# Ling 8700: Lecture Notes 3
## From Distributed Associative Memory to Cued Association Semantics

We have seen how interconnected neurons can define mental states and cued associations.

This lecture will describe how mental states and cued associations can define complex ideas.

## Contents

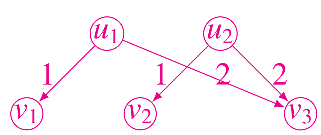
## 3.1 Previously: mental states and cued associations

Recall neural activation patterns and potentiated connection weights:

$$\underbrace{\begin{bmatrix} .71 \\ .0 \\ .71 \\ .0 \\ .0 \end{bmatrix}}_{\text{target } v_1} \underbrace{\begin{bmatrix} .0 & .58 & .0 & .58 & .58 \end{bmatrix}}_{\text{cue } u_1} + \underbrace{\begin{bmatrix} .0 \\ .58 \\ .58 \\ .0 \\ .58 \end{bmatrix}}_{\text{target } v_2} \underbrace{\begin{bmatrix} .0 & .0 & .71 & .71 & .0 \end{bmatrix}}_{\text{cue } u_2} = \underbrace{\begin{bmatrix} .0 & .41 & .0 & .41 & .41 \\ .0 & .0 & .41 & .41 & .0 \\ .0 & .41 & .41 & .82 & .41 \\ .0 & .0 & .0 & .0 & .0 \\ .0 & .0 & .41 & .41 & .0 \end{bmatrix}}_{\text{associations } M_1}$$

$$\underbrace{\begin{bmatrix} .58 \\ .58 \\ .0 \\ .58 \\ .0 \end{bmatrix}}_{\text{target } v_3} \underbrace{\begin{bmatrix} .0 & .58 & .58 & .0 & .58 \end{bmatrix}}_{\text{cue } u_1} + \underbrace{\begin{bmatrix} .58 \\ .58 \\ .0 \\ .58 \\ .0 \end{bmatrix}}_{\text{target } v_3} \underbrace{\begin{bmatrix} .0 & .0 & .71 & .71 & .0 \end{bmatrix}}_{\text{cue } u_2} = \underbrace{\begin{bmatrix} .0 & .33 & .74 & .41 & .33 \\ .0 & .33 & .74 & .41 & .33 \\ .0 & .0 & .0 & .0 & .0 \\ .0 & .33 & .74 & .41 & .33 \\ .0 & .0 & .0 & .0 & .0 \end{bmatrix}}_{\text{associations } M_2}$$

define coordinates of points (mental states) in mental space, linked by cued associations:




## 3.2 Referential states and elementary predications

Cued associations among mental states can be used to build complex ideas:

1. Mental states are inherently **referential** (Karttunen, 1976); associated with stimuli.

   They **generalize** over objects/stimuli that produce the same pattern of activation

   (similar appearance patterns in occipital lobe, smell in olfactory cortex, etc.).

   Referential states are **countable** (Parsons, 1990): can distinguish instances in generalization

   (they have cardinalities of satisfying referents / groups of divisible non-infinitesimal quanta).

2. Some referential states are **elementary predications** (Copestake et al., 2005), with . . .

(a) **predication type constant** — features of predication, notated as function $\mathbf{f_0}$ to type (e.g. **Holding** for predication of kids holding spoons).

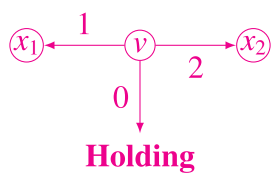
(b) **participants** — cued associations (functions) $\mathbf{f_1}$, $\mathbf{f_2}$ from predication to participant (e.g. referential states for holder, held).

Relation betw. predication & participant/type has same **temporal extent** as predication.

(If you hold a spoon, then change it to fork, it's a new holding; not so for cause/beneficiary.)

This defines what is and is not a direct participant in an elementary predication.

If we draw cued associations as labeled directed edges, elementary predications look like this:




where $x_1$ generalizes over the holder, $x_2$ generalizes over the held things, and $v$ generalizes over elementary predications (events) of type '**Holding**.'

This can also be written $(\mathbf{f_0}\ v) = \mathbf{Holding}, (\mathbf{f_1}\ v) = x_1, (\mathbf{f_2}\ v) = x_2$ or simply $(\mathbf{Holding}\ v\ x_1\ x_2)$.

Elementary predications are referential states, can be described by other elementary predications...

- temporal predications: date, time, duration, speed $(\mathbf{BeingDuring}\ u\ v\ y), (\mathbf{BeingATrip}\ z\ y)$
- implicational predications: one predication causes another $(\mathbf{Causing}\ u\ v\ y), (\mathbf{Breaking}\ y\ x_2)$
- predications about belief: $(\mathbf{Believing}\ u\ y\ v), (\mathbf{BeingAHistorian}\ z\ y)$

Referential states and elementary predications are different from nouns and verbs:

- e.g. elementary predications can be encoded as nouns: *Rome's destruction of Carthage*.

## 3.3 Broad and narrow generalizations in referential states

Complex ideas often define **narrow** generalizations by combining multiple **broad** generalizations.

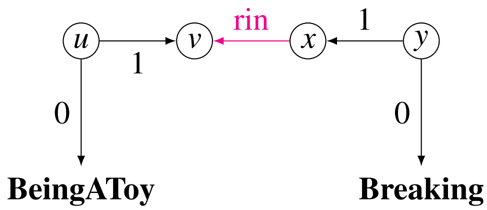
- E.g.: *toys break* (defines a subset of broken toys from toys and brokens [broken things])

Narrow (broken toys) and first broad (toys) generalization are reused, but not the second (brokens):

- *toys break, but they can be fixed* (they = broken toys)
- *toys break, but usually they don't break* (they = toys)

- ?? *toys break, but usually they aren't toys* (they = brokens [broken things])

so we assume just one broad ($v$) and one narrow ($x$) generalization persist as referential states:



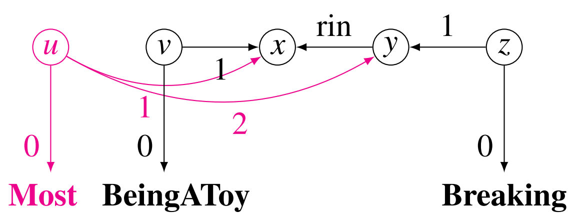

Here $v$,$x$ generalize over toys and broken toys ($u$,$y$ generalize over being a toy and breaking).

The narrow generalization restricts and inherits from the broad one ('restriction inheritance': rin).

## 3.4 Generalized quantifiers (Barwise & Cooper, 1981)

Sometimes narrow and broad generalizations are compared by cardinality.

- E.g.: *'most toys break'* asserts that the set of broken toys includes over half of all toys.




These are elementary predications called **generalized quantifiers** (Barwise & Cooper, 1981).

(The broader generalization is the **restrictor**, and the narrower is the **nuclear scope**.)

Quantifiers are important because they allow abstract reasoning about **conditional probabilities**.

- E.g.: All cholera are digestive problems, but few digestive problems are cholera.

Quantifiers may be inferred from the strength (frequency) of cued associations between referents.

It is difficult to imagine performing logical inference without quantifiers.

Quantifiers are learned late: 'quantifier spreading' (Inhelder & Piaget, 1958; Philip, 1995).

