# LING4400: Study Guide 1

**Practice 2.1:**

How many functions of type $\langle e, t\rangle$ are there in a world with two e's: (**A**, **B**), and two t's?

$2^2 = 4$

**Practice 2.2:**

List all the possible functions of type $\langle e, t\rangle$ in a world with two e's: (**A**, **B**), and two t's.

| input | output |
|---|---|
| **A** | : **False** |
| **B** | : **False** |

| input | output |
|---|---|
| **A** | : **False** |
| **B** | : **True** |

| input | output |
|---|---|
| **A** | : **True** |
| **B** | : **False** |

| input | output |
|---|---|
| **A** | : **True** |
| **B** | : **True** |

**Practice 2.3:**

Write a lambda calculus function that multiplies a number by two and then adds one. You can use the symbols '×' and '+' inside your function.

$\lambda_{x:e}\ (2 \times x) + 1$

**Practice 2.4:**

Write a lambda calculus expression that applies your function above to the number 3. You don't have to show the result.

$(\lambda_{x:e}\ (2 \times x) + 1)\ 3$

**Practice 2.5:**

Beta reduce the following expression:

$$(\lambda_{x:\mathrm{e}}\ (\mathsf{And}\ (\mathsf{Coastal}\ x)\ (\mathsf{Capital}\ x)))\ \mathsf{Laos}$$

(And (Coastal Laos) (Capital Laos)))

**Practice 2.6:**

Beta reduce the following expression:

$$(\lambda_{y:\mathrm{e}}\ \lambda_{x:\mathrm{e}}\ \mathsf{Contain}\ y\ x)\ \mathsf{Laos}\ \mathsf{Asia}$$

Contain Laos Asia

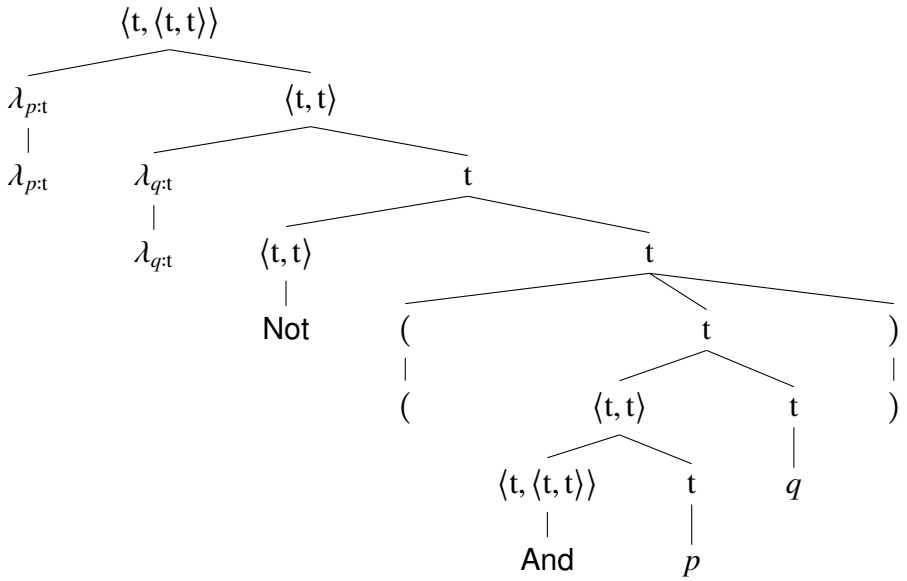
**Practice 3.1:**

What is the interpretation of the expression And True?

| input | output |
|---|---|
| **False** : | **False** |
| **True** : | **True** |

**Practice 3.2:**

Draw a derivation tree showing types for the expression $\lambda_{p:\mathrm{t}}\ \lambda_{q:\mathrm{t}}\ \mathsf{Not}\ (\mathsf{And}\ p\ q)$.




