Group and Phase Velocity (5.7)

Harmonic wave:

\[ \phi = kx - \omega t = \text{constant identifies a point on the wave} \]

\[ \frac{d\phi}{dt} = 0 \quad \Rightarrow \quad k \frac{dx}{dt} - \omega = 0 \quad \Rightarrow \quad \frac{dx}{dt} = \frac{\omega}{k} \]

\[ \boxed{V = \frac{\omega}{k}} \quad \text{Phase velocity} \]

\[ \boxed{\frac{d\phi}{dx} = \frac{\omega}{k} = f_k} \]

Not observable. For waves similar to plane waves but being observable,

Real observable wave, called a wavepacket, turns on and off can be idealized as a pulse:

which is a superposition of harmonic waves.

Let \( \phi(t) = k(x - \omega t) \) be the

phases of each wave compact. Note \( k = k_0 \).

At the peak:

\[ \frac{d\phi}{dt} = 0 \]

\[ \Rightarrow \quad \frac{dx}{dt} = \frac{\omega}{k} \]

\[ x - \frac{dx}{dt} t = 0 \]

\[ \Rightarrow \quad \frac{x - dx}{dt} = 0 \]

\[ \frac{dx}{dt} = \frac{dx}{dt} \]

\[ \Rightarrow \quad \frac{dx}{dt} = \frac{dx}{dt} \]

This is observable, and well defined so long as the wavepacket isn't changing shape too severely.

This is the speed of every transport: \( \frac{dx}{dt} \leq C \).
\[ \omega = \omega(k) \text{ is called a dispersion relation,} \]

\[ \omega = \nu' k = \frac{c}{\eta} k \text{ is the one we know best.} \]

It's a different one for a waveguide.

\[ \nu' = \frac{\omega}{k} = \frac{c}{\eta} \sqrt{\frac{\nu}{k}} \]

\[ \nu = \frac{d\nu}{dk} = \frac{d}{dk} \nu' k = \nu' + k \frac{d\nu'}{dk} \]

\[ \frac{d\nu'}{dk} = \frac{d}{dk} \frac{c}{\eta} = -\frac{c}{\eta^2} \frac{d\eta}{dk} \]

\[ \frac{\eta}{k} = \nu' - \frac{k c}{\eta^2} \frac{d\eta}{dk} \]

\[ \nu = \nu'_0 \left( 1 - \frac{k c}{\eta^2} \frac{d\eta}{dk} \right) \sqrt{\frac{\nu}{k}} \]

Good for use with Sellmeier eqn.

\[ \frac{c}{\nu + \omega_0^2} \equiv \frac{1}{\eta} \sqrt{\frac{\nu}{k}} \]

In analogy to \( \nu' = \nu'_0 \)

See Ex S.10 for a treatment of conductors.

In vacuum media: \( \frac{\partial\eta}{\partial\omega} > 0 \) so \( \eta > 0 \)

and \( \nu' < \nu'_0 \) by about \( \pm 20\% \).
Michelson Interferometer (Simple) in air

\[
\text{source} \rightarrow \text{output}_1 \left\{ \begin{array}{l}
\text{reflected by } s \rightarrow M_1 \rightarrow \text{trans through } l_1 \\
\text{trans through } l_1 \rightarrow M_2 \rightarrow \text{reflected by } s
\end{array} \right.
\]

Assuming flat plane fronts and ignoring phase shifts from reflections and transmissions:

\[
\Delta S = \frac{2\pi (\text{path length difference in air})}{\lambda} = \frac{2\pi (2l_2 - 2l_1)}{\lambda} = \frac{4\pi \Delta l}{\lambda}
\]

\[
\Delta S = N 2\pi \text{ for fully constructive interference}
\]

\[
\Delta h = N \frac{\lambda}{2}
\]

Conservation of energy:

\[
\rho_{in} \rightarrow \rho_{out}^2
\]

\[\sqrt{\rho_{out}^2} = \rho_{in}\]
In general, the source is converging or diverging.

Model this as:

For output, at \( b_2 = 0 \)

But for \( b_2 > b_1 \)

---

Figure 5.23 Fringe patterns as \( \Delta L \) approaches zero. The patterns below the interferogram are the complements of those above. At \( \Delta L = 0 \), the entire field of one interferometer output is bright, and the other field is dark.

If at \( b = 0 \), you see one of your components isn't flat. This is a good way to measure surfaces!
Figure 5.21  (a) Fringe variation in a Michelson interferometer as the mirror is moved by \( \lambda/4 \). (b) Interferogram. The thick segment corresponds to the fringes in (a).

Figure 5.20  A Michelson interferometer diagram showing the light path through the interferometer setup.

Figure 5.24  (a) Interferogram for white light. (b) Zoom detail of (a). (c) Interferograms for different colors combine to give a white fringe at \( \Delta L = 0 \), and again for lesser values of \( \Delta L \).
Multiple-Beam Interference (S.10)

This configuration is often used at normal incidence so we'll solve for that case to simplify things a little. Usually \( n = n' = n \) so we'll put that in shortly.

Round trip phase difference \( \delta_0 = 2 \pi \frac{nd}{\lambda_0} = k_0 \text{ (mod) \rightarrow text uses } k \text{ here} \)

We can ignore the vector character of the wave.

\[
E_z = E_0 x e^{ik_0 x} + E_0 x e^{ik_0 x} + \ldots
\]

\[
= E_0 x e^{ik_0 x} \left[ 1 + n e^{ik_0 x} + (n e^{ik_0 x})^2 + \ldots \right]
\]

By convention, we usually drop this term.

