1997) offer variants of the anaphoric approach wherein the antecedent for the implicit domain restriction of ModOP2 is the content of a previous clause, typically as represented in a Discourse Representation Structure (Kamp 1981). Geurts (1995) and Roberts (1995) show how the presuppositional nature of domain restriction yields a more constrained account of which “antecedents” are available in discourse than a pure accommodation account: given that the anaphoric domain restriction presupposition is triggered by ModOP2, presupposition theory tells us that it must be locally satisfied, either in the restriction on that operator or in the global context. Hence, not just any prior irrealis content can serve as antecedent. Moreover, without a modal to trigger the domain restriction, there is no natural way to accommodate irrealis content to serve as antecedent for anaphoric elements. This explains the pattern Roberts observes in (57) – (59):

(57)  a) An earthquake might hit San Francisco.
     b) That would upset me. It would be frightening.
     b) That upset me. It was frightening.
(59)  a) An earthquake might hit San Francisco.
     b) That upsets me. It is frightening.

In (57), the anaphoric restriction of the domain of would in both clauses in (b) to the irrealis scenario described in (a) permits us to take it to be the irrealis earthquake; that can be understood to refer to the proposition that an earthquake hit San Francisco, the prejacent of might. This is parallel to the non-modal indicative pair in (58), where that and it in (b) are anaphoric to the proposition expressed by (a). (59b) is the same non-modal indicative as (58b), and it is also felicitous. But because there is no modal to facilitate modal subordination, it cannot refer to the irrealis earthquake discussed in (a)—we can only take what is frightening to be the possibility of an earthquake, the proposition expressed by (59a) as a whole. And that can only refer to the same possibility, not to the proposition ‘an earthquake hit San Francisco’. This demonstrates that the mere salience of a proposition—here the irrealis ‘an earthquake hit San Francisco’ in (59a)—does not suffice to make it and its content globally available for anaphora resolution. The domain restriction presupposition is a crucial facilitator.

Frank (1996), Frank & Kamp (1997), van Rooy (2001), and Asher & McCready (2007) introduce more complex contents as the objects of quantification in Discourse Representations, so that modals and other operators relate discourse representations or discourse referents that range not over simple individuals but over world-assignment pairs. Such parameterization permits these accounts to capture the dependencies between modals and indefinites in the relevant domains. This general approach is thus an extension of the parametrized-sum-

Stone (1997,1999), Bittner (2001,2007), and Schlenker (2004,2013) offer another type of anaphoric account, modeled after Partee’s (1984) account of anaphora to Reference Times, in which modal subordination involves anaphora to a particular salient irrealis world or situation. Note that these accounts do not adhere to the general characterization of modal subordination as domain restriction, given above, since they yield not a set of alternatives—as in the usual accounts of domain restriction—but only a single world or situation. As Brasoveanu (2010) discusses, this encounters difficulty when one operator is, say, epistemic, the other deontic: As is well-known, the set of worlds representing what’s deontically desirable needn’t overlap with those which represent what is known to be true: what should be the case often isn’t. So in (60), the set of worlds where there’s a hungry child out there needn’t include any where the interlocutors give it something to eat. Hence, in such circumstances there is no available world to serve as antecedent, to “bridge” the two domains.

(60) There might be a hungry child out there, not a wolf. We ought to give it something to eat.

More recently, Brasoveanu (2007, 2008, 2010) bases his account of modal subordination on anaphora to plural information states, rather than to particular worlds or to discourse referents or representations of particular propositions. Unlike the accounts of Frank et al., these information states themselves are dynamic, so that the relevant quantificational dependencies between possibilities and individuals are updated across discourse and not only under the scope of particular operators. And he argues (2010:521ff) that his approach is superior to accounts like that of Frank & Kamp (1997) because his can successfully address the difference between and distribution of so-called weak vs. strong donkey sentence readings.

It is clear that the more constrained presuppositional view of domain restriction improves on the accommodation-only approach, as argued by Geurts and Roberts. Unfortunately, space precludes a thorough critical comparison of these different anaphoric approaches to modal subordination. But here are some comments about the adequacy of the anaphoric approach in general.

First, note that adoption of the presuppositional, anaphoric approach to domain restriction is not incompatible with the use of accommodation to explain some examples of modal subordination. Just as anaphora resolution generally often requires accommodation, so does the domain restriction that licenses modal subordination. Recall (39) from above:

(39) If Audrey met a sorcerer, she’d be delighted. Sorcerers often have leprechaun companions. Leprechauns sometimes have a pot of gold. If Audrey was really lucky, she might get the sorcerer to get the leprechaun to let her have some of it.

