Ling 5801: Lecture Notes 20
Cued Association Semantics

Natural language sometimes uses complex generalizations over sets of objects.

20.1 Start with Generalized Quantifiers (Barwise & Cooper, 1981)

Here's a simple generalized quantifier analysis for the sentence *Most cars have four wheels*:

$$
\begin{array}{l}
(\text{most } (\lambda_{u_2} \text{ car } u_2) \\
\quad (\lambda_{v_2} \text{ four } (\lambda_{u_5} \text{ wheel } u_5) \\
\qquad (\lambda_{v_5} \text{ have } v_2\ v_5)))
\end{array}
$$

It works like this python program:

```
U = ['c1','c2','c3','w1','w2','w3','w4','w5','w6','w7','w8']

def most(f,g):
  count,count2 = 0,0
  for u in U:
    if f(u): count+=1
    if f(u) and g(u): count2+=1
  return count2 > 0.5*count

def four(f,g):
  count = 0
  for u in U:
    if f(u) and g(u): count+=1
  return count > 4

def car(u): return u in ['c1','c2','c3']

def wheel(u): return u in ['w1','w2','w3','w4','w5','w6','w7','w8']

def have(u,v): return (u,v) in [('c1','w1'),('c1','w2'),('c1','w3'),('c1','w4'),
                                ('c2','w5'),('c2','w6'),('c2','w7'),('c2','w8')]

most( lambda u2: car(u2), lambda v2: four( lambda u5: wheel(u5), lambda v5: have(v2,v5) ) )
```

Here the variables have clear meanings (indices just identify source word by token number):

- u_2 is a variable over cars;
- v_2 is a variable over things that have four wheels;
- u_5 is a variable over wheels;
- v_5 is a variable for each car v_2 over things that car has.

20.2 Eventualities (Davidson, 1967; Parsons, 1990)

Unfortunately this doesn't admit modification yet: *Most cars have four wheels today.*
Fix this by adding variable u_3 over eventualities (where U is the universe of discourse):

$$
\begin{array}{l}
(\text{most } (\lambda_{u_2} \text{ car } u_2) \\
\quad (\lambda_{v_2} \text{ four } (\lambda_{u_5} \text{ wheel } u_5) \\
\qquad (\lambda_{v_5} \text{ some } (\lambda_{u_3} \text{ have } u_3\ v_2\ v_5 \wedge \text{today } u_3)\ U)))
\end{array}
$$

Add eventuality/proposition vars for other predicates (call them e_2, e_5 when u's are taken):

$$\begin{array}{l}(\textsf{most}\ (\lambda_{u_2}\ \textsf{some}\ (\lambda_{e_2}\ \textsf{car}\ e_2\ u_2)\ U)\\ \quad (\lambda_{v_2}\ \textsf{four}\ (\lambda_{u_5}\ \textsf{some}\ (\lambda_{e_5}\ \textsf{wheel}\ e_5\ u_5)\ U)\\ \qquad (\lambda_{v_5}\ \textsf{some}\ (\lambda_{u_3}\ \textsf{have}\ u_3\ v_2\ v_5 \wedge \textsf{some}\ (\lambda_{u_6}\ \textsf{today}\ u_6\ u_3)\ U)\ U)))\end{array}$$

Now we can think of eventualities/propositions as objects (so let's give predicates gerundy names):

$$\begin{array}{l}(\textsf{most}\ (\lambda_{u_2}\ \textsf{some}\ (\lambda_{e_2}\ \textsf{beingACar}\ e_2\ u_2)\ U)\\ \quad (\lambda_{v_2}\ \textsf{four}\ (\lambda_{u_5}\ \textsf{some}\ (\lambda_{e_5}\ \textsf{beingAWheel}\ e_5\ u_5)\ U)\\ \qquad (\lambda_{v_5}\ \textsf{some}\ (\lambda_{u_3}\ \textsf{having}\ u_3\ v_2\ v_5 \wedge \textsf{some}\ (\lambda_{u_6}\ \textsf{beingToday}\ u_6\ u_3)\ U)\ U)))\end{array}$$

The new variables still have (mostly) clear meanings:

- e_2 is a variable for each car u_2 over instances of it being a car (say it's wrecked and recycled);
- e_5 is a variable for each wheel u_5 over instances of it being a wheel (it used to be a doorstop);
- u_3 is a variable for each car v_2 and wheel v_5 over instances of v_2 having v_5;
- u_6 is... not so important (a variable over propositions, mainly just there for consistency).

These extra variables let us quantify over things that happen — *Most cars work twice*:

$$\begin{array}{l}(\textsf{most}\ (\lambda_{u_2}\ \textsf{some}\ (\lambda_{e_2}\ \textsf{beingACar}\ e_2\ u_2)\ U)\\ \quad (\lambda_{v_2}\ \textsf{two}\ (\lambda_{u_3}\ \textsf{working}\ u_3\ v_2)\ U))\end{array}$$

and negate them — *Most cars do not work*:

$$\begin{array}{l}(\textsf{most}\ (\lambda_{u_2}\ \textsf{some}\ (\lambda_{e_2}\ \textsf{beingACar}\ e_2\ u_2)\ U)\\ \quad (\lambda_{v_2}\ \textsf{none}\ (\lambda_{u_3}\ \textsf{working}\ u_3\ v_2)\ U))\end{array}$$

Hobbs (1985) calls this 'reification.'

We can reify quantifier predicates as well (but then those propositions will need quantifiers, etc. . .).

20.3 Cognitive States and Cued Associations (Anderson et al., 1977)

How is meaning implemented in the brain?
We have durable, rapidly-mutable weight-based ('associative') memory (Howard & Kahana, 2002).
This associates cue and target referential cognitive states (generalizations over stimuli), w/o rehersal.
We use it to store sentence meaning, not form (Sachs, 1967; Jarvella, 1971; Bransford & Franks, 1971).

We have cognitive states u_t,v_t and associative memory M, such that:

$$M \stackrel{\text{def}}{=} \sum_t v_t \otimes u_t$$

where: $(v \otimes u)_{[i,j]} \stackrel{\text{def}}{=} v_{[i]} \cdot u_{[j]}$

The associative memory can then be queried using u_t to get (an approximation of) v_t:

$$v_t \approx M\ u_t$$

where: $(M\ u)_{[i]} \stackrel{\text{def}}{=} \sum_{j=1}^{J} M_{[i,j]} \cdot u_{[j]}$

Model dependency relations r_t with label ℓ_t between cue and target u_t and v_t:

$$M \stackrel{\text{def}}{=} \sum_t v_t \otimes r_t + r_t \otimes \ell_t + r_t \otimes u_t$$

Cue with diagonal product:

$$v_t \approx M\ \text{diag}(M\ \ell_t)\ M\ u_t$$

where: $(\text{diag}(v)\ u)_{[i]} \stackrel{\text{def}}{=} v_{[i]} \cdot u_{[i]}$

or more simply (ignoring the approximation):

$$v_t = (\mathbf{f}_{\ell_t}\ u_t)$$

20.4 Semantic Argument Dependencies and Elementary Prediations

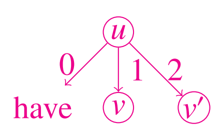
Define **semantic argument dependencies** in elementary predications (Copestake et al., 2005):

$$(f\ u\ v\ v'\ v''\ \ldots) \Leftrightarrow (\mathbf{f}_0\ u){=}\mathbf{v}_f \wedge (\mathbf{f}_1\ u){=}v \wedge (\mathbf{f}_2\ u){=}v' \wedge (\mathbf{f}_3\ u){=}v'' \wedge \ldots$$