Children until about 10 years old don't reliably constrain the restrictor with the noun, etc.:

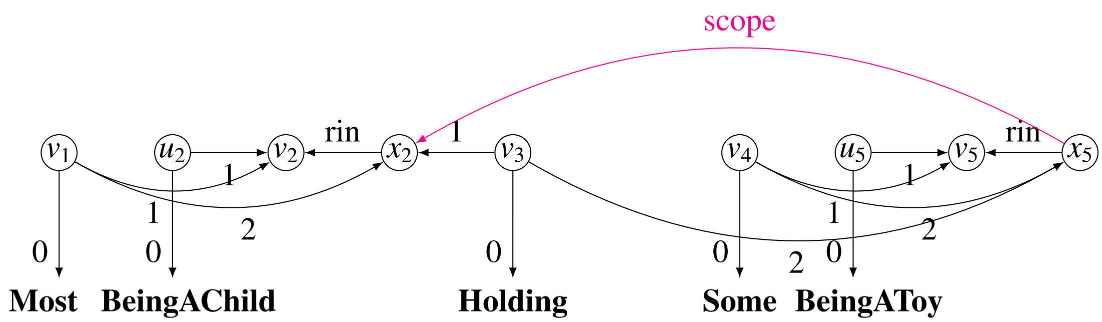
- *Does every bike have a basket?* (shown picture with three bikes, four baskets) → answer: *No.*

But this may indicate a lack of mapping from words to ideas, rather than a lack of this kind of idea.

### 3.5 Quantifier scope dependencies (Schuler & Wheeler, 2014)

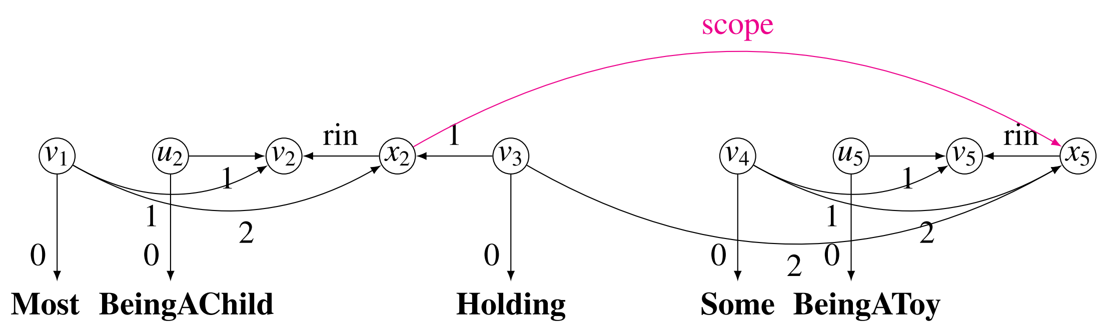
If multiple quantified referents are connected by predications, they must be scoped.

This distinguishes whether there is a set of toys for each child or a set of children for each toy:




In the above example, there is a generalization over held toys for each holder child.

In the below example, there is a generalization over holder children for each held toy (i.e. there is some toy, say the wind-up robot, that most children hold):




### 3.6 Generalized quantifiers in lambda calculus (Barwise & Cooper, 1981)

Philosophers often define complex ideas using logical expressions in lambda calculus.

Lambda expressions $\lambda_x\, \phi$ are just anonymous functions $\phi$ with input parameter $x$.

These anonymous functions can define countable **sets** of inputs $x$ that satisfy $\phi$.

A lambda calculus expression for *Most children hold some toy*:

$$(\textbf{Most}\ (\lambda_{v_2}\ \textbf{BeingAChild}\ v_2)$$
$$(\lambda_{x_2}\ \textbf{Some}\ (\lambda_{v_5}\ \textbf{BeingAToy}\ v_5)$$
$$(\lambda_{x_5}\ \textbf{Holding}\ x_2\ x_5)))$$

then counts these sets, as in this python program:

```
U = ['c1','c2','c3','t1','t2','t3','t4','t5','t6','t7','t8']

def most(f,g):
  count,count2 = 0,0
  for v in U:
    if f(v): count+=1
    if f(v) and g(v): count2+=1
  return count2 > 0.5*count

def some(f,g):
  count = 0
  for v in U:
    if f(v) and g(v): count+=1
  return count > 1

def child(v): return v in ['c1','c2','c3']

def toy(v): return v in ['t1','t2','t3','t4','t5','t6','t7','t8']

def hold(v,x): return (v,x) in [('c1','t1'),('c1','t2'),('c1','t3'),('c1','t4'),
                                ('c2','t5'),('c2','t6'),('c2','t7'),('c2','t8')]

most( lambda v2: child(v2), lambda x2: some( lambda v5: toy(v5), lambda x5: hold(x2,x5) ) )
```

Here the variables have clear meanings (indices just identify source word by token number):

- $v_2$ is a variable over children;
- $x_2$ is a variable over things that hold some toy;
- $v_5$ is a variable over toys;
- $x_5$ is a variable for each child $v_2$ over things that child holds.

## 3.7 Cued associations map to lambda calculus (Schuler & Wheeler, 2014)

The cued association graphs above can be translated into lambda calculus (not a cognitive process).

1. Add a lambda term to $\Gamma$ for each predication in $\Gamma$ with no outscoped variables or inheritances:

$$\frac{\Gamma, (f\ x_0\ x_1\ ..\ x\ ..\ x_N)\ ;\ \Delta}{\Gamma, (\lambda_x\ f\ x_0\ x_1\ ..\ x\ ..\ x_N)\ ;\ \Delta}\ \overbrace{f \notin Q}^{\text{not quant}},\ \overbrace{\forall_y\ (\mathbf{f}_{\mathbf{scope}}\ y){=}x \notin \Gamma}^{\text{no incoming scope}},\ \overbrace{\forall_y\ \forall_{f\in\{\mathbf{f}_{\mathbf{rin}},\mathbf{f}_{\mathbf{cin}},\mathbf{f}_{\mathbf{uin}}\}}\ (f\ x){=}y \notin \Gamma}^{\text{no outgoing inheritance}} \qquad \text{(P)}$$

2. Conjoin lambda terms over the same variable in $\Gamma$ (this combines modifier predications):

$$\frac{\Gamma, (\lambda_x\ \phi), (\lambda_x\ \psi)\ ;\ \Delta}{\Gamma, (\lambda_x\ \phi \wedge \psi)\ ;\ \Delta} \qquad \text{(C)}$$

3. Move terms in $\Gamma$ with no missing predications, outscoped variables or inheritances to $\Delta$:

$$\frac{\Gamma, (\lambda_x\ \psi)\ ;\ \Delta}{\Gamma\ ;\ (\lambda_x\ \psi), \Delta}\ \overbrace{\forall_{f'\notin Q}\ (f'\ ..\ x\ ..) \notin \Gamma}^{\text{no elem. pred. in rest of graph}},\ \overbrace{\forall_y\ (\mathbf{f}_{\mathbf{scope}}\ y){=}x \notin \Gamma}^{\text{no incoming scope}},\ \overbrace{\forall_y\ \forall_{f\in\{\mathbf{f}_{\mathbf{rin}},\mathbf{f}_{\mathbf{cin}},\mathbf{f}_{\mathbf{uin}}\}}\ (f\ x){=}y \notin \Gamma}^{\text{no outgoing inheritance}} \qquad \text{(M)}$$

4. Add translations $\tau_f$ of quantifiers $f$ in $\Gamma$ over complete lambda terms in $\Delta$:

$$\frac{\Gamma, (f\ u\ v\ x)\ ; (\lambda_v\ \phi), (\lambda_x\ \psi), \Delta}{\Gamma, (\tau_f\ (\lambda_v\ \phi)\ (\lambda_x\ \psi))\ ; (\lambda_v\ \phi), (\lambda_x\ \psi), \Delta}\ f \in Q, \overbrace{\forall_{f' \in \{\mathbf{f_{rin}}, \mathbf{f_{cin}}, \mathbf{f_{uin}}\}}\ (f'\ ..)\!=\!x \notin \Gamma}^{\text{no inheritance arriving at nuc. scope}}, \overbrace{\forall_{f' \in Q}\ (f'\ ..\ x\ ..) \notin \Gamma}^{\text{no quant over n. s. in rest of graph}} \qquad \text{(Q)}$$