Frank & Kamp (1997) argue that (39), which would be problematic for their account, should not be taken to involve modal subordination. But I think the anaphoric approach gives us a way of
understanding its felicity without any special stipulations. We can see that the domain restriction for *might* in the last sentence is constructed from content introduced under several preceding operators (in boldface), themselves heterogeneous in type and force: if Audrey is lucky, all these irrealis scenarios will be realized at once, and *then* the possibility in the scope of *might* opens up. Roberts (1995) argues that the felicity of this type of example precludes an explanation of modal subordination as anaphora to the irrealis proposition denoted by a given preceding clause (like the prejacent of a particular modal). It shows that the restriction of a modal auxiliary like *might* need not have already been explicitly under consideration prior to the utterance of the modal which triggers the restriction licensing modal subordination. This is parallel to cases of split antecedents for nominal anaphora:

(61) John told Mary about his problem. *They* agreed that he should resign.

In (61), all the members of the group of John+Mary are salient at the time of utterance of *they*, but the group itself has not been explicitly referred to. (61) is nonetheless perfectly felicitous because that group is weakly familiar (Roberts 2003, 2005), which is to say that its existence can be straightforwardly inferred from prior context. Hence the required plural antecedent is felicitously accommodated. Similarly, the first three sentences in (39) offer a sequence of characterizations of irrealis situations which are in principle compatible. Then in the complement of *might* the locally triggered presuppositions of the underlined definite NPs lead us to accommodate such a merged scenario to simultaneously satisfy the presuppositions triggered by the modal itself (domain restriction) and the anaphoric definites. The construction of the required domain involves accommodation, but it seems no more problematic than the determination of a plural antecedent for *they* in (61). What (61) and (39) show us is that sometimes “antecedents” for anaphora have to be constructed from explicit and implicit content in prior context.

Roberts (1995, 1996) and von Rooij (2005) emphasize the unacceptability of representational accounts of modal subordination that rely too closely on either sentential logical form or the discourse representation of a given clause, in the absence of the consideration of broader pragmatic factors and of the general structure of the discourse in which these utterances occur. And Asher & McCready (2007) argue against anaphoric accounts, offering a range of cases where otherwise unconstrained anaphora to preceding clause(s) over-generates possible interpretations; they argue for the crucial importance of discourse structural constraints on anaphoric accessibility (for both modal subordination and Reference Time retrieval), appealing to the Segmented Discourse Representation Theory of Asher & Lascarides (2003).

If we accept the role of accommodation in presupposition satisfaction generally and in anaphora resolution in particular (Roberts 2003,2005), then the fact that it plays a role in modal subordination shouldn’t surprise us, so long as its power can be suitably constrained. See Roberts (2015) for discussion of a number of ways in which accommodation is constrained in discourse.

Moreover, the kinds of restrictions given by the lexical semantics of a modal operator, as discussed by Heim (1992) and Maier (2015) also play a crucial role in explaining and constraining when modal subordination arises, even in an anaphoric account. For example, consider (56), which Geurts (2012) offered as a criticism of a purely accommodation—based approach:
(56) Wilma may have bought a car, 
#Fred can drive it, too

It seems to me that if an anaphoric approach to modal subordination doesn’t take into account the role of the lexical semantics of the various modal operators realizing ModOP2, (56) will be just as problematic for that approach as for the purely accommodation-based approach. For it seems that the content under the scope of may in the first sentence is salient here and just as available for modal subordination with other continuations, as we see in (62):

(62) Alf: Wilma may have bought a car.
     It would probably be a Ford.
     Dan: Great! Fred could borrow it on weekends so we could go to Coney Island!

Note that the difference between can in (56) and could in (62) seems similar to that between will and would. The first in each pair is realis, requiring a realistic modal base, whereas the second is amenable to an irrealis domain restriction. If this is the case, we can explain the infelicity of (62) by appeal to the lexical constraints on the domain restriction for can: the worlds in which the prejacent complement of may is true do not necessarily contain the actual world; hence it is irrealis, and its scope is therefore not an appropriate antecedent for the domain of can. Recall the pairs (47)/(48) and (49)/(50) where we saw other lexical constraints on domain restriction. This suggests that the lexical semantics of ModOP2 play a more general role in constraining modal subordination, not just the kind of role considered by the entailment-based accounts. These constraints are reminiscent of selectional restrictions on predicates, of the sort we see in Chomsky’s famous Colorless green ideas sleep furiously.

This role of lexical semantics sheds light on the data introduced in Klecha (2011). He considers examples like (63) and (64):

(63) Julia: Someone might be waiting for me.
     Rebekah: #They must know you’re not there.

(64) Tim: There might be blood in there.
     Ezra: He must be the killer.