For example, to define an eventuality u:

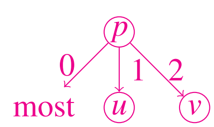
$$(\text{have}\ u\ v\ v') \Leftrightarrow (\mathbf{f}_0\ u){=}\mathbf{v}_{\text{have}} \wedge (\mathbf{f}_1\ u){=}v \wedge (\mathbf{f}_2\ u){=}v'$$

Graphically:



Also, quantifiers (where p is a proposition):

$$(\text{most}\ p\ u\ v) \Leftrightarrow (\mathbf{f}_0\ p){=}\mathbf{v}_{\text{most}} \wedge (\mathbf{f}_1\ p){=}u \wedge (\mathbf{f}_2\ p){=}v$$

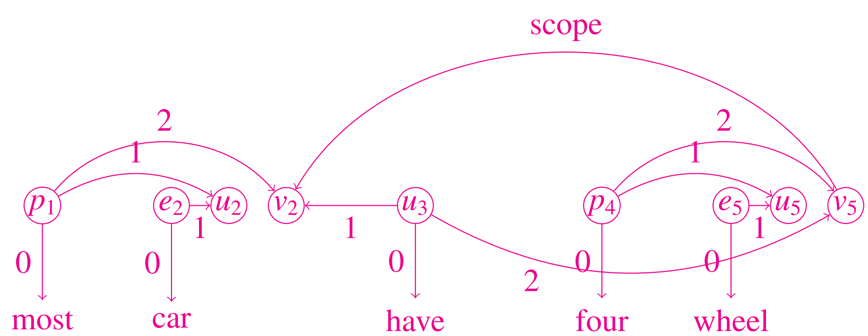


20.5 Scope Dependencies (Schuler & Wheeler, 2014)

Also use cued associations to define **scope dependencies**:

$$(\mathbf{f}_{\text{scope}}\ v){=}v'$$

Now we can define entire expressions, e.g. *Cars usually have four wheels*:



20.6 Simple Translation to Lambda Calculus (Schuler & Wheeler, 2014)

Translate source predications Γ into lambda expression Δ (w. Q as a set of quantifier functions):

1. Add a lambda term to Γ for each predication in Γ with no outscoped variables:

$$\frac{\Gamma, (f\ v_0\ v_1\ ..\ v\ ..\ v_N)\ ;\ \Delta}{\Gamma, (\lambda_v\ f\ v_0\ v_1\ ..\ v\ ..\ v_N)\ ;\ \Delta}\ f \notin Q,\ \forall_u\ (\mathbf{f}_{\text{scope}}\ u){=}v \notin \Gamma \qquad \text{(P)}$$

2. Conjoin lambda terms over the same variable in Γ (this combines modifier predications):

$$\frac{\Gamma, (\lambda_v\ \phi), (\lambda_v\ \psi)\ ;\ \Delta}{\Gamma, (\lambda_v\ \phi \wedge \psi)\ ;\ \Delta} \qquad \text{(C)}$$

3. Move terms in Γ with no missing predications or outscoped variables to Δ:

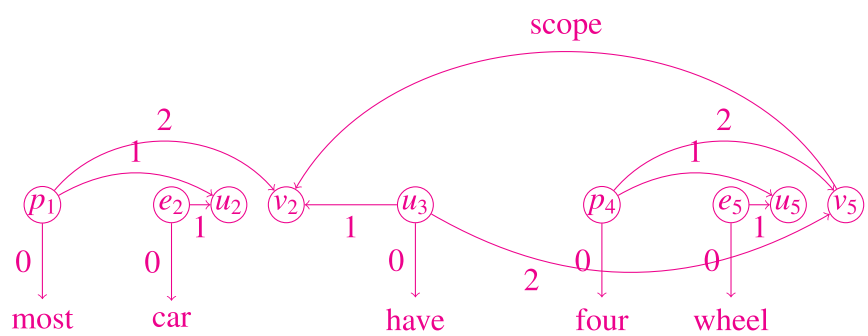
$$\frac{\Gamma, (\lambda_v\ \psi)\ ;\ \Delta}{\Gamma\ ;\ (\lambda_v\ \psi), \Delta}\ \forall_{f \notin Q}\ (f\ ..\ v\ ..) \notin \Gamma,\ \forall_u\ (\mathbf{f}_{\text{scope}}\ u){=}v \notin \Gamma \qquad \text{(M)}$$

4. Add translations τ_f of quantifiers f in Γ over complete lambda terms in Δ:

$$\frac{\Gamma, (f\ p\ u\ v)\ ;\ (\lambda_u\ \phi), (\lambda_v\ \psi), \Delta}{\Gamma, (\tau_f\ (\lambda_u\ \phi)\ (\lambda_v\ \psi))\ ;\ (\lambda_u\ \phi), (\lambda_v\ \psi), \Delta}\ f \in Q \qquad \text{(Q1)}$$

$$\frac{\Gamma, (\mathbf{f}_{\text{scope}}\ v){=}v', (\tau_f\ (\lambda_u\ \phi)\ (\lambda_v\ \psi))\ ;\ \Delta}{\Gamma, (\lambda_{v'}\ \tau_f\ (\lambda_u\ \phi)\ (\lambda_v\ \psi))\ ;\ \Delta} \qquad \text{(Q2)}$$

For example, our graph:



which consists of the following elementary predications and scope dependencies:

$$(\text{most}\ p_1\ u_2\ v_2), (\mathbf{f}_{\text{scope}}\ v_5){=}v_2, (\text{four}\ p_4\ u_5\ v_5), (\text{car}\ e_2\ u_2), (\text{wheel}\ e_5\ u_5), (\text{have}\ u_3\ v_2\ v_5)$$

translates like this (assume unbound eventuality variables have low existential scope):

$$(\text{most}\ p_1\ u_2\ v_2), (\mathbf{f}_{\text{scope}}\ v_5){=}v_2, (\text{four}\ p_4\ u_5\ v_5), (\text{car}\ e_2\ u_2), (\text{wheel}\ e_5\ u_5), (\text{have}\ u_3\ v_2\ v_5)\ ;$$

$$P\ (\text{most}\ p_1\ u_2\ v_2), (\mathbf{f}_{\text{scope}}\ v_5){=}v_2, (\text{four}\ p_4\ u_5\ v_5), (\lambda_{u_2}\ \text{car}\ e_2\ u_2), (\lambda_{u_5}\ \text{wheel}\ e_5\ u_5), (\lambda_{v_5}\ \text{have}\ u_3\ v_2\ v_5)\ ;$$

$$M\ (\text{most}\ p_1\ u_2\ v_2), (\mathbf{f}_{\text{scope}}\ v_5){=}v_2, (\text{four}\ p_4\ u_5\ v_5)\ ;\ (\lambda_{u_2}\ \text{car}\ e_2\ u_2), (\lambda_{u_5}\ \text{wheel}\ e_5\ u_5), (\lambda_{v_5}\ \text{have}\ u_3\ v_2\ v_5)$$

$$Q1\ (\text{most}\ p_1\ u_2\ v_2), (\mathbf{f}_{\text{scope}}\ v_5){=}v_2, (\text{four}\ (\lambda_{u_5}\ \text{wheel}\ e_5\ u_5)\ (\lambda_{v_5}\ \text{have}\ u_3\ v_2\ v_5))\ ;\ (\lambda_{u_2}\ \text{car}\ e_2\ u_2), (\lambda_{u_5}\ ...), (\lambda_{v_5}\ ...)$$