5. Add lambda for outgoing scope:

$$\frac{\Gamma, (\mathbf{f_{scope}}\ y)\!=\!v, (\tau_f\ (\lambda_x\ \phi)\ (\lambda_y\ \psi))\ ;\ \Delta}{\Gamma, (\lambda_v\ \tau_f\ (\lambda_x\ \phi)\ (\lambda_y\ \psi))\ ;\ \Delta}\ \overbrace{\forall_u\ (\mathbf{f_{scope}}\ x)\!=\!u \notin \Gamma}^{\text{no scope departing restrictor}} \qquad \text{(S1)}$$

$$\frac{\Gamma, (\mathbf{f_{scope}}\ x)\!=\!u, (\mathbf{f_{scope}}\ y)\!=\!v, (\tau_f\ (\lambda_x\ \phi)\ (\lambda_y\ \psi))\ ;\ \Delta}{\Gamma, (\mathbf{f_{scope}}\ x)\!=\!u, (\lambda_v\ \tau_f\ (\lambda_x\ \underbrace{(\lambda_u\ \phi)\ v}_{\text{replace } u \text{ with } v})\ (\lambda_y\ \psi))\ ;\ \Delta} \qquad \text{(S2)}$$

6. Add a lambda term to $\Gamma$ for each inheritance that is empty or from complete term in $\Delta$:

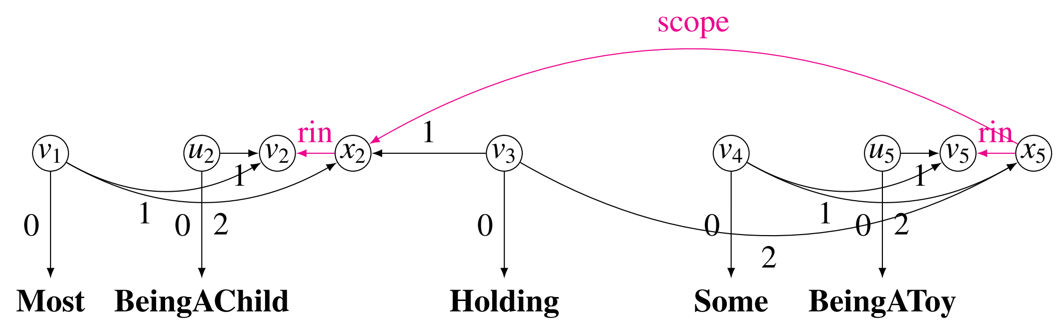
$$\frac{\Gamma, (f\ y)\!=\!x\ ;\ \Delta}{\Gamma, (f\ y)\!=\!x, \underbrace{(\lambda_x\ \mathbf{True})}_{\text{empty inheritance}}\ ;\ \Delta}\ f \in \{\mathbf{f_{rin}}, \mathbf{f_{cin}}\}, \underbrace{\forall_{f' \notin Q}\ (f'\ ..\ x\ ..) \notin \Gamma}_{\text{no elem. pred. on } x \text{ in rest of graph}} \qquad \text{(I1)}$$

$$\frac{\Gamma, (f\ y)\!=\!x\ ;\ (\lambda_x\ \phi), \Delta}{\Gamma, (\lambda_y\ \underbrace{(\lambda_x\ \phi)\ y}_{\text{replace inherited with inheritor}})\ ;\ (\lambda_x\ \phi), \Delta}\ f \in \{\mathbf{f_{rin}}, \mathbf{f_{cin}}\}, \underbrace{\forall_u\ (\mathbf{f_{scope}}\ x)\!=\!u \notin \Gamma}_{\text{no scope departing inherited}} \qquad \text{(I2)}$$

$$\frac{\Gamma, (\mathbf{f_{scope}}\ x)\!=\!u, (f\ y)\!=\!x\ ;\ (\lambda_x\ \phi), \Delta}{\Gamma, (\mathbf{f_{scope}}\ x)\!=\!u, \underbrace{(\mathbf{f_{scope}}\ y)\!=\!u}_{\text{inherit scope}}, (\lambda_y\ \underbrace{(\lambda_x\ \phi)\ y}_{\text{replace inherited with inheritor}})\ ;\ (\lambda_x\ \phi), \Delta}\ f \in \{\mathbf{f_{rin}}, \mathbf{f_{cin}}\}, \underbrace{\forall_v\ (\mathbf{f_{scope}}\ y)\!=\!v \notin \Gamma}_{\text{no scope departing inheritor}} \qquad \text{(I3)}$$

$$\frac{\Gamma, (\mathbf{f_{scope}}\ x)\!=\!u, (\mathbf{f_{scope}}\ y)\!=\!v, (f\ y)\!=\!x\ ;\ (\lambda_x\ \phi), \Delta}{\Gamma, (\mathbf{f_{scope}}\ x)\!=\!u, (\mathbf{f_{scope}}\ y)\!=\!v, (\lambda_y\ \underbrace{(\lambda_u\ \underbrace{(\lambda_x\ \phi)\ y}_{\text{replace inherited with inheritor}}\ v)}_{\text{replace scope of inherited with scope of inheritor}})\ ;\ (\lambda_x\ \phi), \Delta}\ f \in \{\mathbf{f_{rin}}, \mathbf{f_{cin}}\} \qquad \text{(I4)}$$

For example, our graph:



translates into:

$$(\textbf{Most}\ v_1\ v_2\ x_2), (\textbf{f}_{\textbf{rin}}\ x_2){=}v_2, (\textbf{f}_{\textbf{scope}}\ x_5){=}x_2, (\textbf{Some}\ v_4\ v_5\ x_5), (\textbf{f}_{\textbf{rin}}\ x_5){=}v_5, (\textbf{Holding}\ v_3\ x_2\ x_5), (\textbf{BeingAChild}\ u_2\ v_2), (\textbf{BeingAToy}\ u_5\ v_5)\ ;$$

$$P\ (\textbf{Most}\ v_1\ v_2\ x_2), (\textbf{f}_{\textbf{rin}}\ x_2){=}v_2, (\textbf{f}_{\textbf{scope}}\ x_5){=}x_2, (\textbf{Some}\ v_4\ v_5\ x_5), (\textbf{f}_{\textbf{rin}}\ x_5){=}v_5, (\textbf{Holding}\ v_3\ x_2\ x_5), (\lambda_{v_2}\ \textbf{BeingAChild}\ u_2\ v_2), (\lambda_{v_5}\ \textbf{BeingAToy}\ u_5\ v_5)\ ;$$

$$M\ (\textbf{Most}\ v_1\ v_2\ x_2), (\textbf{f}_{\textbf{rin}}\ x_2){=}v_2, (\textbf{f}_{\textbf{scope}}\ x_5){=}x_2, (\textbf{Some}\ v_4\ v_5\ x_5), (\textbf{f}_{\textbf{rin}}\ x_5){=}v_5, (\textbf{Holding}\ v_3\ x_2\ x_5)\ ;\ (\lambda_{v_2}\ \textbf{BeingAChild}\ u_2\ v_2), (\lambda_{v_5}\ \textbf{BeingAToy}\ u_5\ v_5)$$