**Practice 3.3:**

Write an expression to produce the following truth table using conjunction and negation:

| input | output | |
|---|---|---|
| **False** : | input | output |
| | **False** : | **False** |
| | **True** : | **True** |
| **True** : | input | output |
| | **False** : | **False** |
| | **True** : | **False** |

$\lambda_{p:\mathrm{t}}\ \lambda_{q:\mathrm{t}}\ (\mathsf{And}\ (\mathsf{Not}\ p)\ q)$

**Practice 5.1: cardinality of functions**

Given the same denotations for Coastal and Country, what is the denotation of the following expression:

$$[\![\ |\ \lambda_x\ \mathsf{Coastal}\ x \vee \mathsf{Country}\ x\ |\ ]\!]^M$$

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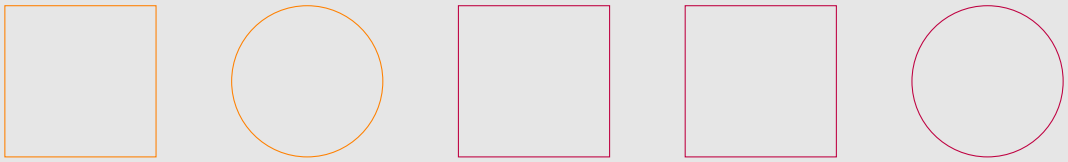
**Practice 5.2: meaning**

Given a world $M$ of Shape entities (where Purple and Square have their usual meanings):



what is the denotation of the following lambda calculus expression?

$$[\![ \text{Most } (\lambda_x \text{ Shape } x \wedge \text{Purple } x)\ (\lambda_x \text{ Square } x) ]\!]^M$$

true

**Practice 5.3: another meaning**

Given the same world of shapes above, what is the denotation of the following lambda calculus expression?

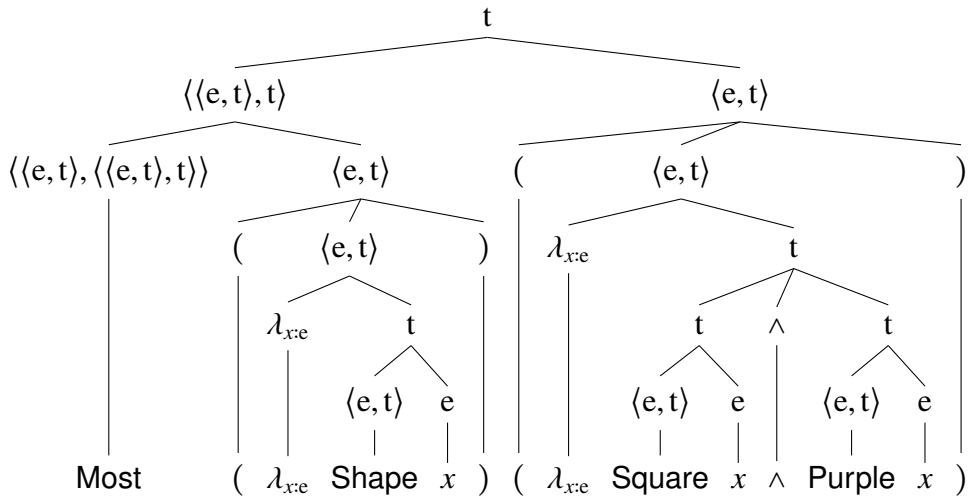
$$[\![ \text{Most } (\lambda_{x:e} \text{ Shape } x)\ (\lambda_{x:e} \text{ Square } x \wedge \text{Purple } x) ]\!]^M$$

false

**Practice 5.4: tree drawing**

Draw a derivation tree for the following expression:

$$\text{Most } (\lambda_{x:e} \text{ Shape } x)\ (\lambda_{x:e} \text{ Square } x \wedge \text{Purple } x)$$




**Practice 5.5:**

Classify the following as cardinal or proportional:

1. *one third*
2. *seven*

1. Proportional
2. Cardinal

**Practice 8.1:**

Assume a world model with two entities: (**A**, **B**), and two truth values.

Draw the truth table for the universal quantifier.