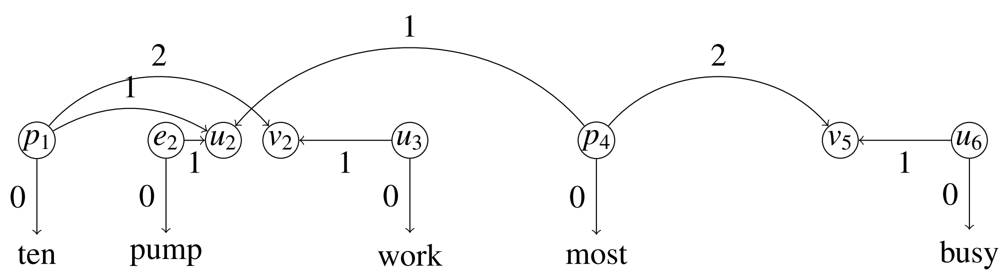
$$Q2\ (\text{most}\ p_1\ u_2\ v_2), (\lambda_{v_2}\ \text{four}\ (\lambda_{u_5}\ \text{wheel}\ e_5\ u_5)\ (\lambda_{v_5}\ \text{have}\ u_3\ v_2\ v_5))\ ;\ (\lambda_{u_2}\ \text{car}\ e_2\ u_2), (\lambda_{u_5}\ ...), (\lambda_{v_5}\ ...)$$

$$M\ (\text{most}\ p_1\ u_2\ v_2)\ ;\ (\lambda_{v_2}\ \text{four}\ (\lambda_{u_5}\ \text{wheel}\ e_5\ u_5)\ (\lambda_{v_5}\ \text{have}\ u_3\ v_2\ v_5)), (\lambda_{u_2}\ \text{car}\ e_2\ u_2), (\lambda_{u_5}\ ...), (\lambda_{v_5}\ ...)$$

$$Q1\ (\text{most}\ (\lambda_{u_2}\ \text{car}\ e_2\ u_2)\ (\lambda_{v_2}\ \text{four}\ (\lambda_{u_5}\ \text{wheel}\ e_5\ u_5)\ (\lambda_{v_5}\ \text{have}\ u_3\ v_2\ v_5)))\ ;\ (\lambda_{u_2}\ ...), (\lambda_{u_5}\ ...), (\lambda_{v_5}\ ...)$$

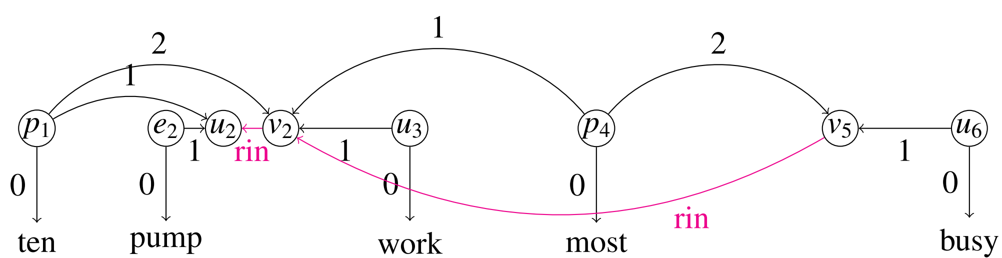
20.7 Restriction Inheritance

We need references to previous restrictor sets: *Ten pumps work. Most of them are busy.*



But preferred reading is often reference to restrictor ∩ nuclear scope sets (*pumps that work*).

We can handle this with inheritance dependencies (‘restriction inheritance,’ rin):

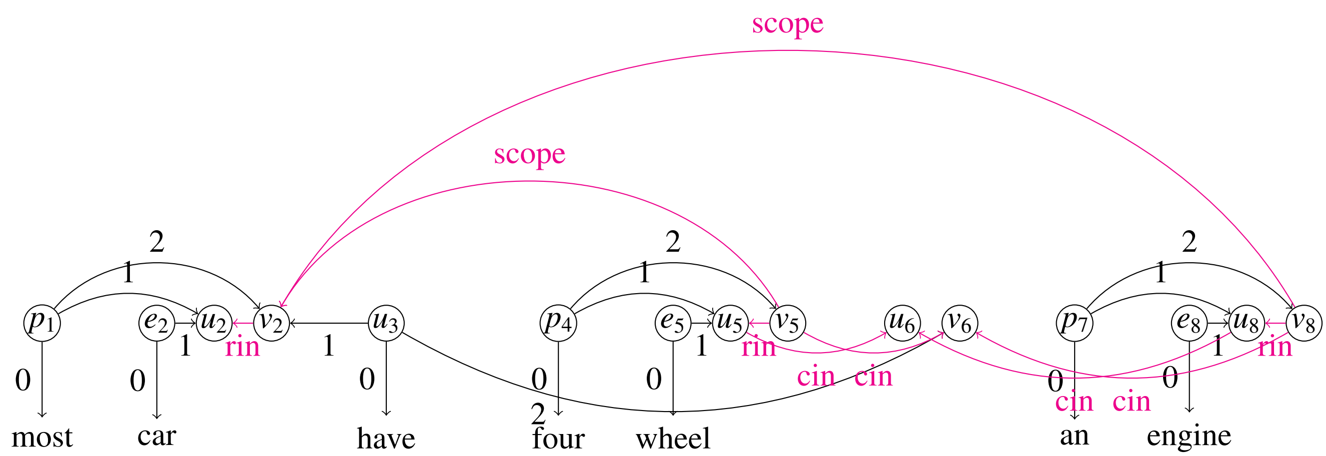


Now v_2 is a variable over the set of *pumps that work*, not just *things that work*.

This is valid because most natural language quantifiers are *conservative*: $Q(R, S) = Q(R, R \cap S)$.

20.8 Conjunction Inheritance

Conjunctions also require inheritance ('conjunction inheritance,' cin):



20.9 Full Translation to Lambda Calculus

Translate source predications Γ into lambda expression Δ (w. Q as a set of quantifier functions):

1. Add a lambda term to Γ for each predication in Γ with no outscoped variables or inheritances:

$$\frac{\Gamma, (f\ v_0\ v_1\ ..\ v\ ..\ v_N)\ ;\ \Delta}{\Gamma, (\lambda_v\ f\ v_0\ v_1\ ..\ v\ ..\ v_N)\ ;\ \Delta}\ f \notin Q,\ \forall_u\ (\mathbf{f}_{\text{scope}}\ u){=}v \notin \Gamma,\ \forall_{f \in \{\mathbf{f}_{\text{rin}},\mathbf{f}_{\text{cin}},\mathbf{f}_{\text{uin}}\}}\ (f\ v){=}u \notin \Gamma \qquad \text{(P)}$$

2. Conjoin lambda terms over the same variable in Γ (this combines modifier predications):

$$\frac{\Gamma, (\lambda_v\ \phi), (\lambda_v\ \psi)\ ;\ \Delta}{\Gamma, (\lambda_v\ \phi \wedge \psi)\ ;\ \Delta} \qquad \text{(C)}$$

