LING4400: Problem Set 2

Due via Carmen dropbox at 11:59 PM 9/26.

1. Using the predicates and operators defined in the lecture notes, draw **derivation trees** that identify the type of each of the following:

   (a) [2 pts.] $2 + 3 = 5$

   (b) [2 pts.] $\lambda_{y:e}\ y = 2 + 3$

   (c) [2 pts.] $|\lambda_{y:e}\ y = 2|$

   (d) [2 pts.] $\lambda_{s:\langle e,t\rangle}\ |s|$

   (e) [2 pts.] $\lambda_{s:\langle e,t\rangle}\ \lambda_{x:e}\ s\ x$

   (f) [2 pts.] $\lambda_{s:\langle e,t\rangle}\ |\lambda_{x:e}\ s\ x|$

2. (a) [3 pts.] Write a **generalized quantifier expression** using the propositional and generalized quantifier functions defined in the lecture notes, as well as predicates Volcano, Coastal and Country of type $\langle e, t\rangle$ and Contain of type $\langle e, \langle e, t\rangle\rangle$, with the same meanings as the sentence *Every country containing no volcano is coastal.* (Note that this sentence is different from the one shown in lecture notes 5 section 3!) It may help to draw the derivation tree first (which is the next problem).

   (b) [3 pts.] Draw a **derivation tree**, with branches corresponding to the notation rules defined in the lecture notes, for the expression you wrote in the previous problem.

3. Given the following world model:

$$D_e^M = \textbf{Laos}, \textbf{Mali}, \textbf{Peru}, \textbf{Pune}, \textbf{Togo}$$

$[\![\text{Coastal}]\!]^M =$

input		output
Laos	:	**False**
Mali	:	**False**
Peru	:	**True**
Pune	:	**True**
Togo	:	**True**

$[\![\text{Country}]\!]^M =$

input		output
Laos	:	**True**
Mali	:	**True**
Peru	:	**True**
Pune	:	**False**
Togo	:	**True**

   (a) [3 pts.] what is the **denotation** of the following expression:

$$[\![\ \lambda_{x:e}\ (\neg\ (\text{Coastal}\ x)) \wedge \text{Country}\ x\ ]\!]^M = ?$$

(b) [1 pts.] what is the **denotation** of the following expression:

$$[\![\mid \lambda_{x:e}\ (\neg\ (\mathsf{Coastal}\ x)) \wedge \mathsf{Country}\ x \mid]\!]^M = ?$$

4. [3 pts.] Draw a **world model** with no more than six objects that satisfies the following equation:

$$[\![\mathsf{ExactlyTwo}\ (\lambda_{x:e}\ \mathsf{Bag}\ x)\ (\lambda_{x:e}\ \mathsf{Half}\ (\lambda_{y:e}\ \mathsf{Blocks}\ y)\ (\lambda_{y:e}\ \mathsf{Contain}\ y\ x))\ ]\!]^M = \textbf{True}$$

   You may draw bags as circles and blocks as squares.

5. What is the **type** of each of the following expressions, assuming predicates Square and Circle of type $\langle e, t\rangle$ (it may help to draw derivation trees with branches corresponding to the notation rules defined in the lecture notes):

   (a) [3 pts.] $(\lambda_{s:\langle e,t\rangle}\ \mathsf{Most}\ (\lambda_{x:e}\ \mathsf{Square}\ x)\ s)$

   (b) [3 pts.] $(\lambda_{s:\langle e,t\rangle}\ \mathsf{Most}\ (\lambda_{x:e}\ \mathsf{Square}\ x)\ s)\ \mathsf{Circle}$

   (c) [3 pts.] $\mathsf{Most}\ (\lambda_{x:e}\ \mathsf{Square}\ x)$

6. [extra credit, replacing question 3 of Problem Set 1] Using the predicates and operators defined in the lecture notes, and assuming variables x and y and constant A are of type e, constant P is of type $\langle e, t\rangle$ and constant R is of type $\langle e, \langle e, t\rangle\rangle$, draw **derivation trees** that identify the type of each of the following:

   (a) [2 pts.] $\lambda_{p:t}\ \mathsf{Not}\ p$

   (b) [2 pts.] $\mathsf{R}\ \mathsf{A}$

   (c) [2 pts.] $\mathsf{R}\ x\ y$

   (d) [2 pts.] $\lambda_{x:e}\ \mathsf{Not}\ (\mathsf{P}\ x)$

   (e) [2 pts.] $\lambda_{p:t}\ \lambda_{q:t}\ \mathsf{And}\ (\mathsf{Not}\ p)\ q$

   (f) [2 pts.] $\lambda_{y:e}\ (\lambda_{x:e}\ \mathsf{P}\ x)\ y$

   (g) [2 pts.] $\mathsf{If}\ (\mathsf{P}\ x)\ \mathsf{True}$

   (h) [2 pts.] $\lambda_{y:e}\ \lambda_{x:e}\ \mathsf{R}\ y$