$[\![\text{Universal}]\!]^M =$

| input | | output |
|---|---|---|
| input | output | |
| **A** | : **False** | : **False** |
| **B** | : **False** | |
| input | output | |
| **A** | : **False** | : **False** |
| **B** | : **True** | |
| input | output | |
| **A** | : **True** | : **False** |
| **B** | : **False** | |
| input | output | |
| **A** | : **True** | : **True** |
| **B** | : **True** | |

**Practice 8.2:**

Translate this expression from first-order logic into English: $\forall_{x:\mathrm{e}}\ \text{City}\ x \rightarrow \text{Capital}\ x$.

*For every thing, if it is a city then it is a capital.*

or

*Every city is a capital.*

**Practice 8.3:**

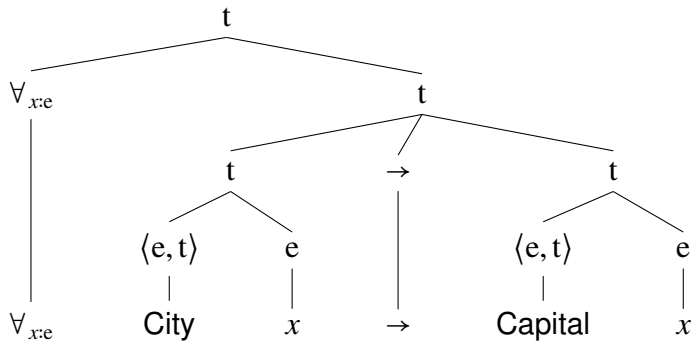
Write a logic expression using the propositional and first-order functions defined in the lecture notes, as well as constant Italy of type e and predicates Volcano of type $\langle \mathrm{e}, \mathrm{t} \rangle$ and Contain of type $\langle \mathrm{e}, \langle \mathrm{e}, \mathrm{t} \rangle \rangle$ stating that *Italy contains a volcano*.

$\exists_{x:\mathrm{e}}\ \text{Volcano}\ x \wedge \text{Contain}\ x\ \text{Italy}$

**Practice 8.4: tree drawing**

Draw a derivation tree for the following expression:

$$\forall_{x:\mathrm{e}}\ \text{City}\ x \rightarrow \text{Capital}\ x$$




**Practice 8.5: translating first-order quantifiers into generalized quantifiers**

Translate the below first-order quantified expression:

$$\forall_{y:e}\ \text{Booth}\ y \rightarrow \exists_{x:e}\ \text{Person}\ x \wedge \text{In}\ y\ x$$

into an expression using only generalized quantifiers Some and All, and predicates Booth, Person and In.

$$\text{All}\ (\lambda_{y:e}\ \text{Booth}\ y)\ (\text{Some}\ (\lambda_{x:e}\ \text{Person}\ x)\ (\text{In}\ y\ x))$$

**Practice 8.6:**

Which of the above classes do the following relations belong to?

1. *intersects*
2. *is next to*
3. *is larger than*

1. reflexive, symmetric, nontransitive
2. irreflexive, symmetric, nontransitive
3. irreflexive, asymmetric, transitive

**Practice 9.1:**

Which of the following are true:

1. $\{\text{Mali}, \text{Togo}\} \subseteq \{\text{Mali}, \text{Togo}\}$
2. $\{\text{Mali}, \text{Togo}\} \not\subset \{\text{Mali}, \text{Togo}\}$
3. $\varnothing \in \{\text{Mali}, \text{Togo}\}$

4. $\varnothing \subset \{\text{Mali, Togo}\}$

1. true
2. true
3. false
4. true

**Practice 9.2:**

Write an expression in ***set notation*** meaning *the set of all sets with no elements*.

$\{ s \mid |s| = 0 \}$

or

$\{ s \mid s = \varnothing \}$

or

$\{\varnothing\}$

**Practice 9.3:**

Write an expression in ***lambda calculus*** meaning *the set of all sets with no elements*.

$\lambda_{s:\langle e,t\rangle}\ s = (\lambda_{x:e}\ \text{False})$