3. Move terms in Γ with no missing predications, outscoped variables or inheritances to Δ:

$$\frac{\Gamma, (\lambda_v\ \psi)\ ;\ \Delta}{\Gamma\ ;\ (\lambda_v\ \psi), \Delta}\ \forall_{f' \notin Q}\ (f' ..\ v\ ..) \notin \Gamma,\ \forall_u\ (\mathbf{f}_{\text{scope}}\ u){=}v \notin \Gamma,\ \forall_{f \in \{\mathbf{f}_{\text{rin}},\mathbf{f}_{\text{cin}},\mathbf{f}_{\text{uin}}\}}\ (f\ v){=}u \notin \Gamma \qquad \text{(M)}$$

4. Add translations τ_f of quantifiers f in Γ over complete lambda terms in Δ:

$$\frac{\Gamma, (f\ p\ u\ v)\ ; (\lambda_u\ \phi), (\lambda_v\ \psi), \Delta}{\Gamma, (\tau_f\ (\lambda_u\ \phi)\ (\lambda_v\ \psi))\ ; (\lambda_u\ \phi), (\lambda_v\ \psi), \Delta}\ f \in Q,\ \forall_{f' \in \{\mathbf{f}_{\text{rin}},\mathbf{f}_{\text{cin}},\mathbf{f}_{\text{uin}}\}}\ (f' ..){=}v \notin \Gamma,\ \forall_{f' \in Q}\ (f' ..\ v\ ..) \notin \Gamma \quad \text{(Q1)}$$

$$\frac{\Gamma, (\mathbf{f}_{\text{scope}}\ v){=}v', (\tau_f\ (\lambda_u\ \phi)\ (\lambda_v\ \psi))\ ;\ \Delta}{\Gamma, (\lambda_{v'}\ \tau_f\ (\lambda_u\ \phi)\ (\lambda_v\ \psi))\ ;\ \Delta}\ \forall_{u'}\ (\mathbf{f}_{\text{scope}}\ u){=}u' \notin \Gamma \tag{Q2}$$

$$\frac{\Gamma, (\mathbf{f}_{\text{scope}}\ u){=}u', (\mathbf{f}_{\text{scope}}\ v){=}v', (\tau_f\ (\lambda_u\ \phi)\ (\lambda_v\ \psi))\ ;\ \Delta}{\Gamma, (\mathbf{f}_{\text{scope}}\ u){=}u', (\lambda_{v'}\ \tau_f\ (\lambda_u\ (\lambda_{u'}\ \phi)\ v')\ (\lambda_v\ \psi))\ ;\ \Delta} \tag{Q3}$$

5. Add a lambda term to Γ for each inheritance that is empty or from complete term in Δ:

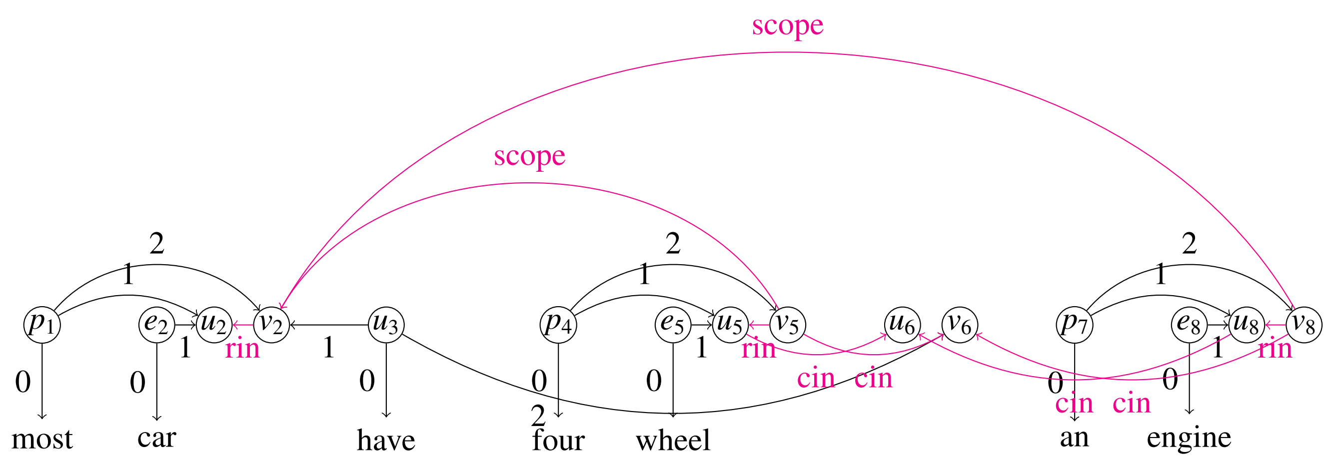
$$\frac{\Gamma, (f\ v){=}u\ ;\ \Delta}{\Gamma, (f\ v){=}u, (\lambda_u\ \mathsf{True})\ ;\ \Delta}\ f \in \{\mathbf{f}_{\text{rin}}, \mathbf{f}_{\text{cin}}\},\ \forall_{f' \notin Q}\ (f' ..\ u\ ..) \notin \Gamma \tag{I1}$$

$$\frac{\Gamma, (f\ v){=}u\ ;\ (\lambda_u\ \phi), \Delta}{\Gamma, (\lambda_v\ (\lambda_u\ \phi)\ v)\ ;\ (\lambda_u\ \phi), \Delta}\ f \in \{\mathbf{f}_{\text{rin}}, \mathbf{f}_{\text{cin}}\},\ \forall_{u'}\ (\mathbf{f}_{\text{scope}}\ u){=}u' \notin \Gamma \tag{I2}$$

$$\frac{\Gamma, (\mathbf{f}_{\text{scope}}\ u){=}u', (f\ v){=}u\ ;\ (\lambda_u\ \phi), \Delta}{\Gamma, (\mathbf{f}_{\text{scope}}\ u){=}u', (\mathbf{f}_{\text{scope}}\ v){=}u', (\lambda_v\ (\lambda_u\ \phi)\ v)\ ;\ (\lambda_u\ \phi), \Delta}\ f \in \{\mathbf{f}_{\text{rin}}, \mathbf{f}_{\text{cin}}\},\ \forall_{v'}\ (\mathbf{f}_{\text{scope}}\ v){=}v' \notin \Gamma \tag{I3}$$

$$\frac{\Gamma, (\mathbf{f}_{\text{scope}}\ u){=}u', (\mathbf{f}_{\text{scope}}\ v){=}v', (f\ v){=}u\ ;\ (\lambda_u\ \phi), \Delta}{\Gamma, (\mathbf{f}_{\text{scope}}\ u){=}u', (\mathbf{f}_{\text{scope}}\ v){=}v', (\lambda_v\ (\lambda_u\ (\lambda_{u'}\ \phi)\ v')\ v)\ ;\ (\lambda_u\ \phi), \Delta}\ f \in \{\mathbf{f}_{\text{rin}}, \mathbf{f}_{\text{cin}}\} \tag{I4}$$

For example, our graph:



which consists of the following elementary predications and scope dependencies:

$(\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\mathbf{f}_\mathrm{s}\ v_5){=}v_2, (\mathbf{f}_\mathrm{s}\ v_8){=}v_2, (\mathsf{f}\ p_4\ u_5\ v_5), (\mathsf{a}\ p_7\ u_8\ v_8), (\mathbf{f}_\mathrm{r}\ v_5){=}u_5, (\mathbf{f}_\mathrm{r}\ v_8){=}u_8, (\mathbf{f}_\mathrm{c}\ v_5){=}v_6, (\mathbf{f}_\mathrm{c}\ v_8){=}v_6, (\mathsf{w}\ e_5\ u_5), (\mathsf{e}\ e_8\ u_8), (\mathbf{f}_\mathrm{c}\ u_5){=}u_6, (\mathbf{f}_\mathrm{c}\ u_8){=}u_6, (\mathsf{c}\ e_2\ u_2), (\mathsf{h}\ e_3\ v_2\ v_6)$

translates like this (assume unbound eventuality variables have low existential scope):

$(\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\mathbf{f}_\mathrm{s}\ v_5){=}v_2, (\mathbf{f}_\mathrm{s}\ v_8){=}v_2, (\mathsf{f}\ p_4\ u_5\ v_5), (\mathsf{a}\ p_7\ u_8\ v_8), (\mathbf{f}_\mathrm{r}\ v_5){=}u_5, (\mathbf{f}_\mathrm{r}\ v_8){=}u_8, (\mathbf{f}_\mathrm{c}\ v_5){=}v_6, (\mathbf{f}_\mathrm{c}\ v_8){=}v_6, (\mathsf{w}\ e_5\ u_5), (\mathsf{e}\ e_8\ u_8), (\mathbf{f}_\mathrm{c}\ u_5){=}u_6, (\mathbf{f}_\mathrm{c}\ u_8){=}u_6, (\mathsf{c}\ e_2\ u_2), (\mathsf{h}\ e_3\ v_2\ v_6)\ ;$

$P\ (\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\mathbf{f}_\mathrm{s}\ v_5){=}v_2, (\mathbf{f}_\mathrm{s}\ v_8){=}v_2, (\mathsf{f}\ p_4\ u_5\ v_5), (\mathsf{a}\ p_7\ u_8\ v_8), (\mathbf{f}_\mathrm{r}\ v_5){=}u_5, (\mathbf{f}_\mathrm{r}\ v_8){=}u_8, (\mathbf{f}_\mathrm{c}\ v_5){=}v_6, (\mathbf{f}_\mathrm{c}\ v_8){=}v_6, (\mathsf{w}\ e_5\ u_5), (\mathsf{e}\ e_8\ u_8), (\mathbf{f}_\mathrm{c}\ u_5){=}u_6, (\mathbf{f}_\mathrm{c}\ u_8){=}u_6, (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6)\ ;$

$I1\ (\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\mathbf{f}_\mathrm{s}\ v_5){=}v_2, (\mathbf{f}_\mathrm{s}\ v_8){=}v_2, (\mathsf{f}\ p_4\ u_5\ v_5), (\mathsf{a}\ p_7\ u_8\ v_8), (\mathbf{f}_\mathrm{r}\ v_5){=}u_5, (\mathbf{f}_\mathrm{r}\ v_8){=}u_8, (\mathbf{f}_\mathrm{c}\ v_5){=}v_6, (\mathbf{f}_\mathrm{c}\ v_8){=}v_6, (\mathsf{w}\ e_5\ u_5), (\mathsf{e}\ e_8\ u_8), (\mathbf{f}_\mathrm{c}\ u_5){=}u_6, (\mathbf{f}_\mathrm{c}\ u_8){=}u_6, (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)\ ;$

$M\ (\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\mathbf{f}_\mathrm{s}\ v_5){=}v_2, (\mathbf{f}_\mathrm{s}\ v_8){=}v_2, (\mathsf{f}\ p_4\ u_5\ v_5), (\mathsf{a}\ p_7\ u_8\ v_8), (\mathbf{f}_\mathrm{r}\ v_5){=}u_5, (\mathbf{f}_\mathrm{r}\ v_8){=}u_8, (\mathbf{f}_\mathrm{c}\ v_5){=}v_6, (\mathbf{f}_\mathrm{c}\ v_8){=}v_6, (\mathsf{w}\ e_5\ u_5), (\mathsf{e}\ e_8\ u_8), (\mathbf{f}_\mathrm{c}\ u_5){=}u_6, (\mathbf{f}_\mathrm{c}\ u_8){=}u_6\ ;\ (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

$I2\ (\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\mathbf{f}_\mathrm{s}\ v_5){=}v_2, (\mathbf{f}_\mathrm{s}\ v_8){=}v_2, (\mathsf{f}\ p_4\ u_5\ v_5), (\mathsf{a}\ p_7\ u_8\ v_8), (\mathbf{f}_\mathrm{r}\ v_5){=}u_5, (\mathbf{f}_\mathrm{r}\ v_8){=}u_8, (\mathbf{f}_\mathrm{c}\ v_5){=}v_6, (\mathbf{f}_\mathrm{c}\ v_8){=}v_6, (\mathsf{w}\ e_5\ u_5), (\mathsf{e}\ e_8\ u_8), (\lambda_{u_5}\ \top), (\lambda_{u_8}\ \top)\ ;\ (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

$P\ (\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\mathbf{f}_\mathrm{s}\ v_5){=}v_2, (\mathbf{f}_\mathrm{s}\ v_8){=}v_2, (\mathsf{f}\ p_4\ u_5\ v_5), (\mathsf{a}\ p_7\ u_8\ v_8), (\mathbf{f}_\mathrm{r}\ v_5){=}u_5, (\mathbf{f}_\mathrm{r}\ v_8){=}u_8, (\mathbf{f}_\mathrm{c}\ v_5){=}v_6, (\mathbf{f}_\mathrm{c}\ v_8){=}v_6, (\lambda_{u_5}\ \mathsf{w}\ e_5\ u_5), (\lambda_{u_8}\ \mathsf{e}\ e_8\ u_8), (\lambda_{u_5}\ \top), (\lambda_{u_8}\ \top)\ ;\ (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

$C\ (\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\mathbf{f}_\mathrm{s}\ v_5){=}v_2, (\mathbf{f}_\mathrm{s}\ v_8){=}v_2, (\mathsf{f}\ p_4\ u_5\ v_5), (\mathsf{a}\ p_7\ u_8\ v_8), (\mathbf{f}_\mathrm{r}\ v_5){=}u_5, (\mathbf{f}_\mathrm{r}\ v_8){=}u_8, (\mathbf{f}_\mathrm{c}\ v_5){=}v_6, (\mathbf{f}_\mathrm{c}\ v_8){=}v_6, (\lambda_{u_5}\ \mathsf{w}\ e_5\ u_5), (\lambda_{u_8}\ \mathsf{e}\ e_8\ u_8)\ ;\ (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

$M\ (\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\mathbf{f}_\mathrm{s}\ v_5){=}v_2, (\mathbf{f}_\mathrm{s}\ v_8){=}v_2, (\mathsf{f}\ p_4\ u_5\ v_5), (\mathsf{a}\ p_7\ u_8\ v_8), (\mathbf{f}_\mathrm{r}\ v_5){=}u_5, (\mathbf{f}_\mathrm{r}\ v_8){=}u_8, (\mathbf{f}_\mathrm{c}\ v_5){=}v_6, (\mathbf{f}_\mathrm{c}\ v_8){=}v_6\ ;\ (\lambda_{u_5}\ \mathsf{w}\ e_5\ u_5), (\lambda_{u_8}\ \mathsf{e}\ e_8\ u_8), (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

