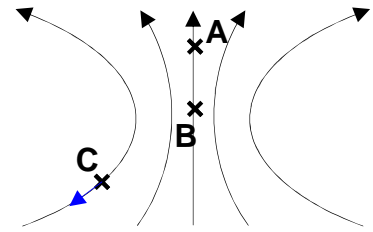


Problem 1 [12 points]. Field lines in a region of space are shown in the figure. Three locations A,B,C are indicated, as well.



(a) The location with the lowest electric potential is (circle one):

☒ A ☐ B ☐ C Can't be determined

Potential decreases as you go down a field line.

(b) An electron would have the highest potential energy at (circle one):

☒ A ☐ B ☐ C Can't be determined

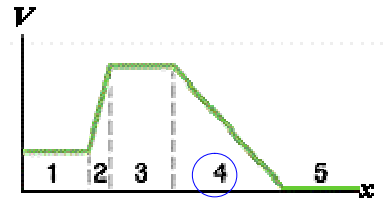
$U = qV$. Since the electron charge is negative, the region of lowest potential yields the highest potential energy.

(c) Suppose I place an electron at C. Clearly indicate the direction of the force on it on the figure.

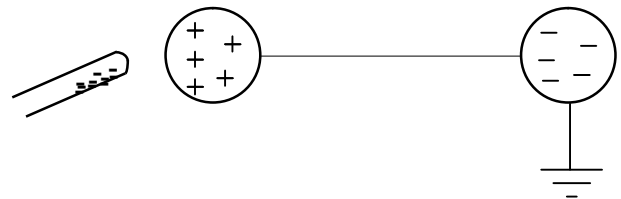
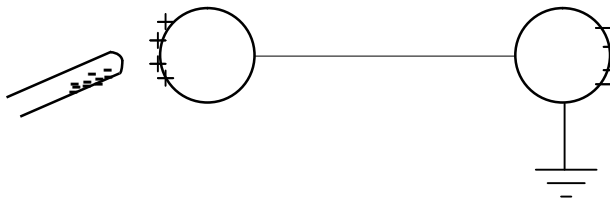
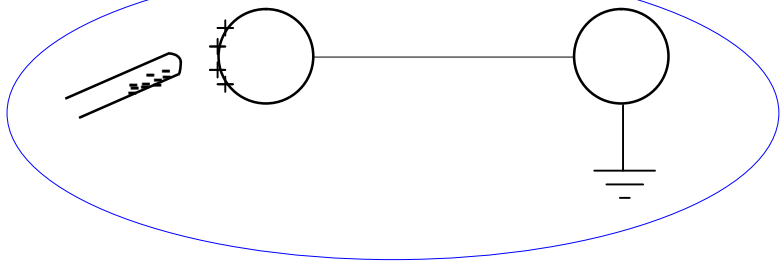
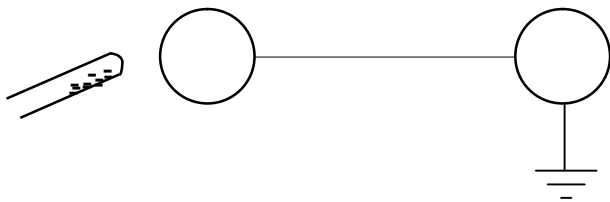
For electrons, the force is opposite to the electric field.

Problem 2 [4 points]. In which region (1-5) would a proton experience a force in the +x direction?

$E_x = -\frac{\partial V}{\partial x}$, so the slope must be negative for the field direction to be positive.



Problem 3 [4 points]. A charged rod and two conducting balls connected by a long wire are placed as shown in cross-section. Circle the figure that best represents the charge distribution on the balls.



Problem 4 [4 points]. Two small, conducting balls are brought into contact and then separated slightly. The electric force between them could be (circle every possible answer): attractive ☒ zero ☒ repulsive.