$$I2\ (\textbf{Most}\ v_1\ v_2\ x_2), (\textbf{f}_{\textbf{rin}}\ x_2){=}v_2, (\textbf{f}_{\textbf{scope}}\ x_5){=}x_2, (\textbf{Some}\ v_4\ v_5\ x_5), (\lambda_{x_5}\ \textbf{BeingAToy}\ u_5\ x_5), (\textbf{Holding}\ v_3\ x_2\ x_5)\ ;\ (\lambda_{v_2}\ \textbf{BeingAChild}\ u_2\ v_2), (\lambda_{v_5}\ \textbf{BeingAToy}\ u_5\ v_5)$$

$$P\ (\textbf{Most}\ v_1\ v_2\ x_2), (\textbf{f}_{\textbf{rin}}\ x_2){=}v_2, (\textbf{f}_{\textbf{scope}}\ x_5){=}x_2, (\textbf{Some}\ v_4\ v_5\ x_5), (\lambda_{x_5}\ \textbf{BeingAToy}\ u_5\ x_5), (\lambda_{x_5}\ \textbf{Holding}\ v_3\ x_2\ x_5)\ ;\ (\lambda_{v_2}\ \textbf{BeingAChild}\ u_2\ v_2), (\lambda_{v_5}\ \textbf{BeingAToy}\ u_5\ v_5)$$

$$C\ (\textbf{Most}\ v_1\ v_2\ x_2), (\textbf{f}_{\textbf{rin}}\ x_2){=}v_2, (\textbf{f}_{\textbf{scope}}\ x_5){=}x_2, (\textbf{Some}\ v_4\ v_5\ x_5), (\lambda_{x_5}\ \textbf{BeingAToy}\ u_5\ x_5 \wedge \textbf{Holding}\ v_3\ x_2\ x_5)\ ;\ (\lambda_{v_2}\ \textbf{BeingAChild}\ u_2\ v_2), (\lambda_{v_5}\ \textbf{BeingAToy}\ u_5\ v_5)$$

$$M\ (\textbf{Most}\ v_1\ v_2\ x_2), (\textbf{f}_{\textbf{rin}}\ x_2){=}v_2, (\textbf{f}_{\textbf{scope}}\ x_5){=}x_2, (\textbf{Some}\ v_4\ v_5\ x_5)\ ;\ (\lambda_{x_5}\ \textbf{BeingAToy}\ u_5\ x_5 \wedge \textbf{Holding}\ v_3\ x_2\ x_5), (\lambda_{v_2}\ \textbf{BeingAChild}\ u_2\ v_2), (\lambda_{v_5}\ \textbf{BeingAToy}\ u_5\ v_5)$$

$$Q\ (\textbf{Most}\ v_1\ v_2\ x_2), (\textbf{f}_{\textbf{rin}}\ x_2){=}v_2, (\textbf{f}_{\textbf{scope}}\ x_5){=}x_2, (\textbf{Some}\ (\lambda_{v_5}\ \textbf{BeingAToy}\ u_5\ v_5)\ (\lambda_{x_5}\ \textbf{BeingAToy}\ u_5\ x_5 \wedge \textbf{Holding}\ v_3\ x_2\ x_5))\ ;\ (\lambda_{v_2}\ \textbf{BeingAChild}\ u_2\ v_2), (\lambda_{v_5}\ ...), (\lambda_{x_5}\ ...)$$

$$S1\ (\textbf{Most}\ v_1\ v_2\ x_2), (\textbf{f}_{\textbf{rin}}\ x_2){=}v_2, (\lambda_{x_2}\ \textbf{Some}\ (\lambda_{v_5}\ \textbf{BeingAToy}\ u_5\ v_5)\ (\lambda_{x_5}\ \textbf{BeingAToy}\ u_5\ x_5 \wedge \textbf{Holding}\ v_3\ x_2\ x_5))\ ;\ (\lambda_{v_2}\ \textbf{BeingAChild}\ u_2\ v_2), (\lambda_{v_5}\ ...), (\lambda_{x_5}\ ...)$$

$$I2\ (\textbf{Most}\ v_1\ v_2\ x_2), (\lambda_{x_2}\ \textbf{BeingAChild}\ u_2\ x_2), (\lambda_{x_2}\ \textbf{Some}\ (\lambda_{v_5}\ \textbf{BeingAToy}\ u_5\ v_5)\ (\lambda_{x_5}\ \textbf{BeingAToy}\ u_5\ x_5 \wedge \textbf{Holding}\ v_3\ x_2\ x_5))\ ;\ (\lambda_{v_2}\ ...), (\lambda_{v_5}\ ...), (\lambda_{x_5}\ ...)$$

$$C\ (\textbf{Most}\ v_1\ v_2\ x_2), (\lambda_{x_2}\ \textbf{BeingAChild}\ u_2\ x_2 \wedge \textbf{Some}\ (\lambda_{v_5}\ \textbf{BeingAToy}\ u_5\ v_5)\ (\lambda_{x_5}\ \textbf{BeingAToy}\ u_5\ x_5 \wedge \textbf{Holding}\ v_3\ x_2\ x_5))\ ;\ (\lambda_{v_2}\ ...), (\lambda_{v_5}\ ...), (\lambda_{x_5}\ ...)$$

$$M\ (\textbf{Most}\ v_1\ v_2\ x_2)\ ;\ (\lambda_{x_2}\ \textbf{BeingAChild}\ u_2\ x_2 \wedge \textbf{Some}\ (\lambda_{v_5}\ \textbf{BeingAToy}\ u_5\ v_5)\ (\lambda_{x_5}\ \textbf{BeingAToy}\ u_5\ x_5 \wedge \textbf{Holding}\ v_3\ x_2\ x_5)), (\lambda_{v_2}\ ...), (\lambda_{v_5}\ ...), (\lambda_{x_5}\ ...)$$

$$Q\ (\textbf{Most}\ (\lambda_{v_2}\ \textbf{BeingAChild}\ u_2\ v_2)\ (\lambda_{x_2}\ \textbf{BeingAChild}\ u_2\ x_2 \wedge \textbf{Some}\ (\lambda_{v_5}\ \textbf{BeingAToy}\ u_5\ v_5)\ (\lambda_{x_5}\ \textbf{BeingAToy}\ u_5\ x_5 \wedge \textbf{Holding}\ v_3\ x_2\ x_5)))\ ;\ (\lambda_{v_2}\ ...), (\lambda_{v_5}\ ...), (\lambda_{x_5}\ ...)$$

## 3.8 How are complex ideas experienced?

Remember, in this model a complex idea is a collection of cued associations in associative memory.

The entire idea is not all active at the same time.

How is such an idea experienced?