The text never mentions it.
Now \( |\ln x \, e^{|z|} - 1| < 1 \) so use
\[
\frac{\xi}{n_0} = 1 \times 1 \times 1 \times 1 \ldots = \frac{1}{n \lambda}
\]
\[
E_x = E_0 \, \delta_x \, \frac{1}{1 - n \lambda e^{i \phi}} = E_0 \, \frac{\xi}{n \lambda} \quad \text{for } r = \frac{1}{r}
\]
\[
E_x = E_0 \, \frac{\xi}{1 - n \lambda \lambda e^{i \phi}}
\]
\[
I_x = I_0 \, \frac{(\xi T)}{1 - (n \lambda e^{i \phi} + n \lambda \lambda e^{i \phi}) + (n \lambda \lambda)^2}
\]

At normal incidence \( T = T \times n \lambda = R \)

Let \( r = n_0 e^{i \phi} \) where \( \phi \) = reflection phase shift
\[
r = n_0 e^{i \phi}
\]
\[
R = n_0 e^{i \phi}
\]
\[
\text{For an uncoated glass slab } \phi = 0 \\
\text{However usually multilayer coated}
\]
\[
\text{slohs are used, so we have } \phi \lambda \text{ in.}
\]
\[
I_x = I_0 \, \frac{T}{1 - (n \lambda e^{i \phi} + n \lambda e^{i \phi}) + R^2}
\]
\[
\text{where } S = \frac{n \lambda e^{i \phi} + n \lambda e^{i \phi}}{\lambda_0} + 2 \phi
\]
\[
= I_0 \, \frac{T}{1 - 2 \lambda \cos \phi + R^2}
\]
This is fine but there's a standard form that's always used so work on this a little more:

\[ 1 - 2R \cos \theta \times R^2 = (1 - R)^2 + 2R(1 - \cos \theta) \]

\[ = (1 - R)^2 + 2R \left( 2 \sin^2 \frac{\theta}{2} \right) \]

\[ = (1 - R)^2 \left[ 1 + \frac{4R}{(1 - R)^2} \sin^2 \frac{\theta}{2} \right] \]

Define the Finesse coefficient \( F = \frac{4R}{(1 - R)^2} \)

\[ T_x = T_0 \frac{T}{(1 - R)^2} \frac{1}{1 + F \sin^2 \frac{\theta}{2}} \]

For no absorption: \( T + R = 1 \)  
(This is not always the case, absorbing media are sometimes deliberately introduced for spectroscopic studies.)

\[ T_x = T_0 \frac{1}{1 + F \sin^2 \frac{\theta}{2}} \]
Figure 5.28  (a) Transmittance of two identical flat parallel mirrors separated by distance \( d \). A mirror reflectance of 0.5 gives \( F = 8 \), and \( R = 0.95 \) gives \( F = 1520 \). (b) Standing waves when the mirror spacing \( d \) is equal to an integer number half wavelengths.

- Or think of graph (a) as \( \frac{I}{I_0} \) vs \( f \) \( \rightarrow \) spectroscopy!
- Even if \( R = 0.95 \), \( \frac{I}{I_0} = 1 \) for some \( f \) !!!

\[
I_x = I_0 \text{ when } \sin \frac{\delta}{2} = 0 \text{ on } \frac{z \phi_R}{\lambda_0} = N \pi \quad N \in \mathbb{Z}
\]

\[
\delta = \frac{\lambda_0}{2\pi n} (N\pi - \phi_R)
\]

\[
\delta_{\\text{max}} - \delta_{\\text{min}} = \frac{\lambda_0}{2\pi n} = \frac{\lambda}{2}
\]

This is a resonance effect: resonant cavity

**Note that a field equal to 1 exists & a concept result:**

\[
\rho (1-R) = I_W \quad \rho = \frac{I_W}{0.05} = 20W
\]
Scanning Fabry-Perot Interferometer

The mirrors must be very flat and very parallel.

Vary $d$ by translating one of the mirrors, often using a piezoelectric transducer.

Blue and red

![Graph showing spectral lines](image)

Filtered by

\[ \frac{I_x}{I_0} = \frac{1}{1 \cdot F \sin^2 \frac{d}{2}} = \frac{1}{2} \]

\[ \sin \frac{d/2}{2} = \frac{1}{F} \]

If we're doing spectroscopy, $F$ is large so $\frac{d}{2}$ is small.

\[ \frac{d}{2} = \frac{2}{F} \]

\[ \text{FWHM} = \frac{\lambda}{V F} \]

\[ \text{Finesse} \ F = \frac{2 \pi}{\text{FWHM}} = \pi \frac{V F}{\lambda} \]

Large Finesse yields better resolution.
It can happen that the spectrum to be measured is to bleed:

\[ \frac{2 \pi n \lambda}{\lambda_0} \text{, } \phi_k = N \pi \]

\[ N = \frac{2\pi n \lambda}{\lambda_0} = \frac{2\lambda}{\lambda} \]

For overlap to just occur:

\[(N+1)\lambda = N(\lambda + \Delta \lambda)\]

\[ \Delta \lambda = \frac{\lambda}{N} \]

Free spectral range: \[ \Delta \lambda_{FSR} = \frac{\lambda^2}{2nd} \]

For \( \lambda \), use \( N \) of the spectrum to be analyzed.

Keep spectral width well below \( \Delta \lambda_{FSR} \).

In frequency:

\[ \Delta f_{FM} = \frac{C}{2nd} \]