LING5702: Problem Set 1
Due via Carmen dropbox at 11:59 PM 1/23.

1. Assume a probability space, as described in the lecture notes, over a seed you are planting, with outcomes:
   - $T$ that the plant turns out to be a tree
   - $S$ that the plant turns out to be a shrub
   - $G$ that the plant turns out to be green
   - $B$ that the plant turns out to be brown

Using the above outcomes . . .

(a) [3 pts.] Write a probability expression expressing that half the time your seed will turn out to be green.

(b) [3 pts.] Write a probability expression expressing that half the time your seed will turn out to be a green tree.

(c) [3 pts.] Write a probability expression expressing that if your seed turns out to be green, half the time it will be a tree.

2. Suppose you have the following probability model over a seed you are planting:
   - a sixth of the time you get a green tree
   - a third of the time you get a brown tree
   - a sixth of the time you get a green shrub
   - a third of the time you get a brown shrub

Using the above model . . .

(a) [3 pts.] What is the probability it will turn out to be a green plant (either a tree or a shrub)?

(b) [3 pts.] If you already know it will turn out to be green, what is the probability it will turn out to be a tree?
3. Using the generalized quantifier functions in the lecture notes on typed lambda calculus and the following predicates:
   - Tree \( x \), meaning that \( x \) is a tree
   - Green \( x \), meaning that \( x \) is green
   - Round \( x \), meaning that \( x \) is round
   - Park \( x \), meaning that \( x \) is a park area
   - In \( x \), meaning that \( x \) is in area \( y \)

   (a) [3 pts.] Write a typed lambda calculus expression stating that half the round trees are green.

   (b) [3 pts.] Write a typed lambda calculus expression stating that half the trees are green and round.

   (c) [3 pts. – difficult!] Write a typed lambda calculus expression stating that all the trees are in some park (possibly a different park for each tree).

4. [difficult!] Using the non-intensional and intensional quantifier functions from the lecture notes on typed lambda calculus:
   - \( \text{Ratio}_{\geq} n R S \), meaning that at least \( n \) (fraction) of \( R \) are in \( S \),
   - \( \text{Count}_{\geq} n R S \), meaning that at least \( n \) (instances) of \( R \) are in \( S \),
   - \( \text{IntensionOfCount}_{\geq} i n R S \), meaning that \( i \) is an intension that at least \( n \) of \( R \) are in \( S \),

   and the following predicates:
   - Kid \( x \), meaning that \( x \) is a kid,
   - Horse \( y \), meaning that \( y \) is a horse,
   - Time \( t \), meaning that \( t \) is a point in time,
   - Ride \( t x y \), meaning that \( x \) rides \( y \) at time \( t \),
   - Want \( t x i \), meaning that \( x \) wants intension \( i \) to be true at time \( t \),

   write a typed lambda calculus expression stating that:

   (a) [2pts.] Every kid wants to ride a horse at some point in time (but they don’t care which horse or when);
(b) [2pts.] For every kid there is a particular real horse that they want to ride at some point in time (but they don’t care when, and it may be a different horse for each kid).

(c) [2pts.] There is a single (presumably famous) horse that every kid wants to ride at some point in time;

Note: only quantifier functions can take lambda functions \((\lambda x \ldots)\) as arguments; all the predicates can take only entity variables as arguments.