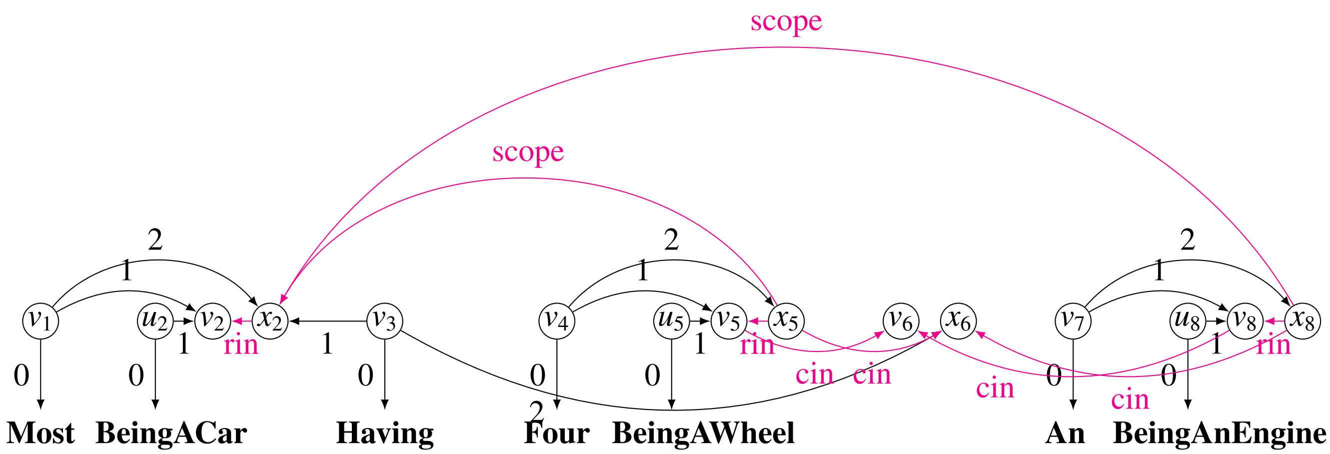
Just as we apprehend visual scenes by saccading from one physical fixation point to another, in this model we apprehend complex ideas by **transitioning** from one referential state to another, via cued associations.

So, as we think about children and playthings, there is always a 'you-are-here pointer' in the graph.

## 3.9 Conjunction Inheritance (Schuler & Wheeler, 2014)

The above translation also supports other kinds of inheritance.

Conjunctions also require inheritance ('conjunction inheritance,' cin):




This consists of the following elementary predications and scope dependencies:

$$(\mathbf{Most}\ v_1\ v_2\ x_2), (\mathbf{f}_{\mathrm{rin}}\ x_2){=}v_2, (\mathbf{f}_{\mathrm{scope}}\ x_5){=}x_2, (\mathbf{f}_{\mathrm{scope}}\ x_8){=}x_2, (\mathbf{Four}\ v_4\ v_5\ x_5), (\mathbf{A}\ v_7\ v_8\ x_8),$$
$$(\mathbf{f}_{\mathrm{rin}}\ x_5){=}v_5, (\mathbf{f}_{\mathrm{rin}}\ x_8){=}v_8, (\mathbf{f}_{\mathrm{cin}}\ x_5){=}x_6, (\mathbf{f}_{\mathrm{cin}}\ x_8){=}x_6, (\mathbf{BeingAWheel}\ u_5\ v_5),$$
$$(\mathbf{BeingAnEngine}\ u_8\ v_8), (\mathbf{f}_{\mathrm{cin}}\ v_5){=}v_6, (\mathbf{f}_{\mathrm{cin}}\ v_8){=}v_6, (\mathbf{BeingACar}\ u_2\ v_2), (\mathbf{Having}\ u_3\ x_2\ x_6)$$

It translates like this (assume unbound elem pred variables have low existential scope):

$(\mathbf{M}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{f}_s\ x_5){=}x_2, (\mathbf{f}_s\ x_8){=}x_2, (\mathbf{F}\ v_4\ v_5\ x_5), (\mathbf{A}\ v_7\ v_8\ x_8), (\mathbf{f}_r\ x_5){=}v_5, (\mathbf{f}_r\ x_8){=}v_8, (\mathbf{f}_c\ x_5){=}x_6, (\mathbf{f}_c\ x_8){=}x_6, (\mathbf{W}\ u_5\ v_5), (\mathbf{E}\ u_8\ v_8), (\mathbf{f}_c\ v_5){=}v_6, (\mathbf{f}_c\ v_8){=}v_6, (\mathbf{C}\ u_2\ v_2), (\mathbf{H}\ u_3\ x_2\ x_6)\ ;$

$P\ (\mathbf{M}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{f}_s\ x_5){=}x_2, (\mathbf{f}_s\ x_8){=}x_2, (\mathbf{F}\ v_4\ v_5\ x_5), (\mathbf{A}\ v_7\ v_8\ x_8), (\mathbf{f}_r\ x_5){=}v_5, (\mathbf{f}_r\ x_8){=}v_8, (\mathbf{f}_c\ x_5){=}x_6, (\mathbf{f}_c\ x_8){=}x_6, (\mathbf{W}\ u_5\ v_5), (\mathbf{E}\ u_8\ v_8), (\mathbf{f}_c\ v_5){=}v_6, (\mathbf{f}_c\ v_8){=}v_6, (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6)\ ;$

$I1\ (\mathbf{M}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{f}_s\ x_5){=}x_2, (\mathbf{f}_s\ x_8){=}x_2, (\mathbf{F}\ v_4\ v_5\ x_5), (\mathbf{A}\ v_7\ v_8\ x_8), (\mathbf{f}_r\ x_5){=}v_5, (\mathbf{f}_r\ x_8){=}v_8, (\mathbf{f}_c\ x_5){=}x_6, (\mathbf{f}_c\ x_8){=}x_6, (\mathbf{W}\ u_5\ v_5), (\mathbf{E}\ u_8\ v_8), (\mathbf{f}_c\ v_5){=}v_6, (\mathbf{f}_c\ v_8){=}v_6, (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)\ ;$

$M\ (\mathbf{M}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{f}_s\ x_5){=}x_2, (\mathbf{f}_s\ x_8){=}x_2, (\mathbf{F}\ v_4\ v_5\ x_5), (\mathbf{A}\ v_7\ v_8\ x_8), (\mathbf{f}_r\ x_5){=}v_5, (\mathbf{f}_r\ x_8){=}v_8, (\mathbf{f}_c\ x_5){=}x_6, (\mathbf{f}_c\ x_8){=}x_6, (\mathbf{W}\ u_5\ v_5), (\mathbf{E}\ u_8\ v_8), (\mathbf{f}_c\ v_5){=}v_6, (\mathbf{f}_c\ v_8){=}v_6\ ;\ (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

$I2\ (\mathbf{M}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{f}_s\ x_5){=}x_2, (\mathbf{f}_s\ x_8){=}x_2, (\mathbf{F}\ v_4\ v_5\ x_5), (\mathbf{A}\ v_7\ v_8\ x_8), (\mathbf{f}_r\ x_5){=}v_5, (\mathbf{f}_r\ x_8){=}v_8, (\mathbf{f}_c\ x_5){=}x_6, (\mathbf{f}_c\ x_8){=}x_6, (\mathbf{W}\ u_5\ v_5), (\mathbf{E}\ u_8\ v_8), (\lambda_{v_5}\ \top), (\lambda_{v_8}\ \top)\ ;\ (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

$P\ (\mathbf{M}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{f}_s\ x_5){=}x_2, (\mathbf{f}_s\ x_8){=}x_2, (\mathbf{F}\ v_4\ v_5\ x_5), (\mathbf{A}\ v_7\ v_8\ x_8), (\mathbf{f}_r\ x_5){=}v_5, (\mathbf{f}_r\ x_8){=}v_8, (\mathbf{f}_c\ x_5){=}x_6, (\mathbf{f}_c\ x_8){=}x_6, (\lambda_{v_5}\ \mathbf{W}\ u_5\ v_5), (\lambda_{v_8}\ \mathbf{E}\ u_8\ v_8), (\lambda_{v_5}\ \top), (\lambda_{v_8}\ \top)\ ;\ (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