$I2\ (\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\mathbf{f}_\mathrm{s}\ v_5){=}v_2, (\mathbf{f}_\mathrm{s}\ v_8){=}v_2, (\mathsf{f}\ p_4\ u_5\ v_5), (\mathsf{a}\ p_7\ u_8\ v_8), (\lambda_{v_5}\ \mathsf{w}\ e_5\ u_5), (\lambda_{v_8}\ \mathsf{e}\ e_8\ u_8), (\lambda_{v_5}\ \mathsf{h}\ e_3\ v_2\ v_5), (\lambda_{v_8}\ \mathsf{h}\ e_3\ v_2\ v_8)\ ;\ (\lambda_{u_5}\ \mathsf{w}\ e_5\ u_5), (\lambda_{u_8}\ \mathsf{e}\ e_8\ u_8), (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

$C\ (\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\mathbf{f}_\mathrm{s}\ v_5){=}v_2, (\mathbf{f}_\mathrm{s}\ v_8){=}v_2, (\mathsf{f}\ p_4\ u_5\ v_5), (\mathsf{a}\ p_7\ u_8\ v_8), (\lambda_{v_5}\ \mathsf{w}\ e_5\ v_5 \wedge \mathsf{h}\ e_3\ v_2\ v_5), (\lambda_{v_8}\ \mathsf{e}\ e_8\ v_8 \wedge \mathsf{h}\ e_3\ v_2\ v_8)\ ;\ (\lambda_{u_5}\ \mathsf{w}\ e_5\ u_5), (\lambda_{u_8}\ \mathsf{e}\ e_8\ u_8), (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

$M\ (\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\mathbf{f}_\mathrm{s}\ v_5){=}v_2, (\mathbf{f}_\mathrm{s}\ v_8){=}v_2, (\mathsf{f}\ p_4\ u_5\ v_5), (\mathsf{a}\ p_7\ u_8\ v_8)\ ;\ (\lambda_{v_5}\ \mathsf{w}\ e_5\ v_5 \wedge \mathsf{h}\ e_3\ v_2\ v_5), (\lambda_{v_8}\ \mathsf{e}\ e_8\ v_8 \wedge \mathsf{h}\ e_3\ v_2\ v_8), (\lambda_{u_5}\ \mathsf{w}\ e_5\ u_5), (\lambda_{u_8}\ \mathsf{e}\ e_8\ u_8), (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

$Q1\ (\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\mathsf{f}\ (\lambda_{u_5}\ \mathsf{w}\ e_5\ u_5)\ (\lambda_{v_5}\ \mathsf{w}\ e_5\ v_5 \wedge \mathsf{h}\ e_3\ v_2\ v_5)), (\mathsf{a}\ (\lambda_{u_8}\ \mathsf{e}\ e_8\ u_8)\ (\lambda_{v_8}\ \mathsf{e}\ e_8\ v_8 \wedge \mathsf{h}\ e_3\ v_2\ v_8))\ ;\ (\lambda_{v_5}\ ...), (\lambda_{v_8}\ ...), (\lambda_{u_5}\ ...), (\lambda_{u_8}\ ...), (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

$Q2\ (\mathsf{m}\ p_1\ u_2\ v_2), (\mathbf{f}_\mathrm{r}\ v_2){=}u_2, (\lambda_{v_2}\ \mathsf{f}\ (\lambda_{u_5}\ \mathsf{w}\ e_5\ u_5)\ (\lambda_{v_5}\ \mathsf{w}\ e_5\ v_5 \wedge \mathsf{h}\ e_3\ v_2\ v_5)), (\lambda_{v_2}\ \mathsf{a}\ (\lambda_{u_8}\ \mathsf{e}\ e_8\ u_8)\ (\lambda_{v_8}\ \mathsf{e}\ e_8\ v_8 \wedge \mathsf{h}\ e_3\ v_2\ v_8))\ ;\ (\lambda_{v_5}\ ...), (\lambda_{v_8}\ ...), (\lambda_{u_5}\ ...), (\lambda_{u_8}\ ...), (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

$I2\ (\mathsf{m}\ p_1\ u_2\ v_2), (\lambda_{v_2}\ \mathsf{c}\ e_2\ v_2), (\lambda_{v_2}\ \mathsf{f}\ (\lambda_{u_5}\ \mathsf{w}\ e_5\ u_5)\ (\lambda_{v_5}\ \mathsf{w}\ e_5\ v_5 \wedge \mathsf{h}\ e_3\ v_2\ v_5)), (\lambda_{v_2}\ \mathsf{a}\ (\lambda_{u_8}\ \mathsf{e}\ e_8\ u_8)\ (\lambda_{v_8}\ \mathsf{e}\ e_8\ v_8 \wedge \mathsf{h}\ e_3\ v_2\ v_8))\ ;\ (\lambda_{v_5}\ ...), (\lambda_{v_8}\ ...), (\lambda_{u_5}\ ...), (\lambda_{u_8}\ ...), (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

$C\ (\mathsf{m}\ p_1\ u_2\ v_2), (\lambda_{v_2}\ \mathsf{c}\ e_2\ v_2 \wedge \mathsf{f}\ (\lambda_{u_5}\ \mathsf{w}\ e_5\ u_5)\ (\lambda_{v_5}\ \mathsf{w}\ e_5\ v_5 \wedge \mathsf{h}\ e_3\ v_2\ v_5) \wedge \mathsf{a}\ (\lambda_{u_8}\ \mathsf{e}\ e_8\ u_8)\ (\lambda_{v_8}\ \mathsf{e}\ e_8\ v_8 \wedge \mathsf{h}\ e_3\ v_2\ v_8))\ ;\ (\lambda_{v_5}\ ...), (\lambda_{v_8}\ ...), (\lambda_{u_5}\ ...), (\lambda_{u_8}\ ...), (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

$M\ (\mathsf{m}\ p_1\ u_2\ v_2)\ ;\ (\lambda_{v_2}\ \mathsf{c}\ e_2\ v_2 \wedge \mathsf{f}\ (\lambda_{u_5}\ \mathsf{w}\ e_5\ u_5)\ (\lambda_{v_5}\ \mathsf{w}\ e_5\ v_5 \wedge \mathsf{h}\ e_3\ v_2\ v_5) \wedge \mathsf{a}\ (\lambda_{u_8}\ \mathsf{e}\ e_8\ u_8)\ (\lambda_{v_8}\ \mathsf{e}\ e_8\ v_8 \wedge \mathsf{h}\ e_3\ v_2\ v_8)), (\lambda_{v_5}\ ...), (\lambda_{v_8}\ ...), (\lambda_{u_5}\ ...), (\lambda_{u_8}\ ...), (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