(63) is infelicitous if we understand they to be the someone whose existence Julia is speculating about. And “Ezra’s utterance in [(64)] cannot mean “If there’s blood in there, he must be the killer”.” (p.16). On the basis of such examples, Klecha concludes that there are modals, including must, that are obligatorily non-subordinated, that is, which “never undergo modal subordination”.

But I think this is the wrong conclusion to draw. Consider modal subordination with epistemic must in (65):

(65) [The speaker is a meteorologist talking about hurricane patterns in his locale, North Carolina, where the hurricanes almost always approach from the sea to the southeast.]
(a) Sometimes when a hurricane approaches at just the right angle, we get lucky and it veers back out to sea after hitting the coast here.

(b) The hurricane must lose power from the direct impact with the land mass, because it usually just peters out after that.

‘if a hurricane hits the coast here and veers out to sea, it must lose power from the impact with the land mass’

Epistemic must is arguably a special kind of evidential (von Fintel & Gillies 2010), so that its prejacent complement is understood to follow from the relevant body of evidence given by its modal base. Roberts (2019b) argues that must is indexically anchored to a salient agent who is privy to that body of evidence, and who is understood to be only committed to the truth of the prejacent conditional on the reliability of the evidence. In (65), the meteorologist is generalizing over her experience with hurricanes (as evidenced by the bare present tense throughout the discourse), using that as evidence for the conclusion she draws in (b). The only way to explain the felicitous resolution of the anaphora triggered by the hurricane is by taking must in (b) to be modally subordinate to the irrealis content considered in (a), as captured by the conditional paraphrase.

And now we can explain Klecha’s examples (63) and (64), as well. In each, the irrealis content in the first utterance is too weak to serve as evidence for drawing the strong conclusion in the second (since it involves the mere possibility operator might); moreover, that evidence is attributed to a different agent, so is not necessarily evidence that the second speaker would subscribe to as reliable. Unless we take Rebekah or Ezra to simply take the prejacent of might as actually true, their utterances are infelicitous. In fact, I can get such an interpretation, but then this is not modal subordination, but just involves globally accommodating that the relevant agent takes ‘someone is waiting for Julia’ (63) or ‘there is blood in there’ (64) to be true.

This brief sketch is just intended to argue, as with (49) and (50) above, that when we look more carefully into the lexical semantics of a modal auxiliary, and in particular the selectional restrictions on what can serve to restrict its domain, we can discover a variety of constraints on how its domain is restricted, and hence develop a more empirically adequate account of modal subordination generally.

So an adequate general account of modal subordination must in the end be complex, involving not only the resolution of an anaphoric domain presupposition, but the lexical semantics of the particular modal operators involved, a variety of pragmatic inferences, and constrained accommodation.

A warning: Do not confuse modal subordination with a closely related phenomenon, dubbed telescoping by Roberts (1987, 1989):

(66) Every frog that saw an insect ate it.
    #It was a fly.
(67) Every frog that saw an insect ate it.
    It disappeared forever.
As predicted by dynamic accounts of discourse anaphora, because the only potential antecedent of the right sort is not available in (66)—*an insect* is under the scope of *every frog*, and so the corresponding discourse referent “expires” with the closure of the scope of *every* in the first sentence—the second sentence is infelicitous in the context of utterance. But in (67), the anaphora is felicitous. One might be tempted to take the second sentence in (67) to involve a covert QAdv, perhaps ‘always’ or ‘typically’. But if such a covert operator is available in (67), why not in (66), since overt addition of one of these QAdvs to (66) makes it acceptable?

Roberts (1987, 1989) and Poesio & Zucchi (1992) show that the telescoping in (67) is much more tightly constrained than modal subordination with an overt QAdv. One of the main constraints is that all information predicated of a singular pronoun in such examples must be entirely generic, not specific to a particular instance or even pertaining to a particular event. To get the flavor of this constraint, consider (68), due to Partee (p.c. to Roberts 1987):

(68)  
   a. Each degree candidate walked to the stage.  
   b. He took his diploma from the Dean and returned to his seat.

Here, the continuation in (68b) is felicitous, and we understand it to be a continued description of what happened in each case. But the follow-up in (68c) is infelicitous:

   c. #He had overcome serious personal challenges to complete the degree.  
   d. They had overcome serious personal challenges to complete their degrees.

Roberts speculated that follow-ups like (c) are unacceptable because they do not sound like a plausible continued description of the generic scenario first sketched in (a). Note that the propositional content of (68c) itself is not problematic—one could imagine a class in which all the graduates had overcome such challenges, and the same proposition is explicitly expressed and felicitous in (68d)—it’s just that in (68c) it doesn’t sound suitably generic or describe the unfolding generic scenario. See Roberts (2005) for discussion of how this bears on so-called “number-neutral” theories of pronominal anaphora in discourse: the difference in acceptability between (c) and (d) argues that pronominal number does matter for anaphora resolution.