$C\ (\mathbf{M}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{f}_s\ x_5){=}x_2, (\mathbf{f}_s\ x_8){=}x_2, (\mathbf{F}\ v_4\ v_5\ x_5), (\mathbf{A}\ v_7\ v_8\ x_8), (\mathbf{f}_r\ x_5){=}v_5, (\mathbf{f}_r\ x_8){=}v_8, (\mathbf{f}_c\ x_5){=}x_6, (\mathbf{f}_c\ x_8){=}x_6, (\lambda_{v_5}\ \mathbf{W}\ u_5\ v_5), (\lambda_{v_8}\ \mathbf{E}\ u_8\ v_8)\ ;\ (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

$M\ (\mathbf{M}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{f}_s\ x_5){=}x_2, (\mathbf{f}_s\ x_8){=}x_2, (\mathbf{F}\ v_4\ v_5\ x_5), (\mathbf{A}\ v_7\ v_8\ x_8), (\mathbf{f}_r\ x_5){=}v_5, (\mathbf{f}_r\ x_8){=}v_8, (\mathbf{f}_c\ x_5){=}x_6, (\mathbf{f}_c\ x_8){=}x_6\ ;\ (\lambda_{v_5}\ \mathbf{W}\ u_5\ v_5), (\lambda_{v_8}\ \mathbf{E}\ u_8\ v_8), (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

$I2\ (\mathbf{M}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{f}_s\ x_5){=}x_2, (\mathbf{f}_s\ x_8){=}x_2, (\mathbf{F}\ v_4\ v_5\ x_5), (\mathbf{A}\ v_7\ v_8\ x_8), (\lambda_{x_5}\ \mathbf{W}\ u_5\ v_5), (\lambda_{x_8}\ \mathbf{E}\ u_8\ v_8), (\lambda_{x_5}\ \mathbf{H}\ u_3\ x_2\ x_5), (\lambda_{x_8}\ \mathbf{H}\ u_3\ x_2\ x_8)\ ;\ (\lambda_{v_5}\ \mathbf{W}\ u_5\ v_5), (\lambda_{v_8}\ \mathbf{E}\ u_8\ v_8), (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

$C\ (\mathbf{M}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{f}_s\ x_5){=}x_2, (\mathbf{f}_s\ x_8){=}x_2, (\mathbf{F}\ v_4\ v_5\ x_5), (\mathbf{A}\ v_7\ v_8\ x_8), (\lambda_{x_5}\ \mathbf{W}\ u_5\ x_5 \wedge \mathbf{H}\ u_3\ x_2\ x_5), (\lambda_{x_8}\ \mathbf{E}\ u_8\ x_8 \wedge \mathbf{H}\ u_3\ x_2\ x_8)\ ;\ (\lambda_{v_5}\ \mathbf{W}\ u_5\ v_5), (\lambda_{v_8}\ \mathbf{E}\ u_8\ v_8), (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

$M\ (\mathbf{M}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{f}_s\ x_5){=}x_2, (\mathbf{f}_s\ x_8){=}x_2, (\mathbf{F}\ v_4\ v_5\ x_5), (\mathbf{A}\ v_7\ v_8\ x_8)\ ;\ (\lambda_{x_5}\ \mathbf{W}\ u_5\ x_5 \wedge \mathbf{H}\ u_3\ x_2\ x_5), (\lambda_{x_8}\ \mathbf{E}\ u_8\ x_8 \wedge \mathbf{H}\ u_3\ x_2\ x_8), (\lambda_{v_5}\ \mathbf{W}\ u_5\ v_5), (\lambda_{v_8}\ \mathbf{E}\ u_8\ v_8), (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

$Q\ (\mathbf{M}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ (\lambda_{v_5}\ \mathbf{W}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{W}\ u_5\ x_5 \wedge \mathbf{H}\ u_3\ x_2\ x_5)), (\mathbf{A}\ (\lambda_{v_8}\ \mathbf{E}\ u_8\ v_8)\ (\lambda_{x_8}\ \mathbf{E}\ u_8\ x_8 \wedge \mathbf{H}\ u_3\ x_2\ x_8))\ ;\ (\lambda_{x_5}\ ...), (\lambda_{x_8}\ ...), (\lambda_{v_5}\ ...), (\lambda_{v_8}\ ...), (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

$S1\ (\mathbf{M}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\lambda_{x_2}\ \mathbf{F}\ (\lambda_{v_5}\ \mathbf{W}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{W}\ u_5\ x_5 \wedge \mathbf{H}\ u_3\ x_2\ x_5)), (\lambda_{x_2}\ \mathbf{A}\ (\lambda_{v_8}\ \mathbf{E}\ u_8\ v_8)\ (\lambda_{x_8}\ \mathbf{E}\ u_8\ x_8 \wedge \mathbf{H}\ u_3\ x_2\ x_8))\ ;\ (\lambda_{x_5}\ ...), (\lambda_{x_8}\ ...), (\lambda_{v_5}\ ...), (\lambda_{v_8}\ ...), (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

$I2\ (\mathbf{M}\ v_1\ v_2\ x_2), (\lambda_{x_2}\ \mathbf{C}\ u_2\ x_2), (\lambda_{x_2}\ \mathbf{F}\ (\lambda_{v_5}\ \mathbf{W}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{W}\ u_5\ x_5 \wedge \mathbf{H}\ u_3\ x_2\ x_5)), (\lambda_{x_2}\ \mathbf{A}\ (\lambda_{v_8}\ \mathbf{E}\ u_8\ v_8)\ (\lambda_{x_8}\ \mathbf{E}\ u_8\ x_8 \wedge \mathbf{H}\ u_3\ x_2\ x_8))\ ;\ (\lambda_{x_5}\ ...), (\lambda_{x_8}\ ...), (\lambda_{v_5}\ ...), (\lambda_{v_8}\ ...), (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

$C\ (\mathbf{M}\ v_1\ v_2\ x_2), (\lambda_{x_2}\ \mathbf{C}\ u_2\ x_2 \wedge \mathbf{F}\ (\lambda_{v_5}\ \mathbf{W}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{W}\ u_5\ x_5 \wedge \mathbf{H}\ u_3\ x_2\ x_5) \wedge \mathbf{A}\ (\lambda_{v_8}\ \mathbf{E}\ u_8\ v_8)\ (\lambda_{x_8}\ \mathbf{E}\ u_8\ x_8 \wedge \mathbf{H}\ u_3\ x_2\ x_8))\ ;\ (\lambda_{x_5}\ ...), (\lambda_{x_8}\ ...), (\lambda_{v_5}\ ...), (\lambda_{v_8}\ ...), (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

$M\ (\mathbf{M}\ v_1\ v_2\ x_2)\ ;\ (\lambda_{x_2}\ \mathbf{C}\ u_2\ x_2 \wedge \mathbf{F}\ (\lambda_{v_5}\ \mathbf{W}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{W}\ u_5\ x_5 \wedge \mathbf{H}\ u_3\ x_2\ x_5) \wedge \mathbf{A}\ (\lambda_{v_8}\ \mathbf{E}\ u_8\ v_8)\ (\lambda_{x_8}\ \mathbf{E}\ u_8\ x_8 \wedge \mathbf{H}\ u_3\ x_2\ x_8)), (\lambda_{x_5}\ ...), (\lambda_{x_8}\ ...), (\lambda_{v_5}\ ...), (\lambda_{v_8}\ ...), (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