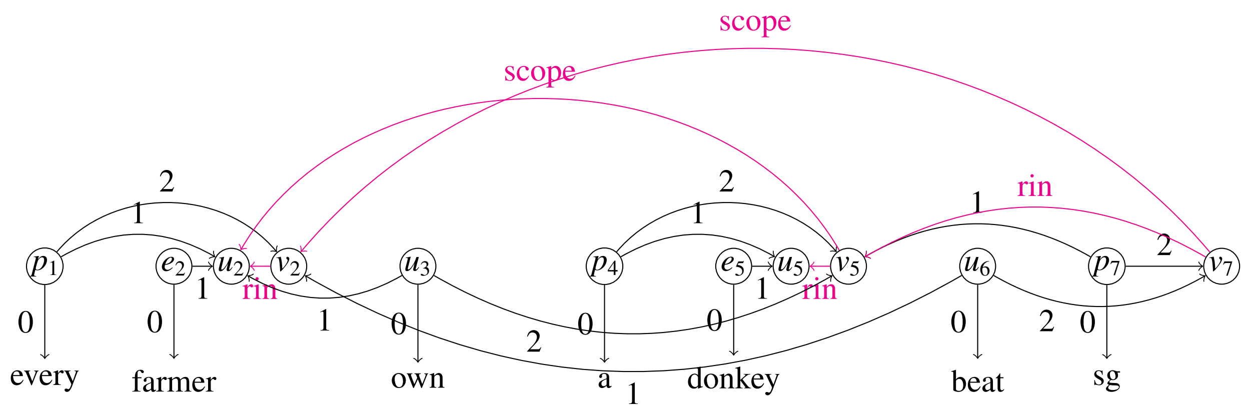
$Q1\ (\mathsf{m}\ (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2)\ (\lambda_{v_2}\ \mathsf{c}\ e_2\ v_2 \wedge \mathsf{f}\ (\lambda_{u_5}\ \mathsf{w}\ e_5\ u_5)\ (\lambda_{v_5}\ \mathsf{w}\ e_5\ v_5 \wedge \mathsf{h}\ e_3\ v_2\ v_5) \wedge \mathsf{a}\ (\lambda_{u_8}\ \mathsf{e}\ e_8\ u_8)\ (\lambda_{v_8}\ \mathsf{e}\ e_8\ v_8 \wedge \mathsf{h}\ e_3\ v_2\ v_8)))\ ;\ (\lambda_{v_2}\ ...), (\lambda_{v_5}\ ...), (\lambda_{v_8}\ ...), (\lambda_{u_5}\ ...), (\lambda_{u_8}\ ...), (\lambda_{u_2}\ \mathsf{c}\ e_2\ u_2), (\lambda_{v_6}\ \mathsf{h}\ e_3\ v_2\ v_6), (\lambda_{u_6}\ \top)$

The result, with existential quantifiers over eventualities filled in (note inheritances now explicit):

$$
\begin{aligned}
&(\mathsf{most}\ (\lambda_{u_2}\ \mathsf{some}\ (\lambda_{e_2}\ \mathsf{car}\ e_2\ u_2))\\
&\quad (\lambda_{v_2}\ \mathsf{some}\ (\lambda_{e_2}\ \mathsf{car}\ e_2\ v_2) \wedge \mathsf{four}\ (\lambda_{u_5}\ \mathsf{some}\ (\lambda_{e_5}\ \mathsf{wheel}\ e_5\ u_5))\\
&\qquad\qquad (\lambda_{v_5}\ \mathsf{some}\ (\lambda_{e_5}\ \mathsf{wheel}\ e_5\ v_5) \wedge \mathsf{some}\ (\lambda_{e_3}\ \mathsf{have}\ e_3\ v_2\ v_5))\\
&\qquad \wedge \mathsf{an}\ \ (\lambda_{u_8}\ \mathsf{some}\ (\lambda_{e_8}\ \mathsf{engine}\ e_8\ u_8))\\
&\qquad\qquad (\lambda_{v_8}\ \mathsf{some}\ (\lambda_{e_8}\ \mathsf{engine}\ e_8\ v_8) \wedge \mathsf{some}\ (\lambda_{e_3}\ \mathsf{have}\ e_3\ v_2\ v_8))))
\end{aligned}
$$

20.10 Donkey sentences (Kamp, 1981)

E.g. *Every farmer who owns a donkey beats it*:



which consists of the following elementary predications and scope dependencies:

$(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\mathbf{f}_\mathrm{s}\;v_7)=v_2, (\mathsf{s}\;p_7\;v_5\;v_7), (\mathsf{b}\;u_6\;v_2\;v_7), (\mathbf{f}_\mathrm{r}\;v_7)=v_5, (\mathsf{o}\;u_3\;u_2\;v_5), (\mathbf{f}_\mathrm{r}\;v_5)=u_5, (\mathsf{d}\;e_5\;u_5)$

translates like this (assume unbound eventuality variables have low existential scope):

$(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\mathbf{f}_\mathrm{s}\;v_7)=v_2, (\mathsf{s}\;p_7\;v_5\;v_7), (\mathsf{b}\;u_6\;v_2\;v_7), (\mathbf{f}_\mathrm{r}\;v_7)=v_5, (\mathsf{o}\;u_3\;u_2\;v_5), (\mathbf{f}_\mathrm{r}\;v_5)=u_5, (\mathsf{d}\;e_5\;u_5)\;;$

$P\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\mathbf{f}_\mathrm{s}\;v_7)=v_2, (\mathsf{s}\;p_7\;v_5\;v_7), (\mathsf{b}\;u_6\;v_2\;v_7), (\mathbf{f}_\mathrm{r}\;v_7)=v_5, (\mathsf{o}\;u_3\;u_2\;v_5), (\mathbf{f}_\mathrm{r}\;v_5)=u_5, (\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)\;;$

$M\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\mathbf{f}_\mathrm{s}\;v_7)=v_2, (\mathsf{s}\;p_7\;v_5\;v_7), (\mathsf{b}\;u_6\;v_2\;v_7), (\mathbf{f}_\mathrm{r}\;v_7)=v_5, (\mathsf{o}\;u_3\;u_2\;v_5), (\mathbf{f}_\mathrm{r}\;v_5)=u_5\;;\;(\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)$

$I2\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\mathbf{f}_\mathrm{s}\;v_7)=v_2, (\mathsf{s}\;p_7\;v_5\;v_7), (\mathsf{b}\;u_6\;v_2\;v_7), (\mathbf{f}_\mathrm{r}\;v_7)=v_5, (\mathsf{o}\;u_3\;u_2\;v_5), (\lambda_{v_5}\;\mathsf{d}\;e_5\;v_5)\;;\;(\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)$

$P\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\mathbf{f}_\mathrm{s}\;v_7)=v_2, (\mathsf{s}\;p_7\;v_5\;v_7), (\mathsf{b}\;u_6\;v_2\;v_7), (\mathbf{f}_\mathrm{r}\;v_7)=v_5, (\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5), (\lambda_{v_5}\;\mathsf{d}\;e_5\;v_5)\;;\;(\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)$

$C\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\mathbf{f}_\mathrm{s}\;v_7)=v_2, (\mathsf{s}\;p_7\;v_5\;v_7), (\mathsf{b}\;u_6\;v_2\;v_7), (\mathbf{f}_\mathrm{r}\;v_7)=v_5, (\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5)\;;\;(\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)$

$M\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\mathbf{f}_\mathrm{s}\;v_7)=v_2, (\mathsf{s}\;p_7\;v_5\;v_7), (\mathsf{b}\;u_6\;v_2\;v_7), (\mathbf{f}_\mathrm{r}\;v_7)=v_5\;;\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5), (\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)$

$I4\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\mathbf{f}_\mathrm{s}\;v_7)=v_2, (\mathsf{s}\;p_7\;v_5\;v_7), (\mathsf{b}\;u_6\;v_2\;v_7), (\lambda_{v_7}\;\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7)\;;\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5), (\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)$

$P\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\mathbf{f}_\mathrm{s}\;v_7)=v_2, (\mathsf{s}\;p_7\;v_5\;v_7), (\lambda_{v_7}\;\mathsf{b}\;u_6\;v_2\;v_7), (\lambda_{v_7}\;\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7)\;;\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5), (\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)$