I still know of no adequate account of telescoping, which is reminiscent of universal instantiation in the predicate calculus. The point here is to caution that we should not conflate this phenomenon with modal subordination.

Another caution: Most authors in the relevant literature tend to focus on a relatively limited range of types of examples of modal subordination. Any adequate account of modal subordination should address the full range of cases considered in §1 above, and the general schema offered at the end of that section. Of course, some inclusions may be controversial, as we saw with (39) above. But if they are excluded from the range of phenomena accounted for, this needs to be on principled grounds.

In this connection, Geurts (2012) argues that most of the literature on modal subordination fails to consider an even broader class of relevant phenomena, and that restricting consideration to only the kinds of cases given above is misleading. Several of his cases arguably involve
telescoping, without considering the distinguishing features of such examples just discussed. Others involve QAdvs, which I have argued fall under the schema in §1 and the accounts discussed in the present section. The other type of example Geurts offers is illustrated by his (69), which arguably involves no modal element:

(69) Last week, all employees received a letter. Most of them read it right away.

He calls the phenomenon illustrated by this type of example “piggyback anaphora”, takes it to subsume modal subordination and telescoping cases as well, and argues that it can be explained as a species of Bridging (Clark 1975). I would characterize it as involving quantificational subordination, here with quantificational determiners all and most. The subject of the second sentence has a partitive interpretation (possible even without explicit of them), and the partitive complement them is understood to be the domain of the previous all, the weakly familiar set of employees. Since, as Geurts notes, the context already saliently entails that each of these individuals has received a letter, then a discourse referent for the salient weakly familiar letter received by the arbitrary employee is available in the restriction of most, licensing use of it in the quantifier’s scope. The parallel with the cases of modal subordination is quite clear.

There does seem to be a broader class of phenomena which include examples like (69). von Fintel (1994) and Roberts (1995) argue that all types of operators presuppose domain restriction: this includes quantificational determiners like those in (69) and even, per Roberts, Priorean tense operators (whose restriction serves as the Reference Time), as well as those considered in §1: modal auxiliaries, QAdvs, and the modal operators implicit in the lexical semantics of attitude predicates. In modal subordination, it is what we know about the restricting situations from prior context that makes the entities in them weakly familiar for the resolution of anaphora in the scope of the restricted operator, just as in (69).

So what’s special about modal subordination? Should we be talking instead about quantificational subordination? Yes and no. It’s useful to observe that modal subordination is just one subtype of a more general phenomenon of domain restriction. And since the most successful accounts sketched in this section are based on domain restriction, they tacitly acknowledge its relationship to this broader class of phenomena. But there are special features of interest in the modal cases: In restricting the domain of a modal, what’s involved isn’t a set of entities, as in (69), but of worlds or situations, which aren’t necessarily those that are candidates for current reality. And the information states corresponding to such irrealis circumstances (Brasoveanu 2010 etc.) are themselves complex, involving information about various other types of entities and relations among them, as well as of the relevant kinds of modal accessibility relations across worlds (as we saw in the discussion of (60)). So what’s of special interest, and what’s at issue in distinguishing between the anaphoric accounts on offer, is how we dynamically track all this complex information across discourse. It is a much more complex anaphoric phenomenon than most, and hence the puzzle more difficult and compelling.

§5. Conclusions and open questions

Discourse anaphora is fairly simple. Its resolution requires only that there be exactly one discourse referent (of the right type) which (i) is logically accessible (given scope constraints),
(ii) satisfies the anaphoric trigger’s descriptive content, and (iii) is more salient at the time of utterance than any other discourse referent satisfying the first two conditions. But discourse itself is quite complicated, its structure is generally at least in part implicit, and the shared information which bears on interpretation itself is both dynamic and rich.17 Nothing 
demonstrates the complexity of discourse better than the attested constraints on anaphora in discourse, and no anaphora is more subtly licensed and constrained than that displayed in the phenomenon of modal subordination.

Hence, modal subordination, wherein a pronoun or other presupposition trigger under the scope of a modal operator seems to take as antecedent a constituent which itself has narrow scope under a modal or other intensional operator in a previous clause (a scope island), serves as a very useful probe for the structure of discourse, for the investigation of domain restriction, and for theories of presupposition satisfaction and projection.

References:


17 Again, space precludes proper discussion of these claims. See Roberts (2917,2019) for discussion.


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