$Q\ (\mathbf{M}\ (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2)\ (\lambda_{x_2}\ \mathbf{C}\ u_2\ x_2 \wedge \mathbf{F}\ (\lambda_{v_5}\ \mathbf{W}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{W}\ u_5\ x_5 \wedge \mathbf{H}\ u_3\ x_2\ x_5) \wedge \mathbf{A}\ (\lambda_{v_8}\ \mathbf{E}\ u_8\ v_8)\ (\lambda_{x_8}\ \mathbf{E}\ u_8\ x_8 \wedge \mathbf{H}\ u_3\ x_2\ x_8)))\ ;\ (\lambda_{x_2}\ ...), (\lambda_{x_5}\ ...), (\lambda_{x_8}\ ...), (\lambda_{v_5}\ ...), (\lambda_{v_8}\ ...), (\lambda_{v_2}\ \mathbf{C}\ u_2\ v_2), (\lambda_{x_6}\ \mathbf{H}\ u_3\ x_2\ x_6), (\lambda_{v_6}\ \top)$

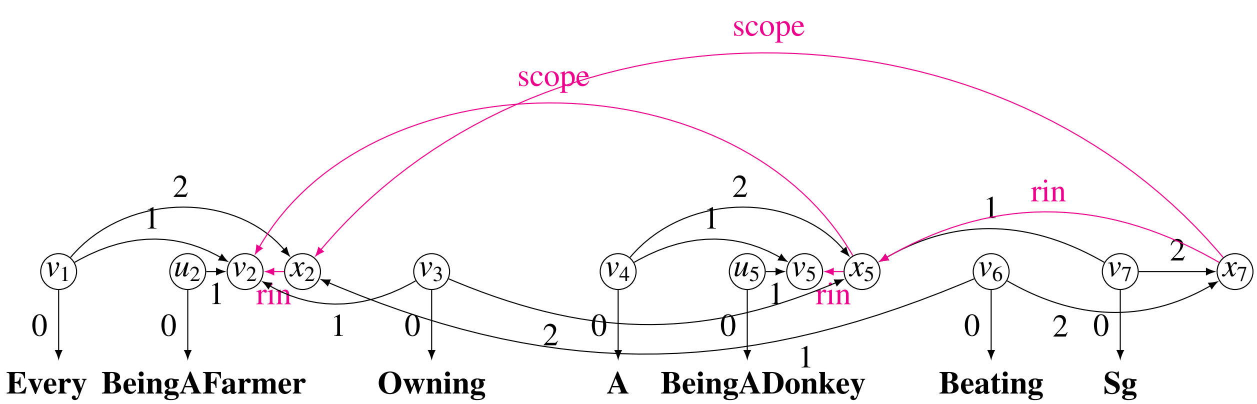
The result, with existential quantifiers over elem preds filled in (note inheritances now explicit):

$$\begin{aligned}
&(\mathbf{Most}\ (\lambda_{v_2}\ \mathbf{Some}\ (\lambda_{u_2}\ \mathbf{BeingACar}\ u_2\ v_2))\\
&\quad (\lambda_{x_2}\ \mathbf{Some}\ (\lambda_{u_2}\ \mathbf{BeingACar}\ u_2\ x_2) \wedge \mathbf{Four}\ (\lambda_{v_5}\ \mathbf{Some}\ (\lambda_{u_5}\ \mathbf{BeingAWheel}\ u_5\ v_5))\\
&\qquad\qquad (\lambda_{x_5}\ \mathbf{Some}\ (\lambda_{u_5}\ \mathbf{BeingAWheel}\ u_5\ x_5) \wedge \mathbf{Some}\ (\lambda_{u_3}\ \mathbf{Having}\ u_3\ x_2\ x_5))\\
&\qquad \wedge \mathbf{An}\ \ (\lambda_{v_8}\ \mathbf{Some}\ (\lambda_{u_8}\ \mathbf{BeingAnEngine}\ u_8\ v_8))\\
&\qquad\qquad (\lambda_{x_8}\ \mathbf{Some}\ (\lambda_{u_8}\ \mathbf{BeingAnEngine}\ u_8\ x_8) \wedge \mathbf{Some}\ (\lambda_{u_3}\ \mathbf{Having}\ u_3\ x_2\ x_8))))
\end{aligned}$$

### 3.10 Donkey sentences (Kamp, 1981)

The above translation also supports ‘donkey sentences.’

E.g. *Every farmer who owns a donkey beats it*:




This consists of the following elementary predications and scope dependencies:

$(\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\mathbf{f}_s\ x_7){=}x_2, (\mathbf{S}\ v_7\ x_5\ x_7), (\mathbf{B}\ v_6\ x_2\ x_7), (\mathbf{f}_r\ x_7){=}x_5, (\mathbf{O}\ v_3\ v_2\ x_5), (\mathbf{f}_r\ x_5){=}v_5, (\mathbf{D}\ u_5\ v_5)$

It translates like this (assume unbound eventuality variables have low existential scope):

$(\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\mathbf{f}_s\ x_7){=}x_2, (\mathbf{S}\ v_7\ x_5\ x_7), (\mathbf{B}\ v_6\ x_2\ x_7), (\mathbf{f}_r\ x_7){=}x_5, (\mathbf{O}\ v_3\ v_2\ x_5), (\mathbf{f}_r\ x_5){=}v_5, (\mathbf{D}\ u_5\ v_5)\ ;$

$P\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\mathbf{f}_s\ x_7){=}x_2, (\mathbf{S}\ v_7\ x_5\ x_7), (\mathbf{B}\ v_6\ x_2\ x_7), (\mathbf{f}_r\ x_7){=}x_5, (\mathbf{O}\ v_3\ v_2\ x_5), (\mathbf{f}_r\ x_5){=}v_5, (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)\ ;$

$M\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\mathbf{f}_s\ x_7){=}x_2, (\mathbf{S}\ v_7\ x_5\ x_7), (\mathbf{B}\ v_6\ x_2\ x_7), (\mathbf{f}_r\ x_7){=}x_5, (\mathbf{O}\ v_3\ v_2\ x_5), (\mathbf{f}_r\ x_5){=}v_5\ ;\ (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)$

$I2\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\mathbf{f}_s\ x_7){=}x_2, (\mathbf{S}\ v_7\ x_5\ x_7), (\mathbf{B}\ v_6\ x_2\ x_7), (\mathbf{f}_r\ x_7){=}x_5, (\mathbf{O}\ v_3\ v_2\ x_5), (\lambda_{x_5}\ \mathbf{D}\ u_5\ x_5)\ ;\ (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)$

$P\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\mathbf{f}_s\ x_7){=}x_2, (\mathbf{S}\ v_7\ x_5\ x_7), (\mathbf{B}\ v_6\ x_2\ x_7), (\mathbf{f}_r\ x_7){=}x_5, (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5), (\lambda_{x_5}\ \mathbf{D}\ u_5\ x_5)\ ;\ (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)$

$C\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\mathbf{f}_s\ x_7){=}x_2, (\mathbf{S}\ v_7\ x_5\ x_7), (\mathbf{B}\ v_6\ x_2\ x_7), (\mathbf{f}_r\ x_7){=}x_5, (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5)\ ;\ (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)$

$M\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\mathbf{f}_s\ x_7){=}x_2, (\mathbf{S}\ v_7\ x_5\ x_7), (\mathbf{B}\ v_6\ x_2\ x_7), (\mathbf{f}_r\ x_7){=}x_5\ ;\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5), (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)$

$I4\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\mathbf{f}_s\ x_7){=}x_2, (\mathbf{S}\ v_7\ x_5\ x_7), (\mathbf{B}\ v_6\ x_2\ x_7), (\lambda_{x_7}\ \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7)\ ;\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5), (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)$

$P\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\mathbf{f}_s\ x_7){=}x_2, (\mathbf{S}\ v_7\ x_5\ x_7), (\lambda_{x_7}\ \mathbf{B}\ v_6\ x_2\ x_7), (\lambda_{x_7}\ \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7)\ ;\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5), (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)$