$C\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\mathbf{f}_\mathrm{s}\;v_7)=v_2, (\mathsf{s}\;p_7\;v_5\;v_7), (\lambda_{v_7}\;\mathsf{b}\;u_6\;v_2\;v_7\wedge\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7)\;;\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5), (\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)$

$M\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\mathbf{f}_\mathrm{s}\;v_7)=v_2, (\mathsf{s}\;p_7\;v_5\;v_7)\;;\;(\lambda_{v_7}\;\mathsf{b}\;u_6\;v_2\;v_7\wedge\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7), (\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5), (\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)$

$Q1\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\mathbf{f}_\mathrm{s}\;v_7)=v_2, (\mathsf{s}\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_7}\;\mathsf{b}\;u_6\;v_2\;v_7\wedge\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7))\;;\;(\lambda_{v_7}\;...), (\lambda_{v_5}\;...), (\lambda_{u_5}\;...)$

$Q3\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;p_4\;u_5\;v_5), (\lambda_{v_2}\;\mathsf{s}\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;v_2\;v_5\wedge\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_7}\;\mathsf{b}\;u_6\;v_2\;v_7\wedge\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7))\;;\;(\lambda_{v_7}\;...), (\lambda_{v_5}\;...), (\lambda_{u_5}\;...)$

$Q1\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\mathbf{f}_\mathrm{s}\;v_5)=u_2, (\mathsf{a}\;(\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5)), (\lambda_{v_2}\;\mathsf{s}\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;v_2\;v_5\wedge\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_7}\;\mathsf{b}\;u_6\;v_2\;v_7\wedge\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7))\;;\;(\lambda_{v_7}\;...), (\lambda_{v_5}\;...), (\lambda_{u_5}\;...)$

$Q2\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\mathsf{f}\;e_2\;u_2), (\lambda_{u_2}\;\mathsf{a}\;(\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5)), (\lambda_{v_2}\;\mathsf{s}\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;v_2\;v_5\wedge\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_7}\;\mathsf{b}\;u_6\;v_2\;v_7\wedge\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7))\;;\;(\lambda_{v_7}\;...), (\lambda_{v_5}\;...), (\lambda_{u_5}\;...)$

$P\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\lambda_{u_2}\;\mathsf{f}\;e_2\;u_2), (\lambda_{u_2}\;\mathsf{a}\;(\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5)), (\lambda_{v_2}\;\mathsf{s}\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;v_2\;v_5\wedge\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_7}\;\mathsf{b}\;u_6\;v_2\;v_7\wedge\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7))\;;\;(\lambda_{v_7}\;...), (\lambda_{v_5}\;...), (\lambda_{u_5}\;...)$

$C\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\lambda_{u_2}\;\mathsf{f}\;e_2\;u_2\wedge\mathsf{a}\;(\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5)), (\lambda_{v_2}\;\mathsf{s}\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;v_2\;v_5\wedge\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_7}\;\mathsf{b}\;u_6\;v_2\;v_7\wedge\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7))\;;\;(\lambda_{v_7}\;...), (\lambda_{v_5}\;...), (\lambda_{u_5}\;...)$

$M\;(\mathsf{e}\;p_1\;u_2\;v_2), (\mathbf{f}_\mathrm{r}\;v_2)=u_2, (\lambda_{v_2}\;\mathsf{s}\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;v_2\;v_5\wedge\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_7}\;\mathsf{b}\;u_6\;v_2\;v_7\wedge\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7))\;;\;(\lambda_{u_2}\;\mathsf{f}\;e_2\;u_2\wedge\mathsf{a}\;(\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5)), (\lambda_{v_7}\;...), (\lambda_{v_5}\;...), (\lambda_{u_5}\;...)$

$I2\;(\mathsf{e}\;p_1\;u_2\;v_2), (\lambda_{v_2}\;\mathsf{f}\;e_2\;u_2\wedge...\wedge\mathsf{s}\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;v_2\;v_5\wedge\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_7}\;\mathsf{b}\;u_6\;v_2\;v_7\wedge\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7))\;;\;(\lambda_{u_2}\;\mathsf{f}\;e_2\;u_2\wedge\mathsf{a}\;(\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5)), (\lambda_{v_7}\;...), (\lambda_{v_5}\;...), (\lambda_{u_5}\;...)$

$M\;(\mathsf{e}\;p_1\;u_2\;v_2)\;;\;(\lambda_{v_2}\;\mathsf{f}\;e_2\;u_2\wedge...\wedge\mathsf{s}\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;v_2\;v_5\wedge\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_7}\;\mathsf{b}\;u_6\;v_2\;v_7\wedge\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7)), (\lambda_{u_2}\;\mathsf{f}\;e_2\;u_2\wedge\mathsf{a}\;(\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5)), (\lambda_{v_7}\;...), (\lambda_{v_5}\;...), (\lambda_{u_5}\;...)$

$Q1\;(\mathsf{e}\;(\lambda_{u_2}\;\mathsf{f}\;e_2\;u_2\wedge\mathsf{a}\;(\lambda_{u_5}\;\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;u_2\;v_5\wedge\mathsf{d}\;e_5\;v_5))\;(\lambda_{v_2}\;\mathsf{f}\;e_2\;u_2\wedge...\wedge\mathsf{s}\;(\lambda_{v_5}\;\mathsf{o}\;u_3\;v_2\;v_5\wedge\mathsf{d}\;e_5\;u_5)\;(\lambda_{v_7}\;\mathsf{b}\;u_6\;v_2\;v_7\wedge\mathsf{o}\;u_3\;v_2\;v_7\wedge\mathsf{d}\;e_5\;v_7)))\;;\;(\lambda_{v_2}\;...), (\lambda_{u_2}\;...), (\lambda_{v_7}\;...), (\lambda_{v_5}\;...), (\lambda_{u_5}\;...)$

Not satisfied when farmers own multiple donkeys because singular pronoun unsatisfied.
Generates 'strong' reading with plural ('Every farmer that owns some donkeys beats them').

20.11 Summation anaphora (Kamp, 1981)

6. Add union of conjunct constraints to complete lambda terms in Δ:

$$\frac{\Gamma, (\mathbf{f}_{\text{uin}}\ v){=}u\ ;\ \Delta}{\Gamma, (\mathbf{f}_{\text{cin}}\ v){=}u, (\mathbf{f}_{\text{union}}\ v){=}u\ ;\ \Delta} \tag{U1}$$

$$\frac{\Gamma, (\mathbf{f}_{\text{union}}\ v){=}u\ ;\ (\lambda_v\ \psi), \Delta}{\Gamma, (\lambda_u\ (\lambda_v\ \psi)\ u)\ ;\ (\lambda_v\ \psi), \Delta}\ (\lambda_u\ \ldots) \notin \Gamma \tag{U2}$$

$$\frac{\Gamma, (\lambda_u\ \phi), (\mathbf{f}_{\text{union}}\ v){=}u\ ;\ (\lambda_v\ \psi), \Delta}{\Gamma, (\lambda_u\ \phi \vee (\lambda_v\ \psi)\ u)\ ;\ (\lambda_v\ \psi), \Delta} \tag{U3}$$

$$\frac{\Gamma, (\lambda_u\ \phi)\ ;\ (\lambda_u\ \psi), \Delta}{\Gamma\ ;\ (\lambda_u\ \phi \wedge \psi), \Delta}\ \forall_v\ (\mathbf{f}_{\text{union}}\ v){=}u \notin \Gamma \tag{U4}$$

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