

(1) The continuity of phase.

(a) To get started, consider this relation involving only a single, independent variable, x :

$$Ae^{iax} + Be^{ibx} = Ce^{icx}$$

with A, B, C, a, b, c real. Given this relation, show that $a = b = c$.

(b) We made an argument on physical grounds that the incident, reflected, and transmitted EM waves must all have the same phase at the interface between two media given only that linear boundary conditions exist of some kind. Here you'll show this explicitly.

Suppose we had simply applied Maxwell's eqns. without first deriving anything about the phases. Our four boundary conditions applied to the three fields would have yielded different relations, but all of the form:

$$A e^{i(\vec{k}_i \cdot \vec{r} - \omega_i t)} + B e^{i(\vec{k}_r \cdot \vec{r} - \omega_r t)} = C e^{i(\vec{k}_t \cdot \vec{r} - \omega_t t)}$$

for different E and B field components, and valid for any time or position on the interface.

Given a relation of this sort, show that:

$$\omega_i = \omega_r = \omega_t \text{ and } \vec{k}_i \cdot \vec{r} = \vec{k}_r \cdot \vec{r} = \vec{k}_t \cdot \vec{r} .$$

(2) (a) text 3.10.

(b) text 3.11.

(3) text 3.12.

(4) text 3.16.

(5) text 3.36.