$C\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\mathbf{f}_s\ x_7){=}x_2, (\mathbf{S}\ v_7\ x_5\ x_7), (\lambda_{x_7}\ \mathbf{B}\ v_6\ x_2\ x_7 \wedge \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7)\ ;\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5), (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)$

$M\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\mathbf{f}_s\ x_7){=}x_2, (\mathbf{S}\ v_7\ x_5\ x_7)\ ;\ (\lambda_{x_7}\ \mathbf{B}\ v_6\ x_2\ x_7 \wedge \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7), (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5), (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)$

$Q\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\mathbf{f}_s\ x_7){=}x_2, (\mathbf{S}\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_7}\ \mathbf{B}\ v_6\ x_2\ x_7 \wedge \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7))\ ;\ (\lambda_{x_7}\ ...), (\lambda_{x_5}\ ...), (\lambda_{v_5}\ ...)$

$S2\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ v_4\ v_5\ x_5), (\lambda_{x_2}\ \mathbf{S}\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ x_2\ x_5 \wedge \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_7}\ \mathbf{B}\ v_6\ x_2\ x_7 \wedge \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7))\ ;\ (\lambda_{x_7}\ ...), (\lambda_{x_5}\ ...), (\lambda_{v_5}\ ...)$

$Q\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\mathbf{f}_s\ x_5){=}v_2, (\mathbf{A}\ (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5)), (\lambda_{x_2}\ \mathbf{S}\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ x_2\ x_5 \wedge \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_7}\ \mathbf{B}\ v_6\ x_2\ x_7 \wedge \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7))\ ;\ (\lambda_{x_7}\ ...), (\lambda_{x_5}\ ...), (\lambda_{v_5}\ ...)$

$S1\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\mathbf{F}\ u_2\ v_2), (\lambda_{v_2}\ \mathbf{A}\ (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5)), (\lambda_{x_2}\ \mathbf{S}\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ x_2\ x_5 \wedge \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_7}\ \mathbf{B}\ v_6\ x_2\ x_7 \wedge \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7))\ ;\ (\lambda_{x_7}\ ...), (\lambda_{x_5}\ ...), (\lambda_{v_5}\ ...)$

$P\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\lambda_{v_2}\ \mathbf{F}\ u_2\ v_2), (\lambda_{v_2}\ \mathbf{A}\ (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5)), (\lambda_{x_2}\ \mathbf{S}\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ x_2\ x_5 \wedge \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_7}\ \mathbf{B}\ v_6\ x_2\ x_7 \wedge \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7))\ ;\ (\lambda_{x_7}\ ...), (\lambda_{x_5}\ ...), (\lambda_{v_5}\ ...)$

$C\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\lambda_{v_2}\ \mathbf{F}\ u_2\ v_2 \wedge \mathbf{A}\ (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5)), (\lambda_{x_2}\ \mathbf{S}\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ x_2\ x_5 \wedge \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_7}\ \mathbf{B}\ v_6\ x_2\ x_7 \wedge \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7))\ ;\ (\lambda_{x_7}\ ...), (\lambda_{x_5}\ ...), (\lambda_{v_5}\ ...)$

$M\ (\mathbf{E}\ v_1\ v_2\ x_2), (\mathbf{f}_r\ x_2){=}v_2, (\lambda_{x_2}\ \mathbf{S}\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ x_2\ x_5 \wedge \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_7}\ \mathbf{B}\ v_6\ x_2\ x_7 \wedge \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7))\ ;\ (\lambda_{v_2}\ \mathbf{F}\ u_2\ v_2 \wedge \mathbf{A}\ (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5)), (\lambda_{x_7}\ ...), (\lambda_{x_5}\ ...), (\lambda_{v_5}\ ...)$

$I2\ (\mathbf{E}\ v_1\ v_2\ x_2), (\lambda_{x_2}\ \mathbf{F}\ u_2\ v_2 \wedge ... \wedge \mathbf{S}\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ x_2\ x_5 \wedge \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_7}\ \mathbf{B}\ v_6\ x_2\ x_7 \wedge \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7))\ ;\ (\lambda_{v_2}\ \mathbf{F}\ u_2\ v_2 \wedge \mathbf{A}\ (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5)), (\lambda_{x_7}\ ...), (\lambda_{x_5}\ ...), (\lambda_{v_5}\ ...)$

$M\ (\mathbf{E}\ v_1\ v_2\ x_2)\ ;\ (\lambda_{x_2}\ \mathbf{F}\ u_2\ v_2 \wedge ... \wedge \mathbf{S}\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ x_2\ x_5 \wedge \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_7}\ \mathbf{B}\ v_6\ x_2\ x_7 \wedge \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7)), (\lambda_{v_2}\ \mathbf{F}\ u_2\ v_2 \wedge \mathbf{A}\ (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5)), (\lambda_{x_7}\ ...), (\lambda_{x_5}\ ...), (\lambda_{v_5}\ ...)$

$Q\ (\mathbf{E}\ (\lambda_{v_2}\ \mathbf{F}\ u_2\ v_2 \wedge \mathbf{A}\ (\lambda_{v_5}\ \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ v_2\ x_5 \wedge \mathbf{D}\ u_5\ x_5))\ (\lambda_{x_2}\ \mathbf{F}\ u_2\ v_2 \wedge ... \wedge \mathbf{S}\ (\lambda_{x_5}\ \mathbf{O}\ v_3\ x_2\ x_5 \wedge \mathbf{D}\ u_5\ v_5)\ (\lambda_{x_7}\ \mathbf{B}\ v_6\ x_2\ x_7 \wedge \mathbf{O}\ v_3\ x_2\ x_7 \wedge \mathbf{D}\ u_5\ x_7)))\ ;\ (\lambda_{x_2}\ ...), (\lambda_{v_2}\ ...), (\lambda_{x_7}\ ...), (\lambda_{x_5}\ ...), (\lambda_{v_5}\ ...)$

Not satisfied when farmers own multiple donkeys because singular pronoun unsatisfied.

Generates ‘strong’ reading with plural (‘Every farmer that owns some donkeys beats them’).

### 3.11 Summation anaphora (Kamp, 1981)

7. Add union of conjunct constraints to complete lambda terms in $\Delta$:

$$\frac{\Gamma, (\mathbf{f}_{\text{uin}}\ v){=}u\ ;\ \Delta}{\Gamma, (\mathbf{f}_{\text{cin}}\ v){=}u, (\mathbf{f}_{\text{union}}\ v){=}u\ ;\ \Delta} \tag{U1}$$

$$\frac{\Gamma, (\mathbf{f}_{\text{union}}\ v){=}u\ ;\ (\lambda_v\ \psi), \Delta}{\Gamma, (\lambda_u\ (\lambda_v\ \psi)\ u)\ ;\ (\lambda_v\ \psi), \Delta}\ (\lambda_u\ \ldots) \notin \Gamma \tag{U2}$$

$$\frac{\Gamma, (\lambda_u\ \phi), (\mathbf{f}_{\text{union}}\ v){=}u\ ;\ (\lambda_v\ \psi), \Delta}{\Gamma, (\lambda_u\ \phi \vee (\lambda_v\ \psi)\ u)\ ;\ (\lambda_v\ \psi), \Delta} \tag{U3}$$

$$\frac{\Gamma, (\lambda_u\ \phi)\ ;\ (\lambda_u\ \psi), \Delta}{\Gamma\ ;\ (\lambda_u\ \phi \wedge \psi), \Delta}\ \forall_v\ (\mathbf{f}_{\text{union}}\ v){=}u \notin \Gamma \tag{U4}$$

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