1. [10 pts.] Suppose you think the sentences *bu la bu* and *bu bu la bu* are generated by a context-free grammar with the following derivations:

Given just these two derivations, what probabilities would you estimate for this grammar?

- \( P(1 \rightarrow 2 \ 3 \ | \ 1) = \)
- \( P(1 \rightarrow 3 \ 2 \ | \ 1) = \)
- \( P(1 \rightarrow bu \ | \ 1) = \)
- \( P(1 \rightarrow la \ | \ 1) = \)
- \( P(2 \rightarrow 2 \ 3 \ | \ 2) = \)
- \( P(2 \rightarrow 3 \ 2 \ | \ 2) = \)
- \( P(2 \rightarrow bu \ | \ 2) = \)
- \( P(2 \rightarrow la \ | \ 2) = \)
- \( P(3 \rightarrow 2 \ 3 \ | \ 3) = \)
- \( P(3 \rightarrow 3 \ 2 \ | \ 3) = \)
- \( P(3 \rightarrow bu \ | \ 3) = \)
- \( P(3 \rightarrow la \ | \ 3) = \)

2. [10 pts.] What is the joint probability of the above two derivations given the grammar probabilities you estimated? (Show your work for partial credit.)
3. [10 pts.] Suppose you think the (same) sentences \textit{bu la bu} and \textit{bu bu la bu} are generated by a context-free grammar with the following derivations:

![Tree diagrams](image)

Given just these two derivations, what probabilities would you estimate for this grammar?

\[
\begin{align*}
P(1 \rightarrow 1 \ 3 \ | \ 1) = \\
P(1 \rightarrow 1 \ 2 \ | \ 1) = \\
P(1 \rightarrow bu \ | \ 1) = \\
P(1 \rightarrow la \ | \ 1) = \\
P(2 \rightarrow 1 \ 3 \ | \ 2) = \\
P(2 \rightarrow 1 \ 2 \ | \ 2) = \\
P(2 \rightarrow bu \ | \ 2) = \\
P(2 \rightarrow la \ | \ 2) = \\
P(3 \rightarrow 1 \ 3 \ | \ 3) = \\
P(3 \rightarrow 1 \ 2 \ | \ 3) = \\
P(3 \rightarrow bu \ | \ 3) = \\
P(3 \rightarrow la \ | \ 3) =
\end{align*}
\]

4. (a) [5 pts.] What is the joint probability of the above two derivations given the grammar probabilities you estimated? (Show your work for partial credit.)

(b) [5 pts.] Which of the grammars in Question 1 and Question 3 is more probable?