Problem 5 [28 points]. A long, thick, cylindrical shell of positive charge is shown in cross-section in the figure. It has inner radius, A , and outer radius, B . Cylindrical shells can be described using the volume charge density, ρ , or the linear charge density, λ . You may use either of these or both for parts a-c.

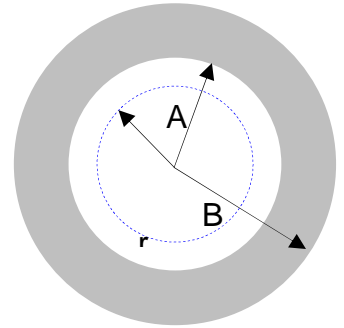
(a) What is the magnitude of the electric field for $r < A$?

Gaussian surface: cylinder of radius r , length L , as usual.

$$\Phi = E(2\pi rL) = q_{enc}/\epsilon_0$$

$$E = 1/(2\pi\epsilon_0) q_{enc}/rL$$

$$q_{enc} = 0 \rightarrow E = 0 \quad \checkmark$$



(b) What is the magnitude of the electric field for $A < r < B$?

$$q_{enc} = \rho (\pi r^2 L - \pi A^2 L)$$

$$E = 1/(2\pi\epsilon_0) \rho (\pi r^2 - \pi A^2) / r \quad \checkmark$$

(c) What is the magnitude of the electric field for $r > B$?

$$q_{enc} = \lambda L$$

$$E = 1/(2\pi\epsilon_0) \lambda / r \quad \checkmark$$

(d) Find the linear charge density, λ , in terms of the volume charge density, ρ . [Hint: Imagine a section of the cylindrical shell of length L . How much charge is in this section?]

Consider a length L of the cylinder. The charge q in this section is:

$$q = \lambda L$$

but also:

$$q = \rho (\pi B^2 L - \pi A^2 L)$$

Equating these:

$$\lambda = \rho (\pi B^2 - \pi A^2) \quad \checkmark$$

Name: KEY

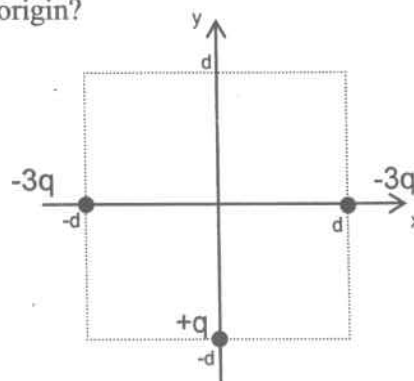
Recitation Instructor: Cochran Coburn Hupe Salzwedel

Problem 5 [25 points]. In the figure, the $-3q$ charges can't move but the $+q$ charge can. The $+q$ charge, mass m , is initially at rest. You may solve this problem symbolically, if you wish. If you'd like to use numbers, use: $q = 400 \mu\text{C}$, $m = 3.0 \text{ g}$, $d = 2.3 \text{ mm}$

15 (a) What is the change in kinetic energy of the $+q$ charge when it reaches the origin?

5 (b) How fast is it moving when it reaches the origin?

5 (c) How fast is it moving when it is at $(x,y) = (0,d)$?



$$a) \Delta KE + \Delta U = 0 \rightarrow \Delta KE = -\Delta U$$

$$U = qV \rightarrow \Delta U = q\Delta V$$

$$\Delta V = V_f - V_i \quad V = \frac{kq}{r}$$

$$V_i = V_{1i} + V_{2i} = \frac{-3kq}{\sqrt{2}d} + \frac{-3kq}{\sqrt{2}d} = \frac{-6kq}{\sqrt{2}d} = -6.64 \times 10^9 \text{ V}$$

$$V_f = V_{1f} + V_{2f} = \frac{-3kq}{d} + \frac{-3kq}{d} = \frac{-6kq}{d} = -9.36 \times 10^9 \text{ V}$$

