LING5702: Lecture Notes 17
Quantifier Scope

The last step in obtaining complex ideas from sounds and gestures is quantifier scope.

Contents

17.1 Simple scope disambiguation [Schuler & Wheeler, 2014]

We'll assume the following constants (with a localist representation: referential states are δ_v):

1. $V \in \mathbb{R}$: a maximum number of referential states (variables in lambda calculus expressions);
2. $\mathbf{q} \in \{0, 1\}^V$: a vector of zeros or ones indicating if each referential state is a quantification;
3. $\mathbf{v} \in \mathbb{R}^V$: a vector of precedence ('readiness') values for each referential state, based on:
   (a) quantifier type (e.g. Each has low precedence, so it usually scopes last/highest)
   (b) participated-in predicates (e.g. y in In x y will scope higher than x)
   (c) order in sentence (this enforces a preference for in-situ scope)
4. $\mathbf{E}_n \in \mathbb{R}^{V \times V}$: a matrix of associations from functions to arguments numbered by n;

We'll also assume **inheritance** associations ('rin') from the lecture notes on sentence processing:

$$\mathbf{E}_{\text{rin}} = \mathbf{E}_1 \operatorname{diag}(\mathbf{q}) \, \mathbf{E}_2^\top$$

We'll need **closure** matrices directly associating states connected by any number of associations:

$$\mathbf{E}_{\text{P}} = \mathbf{I} + \sum_{n=1}^{N} \prod_{i=1}^{n} \sum_{\ell \in \{1,2,3,\dots\}} \mathbf{E}_\ell \operatorname{diag}(\mathbf{1}-\mathbf{q}) + \operatorname{diag}(\mathbf{1}-\mathbf{q}) \, \mathbf{E}_\ell^\top$$

$$\mathbf{E}_{\text{I}} = \mathbf{I} + \sum_{n=1}^{N} \prod_{i=1}^{n} \sum_{\ell \in \{\text{cin},\text{ein},\text{rin}\}} \mathbf{E}_\ell + \mathbf{E}_\ell^\top$$

First, initialize iteration-dependent variables:

1. $\mathbf{Q}_0 = \mathbf{0}^{V \times V}$: an initially empty matrix of immediate outscopings;
2. $\mathbf{P}_0 = \mathbf{E}_{\text{P}} + \mathbf{I} - \operatorname{diag}(\mathbf{E}_{\text{P}})$: a matrix of fully-connected partitions, starting with no inheritances;
3. $\mathbf{u}_0 = \sum_{v \text{ s.t. } v = \operatorname{argmax} \operatorname{diag}(\mathbf{v}) \, \mathbf{P}_0 \, \delta_v} \delta_v$: a vector of used referential states, starting with the readiest.

Then, for each iteration $i \in \{1, 2, 3, \ldots\}$ such that some states remain un-used ($\mathbf{u}_{i-1} \neq \mathbf{1}$):

1. $u_i = \text{argmax}\, \text{diag}(\mathbf{v})\, \underbrace{\text{diag}(\mathbf{1}{-}\mathbf{E}_\text{I}\,(\mathbf{1}{-}\mathbf{u}_{i-1}))}_{\text{not connected to unused}}\, (\mathbf{1}{-}\mathbf{Q}_{i-1}{}^\top \mathbf{1})$: get readiest used un-scoped state;
2. $\mathbf{P}_i = \mathbf{a}\,\mathbf{a}^\top + \underbrace{\mathbf{P}_{i-1}\, \text{diag}(\mathbf{1}{-}\mathbf{a})}_{\text{copy non-merged partitions}}$ where $\mathbf{a} = \mathbf{P}_{i-1}\, \mathbf{E}_\text{I}\, \delta_{u_i}$: merge partitions connected via u_i;
3. $v_i = \text{argmax}\, \text{diag}(\mathbf{v})\, \text{diag}(\mathbf{1}{-}\mathbf{u}_{i-1})\, \mathbf{P}_i\, \delta_{u_i}$: find readiest unused state in new partition;
4. $\mathbf{Q}_i = \mathbf{Q}_{i-1} + \delta_{v_i}\, \delta_{u_i}{}^\top \mathbf{E}_\text{I}$: associate referential states in scope matrix;
5. $\mathbf{u}_i = \mathbf{u}_{i-1} + \delta_{v_i}$: add v_i as used.

Participant and scope associations define lambda calculus expressions as described earlier.

17.2 Evidence for explicit scoping [Dotlačil & Brasoveanu, 2015]

It does seem that scope is explicitly calculated like this (i.e. doesn't remain underspecified):

- **stimuli:** sentences presented in eye-tracking:
  (a) *A caregiver comforted a child every night. The caregivers wanted the children to...*
  (b) *A caregiver comforted a child every night. The caregivers wanted the child to...*
  (c) *A caregiver comforted a child every night. The caregiver wanted the children to...*
  (d) *A caregiver comforted a child every night. The caregiver wanted the child to...*

  These analyses are eliminated at *caregiver*, but neither is the preferred in-situ analysis:

  All (λ_t Night t)
  (λ_t Some (λ_k Caregiver k)
  (λ_k Some (λ_c Child c)
  (λ_c Comfort t k c)))

  Some (λ_c Child c)
  (λ_c All (λ_t Night t)
  (λ_t Some (λ_k Caregiver k)
  (λ_k Comfort t k c)))

  The preferred in-situ (first) analysis is eliminated at *children*:

  Some (λ_k Caregiver k)
  (λ_k Some (λ_c Child c)
  (λ_c All (λ_t Night t)
  (λ_t Comfort t k c)))

  Some (λ_k Caregiver k)
  (λ_k All (λ_t Night t)
  (λ_t Some (λ_c Child c)
  (λ_c Comfort t k c)))

- **measure:** eye-tracking fixation durations at *children* (and spillover word).
- **results:** singular-plural (c) is slowest at *children*, suggests dynamic reanalysis there.

(We don't have a scope re-analysis model, though.)

References

[Dotlačil & Brasoveanu, 2015] Dotlačil, J. & Brasoveanu, A. (2015). The manner and time course of updating quantifier scope representations in discourse. *Language, Cognition and Neuroscience*, 30(3), 305–323.

[Schuler & Wheeler, 2014] Schuler, W. & Wheeler, A. (2014). Cognitive compositional semantics using continuation dependencies. In *Third Joint Conference on Lexical and Computational Semantics (*SEM'14)*.