$$\rightarrow \Delta V = \frac{-6kq}{d} \left(1 - \frac{1}{\sqrt{2}}\right) = -1.76 \frac{kq}{d} = -2.75 \times 10^9 \text{ V}$$

$$\rightarrow \Delta KE = -q\Delta V = \frac{6kq^2}{d} \left(1 - \frac{1}{\sqrt{2}}\right) = 1.76 \frac{kq^2}{d} = 1.1 \times 10^6 \text{ J}$$

$$b) KE = \frac{1}{2}mv^2 \rightarrow v = \left(\frac{2(KE)}{m}\right)^{1/2} = \left[\frac{6kq^2}{d} \left(1 - \frac{1}{\sqrt{2}}\right)\right]^{1/2} \\ = \left(3.52 \frac{kq^2}{md}\right)^{1/2} = 2.71 \times 10^4 \text{ m/s}$$

$$c) U_f = U_i \rightarrow KE = 0 \rightarrow v = 0$$

Problem 7 [23 points]. The figure shows two thin, large, non-conducting sheets of charge in cross-section. The electric field between them has magnitude $8.0 \times 10^6 \text{ V/m}$.

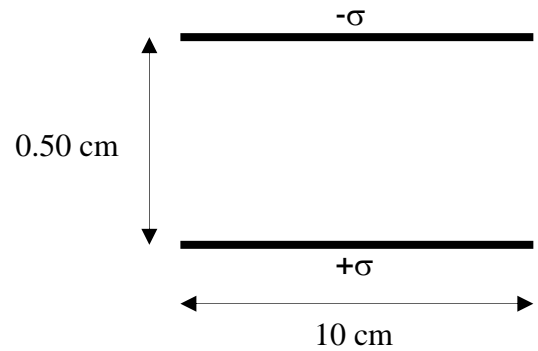
(a) What is the charge density, σ ?

Each sheet contributes $E_{\text{sheet}} = \sigma/2\epsilon_0$ (upward).

The total field is: $E = 2 E_{\text{sheet}} = \sigma/\epsilon_0$.

$$\sigma = E \epsilon_0$$

$$\sigma = (8.0 \times 10^6 \text{ V/m})(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2) = 71 \mu\text{C/m}^2 \quad \checkmark$$



(b) A 5.0 g point mass is added to the above setup on the left, moving to the right, as shown. After a time of $t = 0.010 \text{ s}$, the particle arrives at the "x". What was the charge of the particle?

Neglect fringe fields and gravity. Some kinematic equations are provided that may or may not be useful.

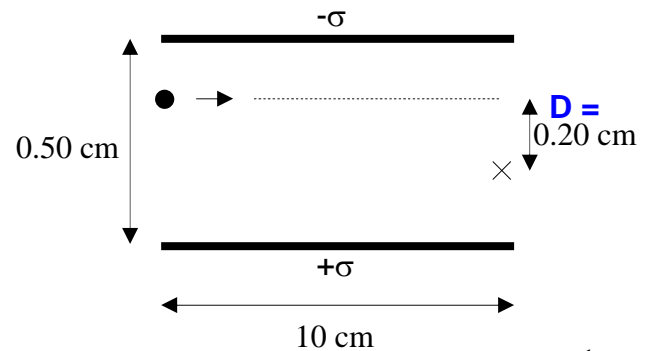
Let the y-axis point upwards.

$$a_y = 2D/t^2 = 2(-0.0020\text{m})/(0.010\text{s})^2 = -40 \text{ m/s}^2$$

$$F_y = ma_y = -0.20 \text{ N}$$

$$F_y = qE_y \text{ so,}$$

$$q = F_y/E_y = (-0.20 \text{ N})/(8.0 \times 10^6 \text{ V/m}) = -2.5 \times 10^{-8} \text{ C} \quad \checkmark$$



$$x = x_o + v_o t + \frac{1}{2} a t^2$$

$$v = v_o + a t$$

$$v^2 - v_o^2 = 2a(x - x_o)$$