<u>Output power</u>

Now, $$\frac{d\phi}{dt} = \left(BV_s N - \frac{1}{\tau_c}\right)\phi$$

but, $$\frac{1}{\tau_c} = \frac{\gamma_0 c}{L_{opt}} + \frac{\gamma_1 c}{2L_{opt}} + \frac{\gamma_2 c}{2L_{opt}}$$

Suppose we use mirror #2 to couple out light.

$$\left.\frac{d\phi}{dt}\right|_{\#2} = -\frac{\gamma_2 c}{2L_{opt}}\phi$$

$$\phi = \frac{1}{B\tau}\left(\frac{R_p}{R_{pc}} - 1\right)$$

$$= \left(\frac{V_0 L_{opt}}{\sigma \ell c}\right)\frac{1}{\tau}\left(\frac{R_p}{R_{pc}} - 1\right)$$

$$= \frac{V_0 \gamma}{\sigma \ell}\frac{\tau_c}{\tau}\left(\frac{R_p}{R_{pc}} - 1\right)$$

$$= \frac{A_b \gamma}{\sigma}\frac{\tau_c}{\tau}\left(\frac{R_p}{R_{pc}} - 1\right)$$

$$P_{out} = \frac{\gamma_2 c}{2L_{opt}}\phi(h\nu) \checkmark$$

$$P_{out} = \left(\frac{\gamma_2 c}{2L_{opt}}h\nu\right)\left(\frac{A_b \gamma}{\sigma}\frac{\tau_c}{\tau}\left[\frac{R_p}{R_{pc}} - 1\right]\right)$$

$$= \underbrace{\left(\frac{h\nu}{\sigma\tau}\right)}_{\equiv I_{sat}}\frac{\gamma_2}{2}A_b\underbrace{\frac{c\gamma\tau_c}{L_{opt}}}_{=1}\left[\frac{R_p}{R_{pc}} - 1\right]$$

b/c $\tau_c = \frac{L_{opt}}{c\gamma}$

$$P_{out} = A_b I_{sat}\frac{\gamma_2}{2}\left[\frac{R_p}{R_{pc}} - 1\right]$$

$$\boxed{P_{out} = A_b I_{sat}\frac{\gamma_2}{2}\left[\frac{P_p}{P_{pc}} - 1\right]}$$

The "slope efficiency" is defined as $\eta_s = \frac{dP_{out}}{dP_p}$

$= A_b I_s \frac{\gamma_2}{2} \frac{1}{P_{pc}}$

see p96 Lamp pumping

$$R_p = \eta_p \frac{P_p}{A l h \nu_{mp}} \qquad \nu_{mp} = \frac{E_2 - E_0}{h}$$

Should be ν_p

$P_{pc} = \ldots$

$$R_{pc} = \frac{\gamma}{\sigma l \tau}$$

$R_{pc} \leftrightarrow P_{pc}$

$$P_{pc} = \frac{A \cancel{l} h \nu_{mp}}{\eta_p} \frac{\gamma}{\sigma \cancel{l} \tau} = \frac{\gamma}{\eta_p} \left(\frac{h\nu_{mp}}{\tau}\right) \frac{A}{\sigma}$$

$$\boxed{\eta_s = \eta_p \left(\frac{\gamma_2}{2\gamma}\right) \left(\frac{h\nu}{h\nu_{mp}}\right) \left(\frac{A_b}{A}\right)}$$

% of the gain medium filled

pump efficiency from Ch 6

laser quantum efficiency (some of the energy of the excited state is lost : $E_1 - E_0$.)

output coupling efficiency

$$= \frac{\text{rate of photons output}}{\text{rate of photons lost from cavity}}$$

max value = 1.

$\boxed{\text{Read Example 7.2}}$

also 7.3 on a CO_2 laser

It puts many things together. It discusses Nd:YAG and uses information that we've assemble about Nd:YAG over the course of the class including figures and HW that I've emphasized.

In the example: $\frac{N_t}{} = 4 \cdot 10^{-4}$ N_t = total Nd concentration

Optimum Output Coupling

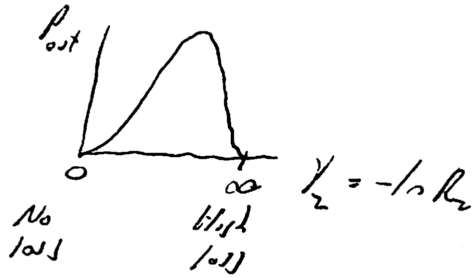
Suppose we only couple out of mirror #2.

$R_2 = 1 \Rightarrow I_{out} = 0$ although Q is a maximum

$R_2 = 0 \quad I_{out} = 0$ because $Q = 0$.

$$P_{out} = A_g I_s \frac{\gamma_2}{2}\left(\frac{P_p}{P_{pc}} - 1\right)$$

we expect



$$\frac{dP_{out}}{d\gamma_2} = 0 \qquad \text{but} \quad P_{pc} = \frac{\gamma}{\eta_p} \frac{h\nu_{np}}{T} \frac{A}{\sigma}$$

depends on γ and thus on γ_2

⋮

Let $P_{mth} \equiv$ threshold for $R_2 = 1$ ("minimum" threshold)

$x_m \equiv \dfrac{P_p}{P_{mth}}$ so this is a measure of how far above this min.

Careful, the laser won't lase unless

$$\boxed{P_p > P_{th} = P_{mth} \frac{\gamma_a + \frac{\gamma_1}{2} + \frac{\gamma_2}{2}}{\gamma_a + \frac{\gamma_1}{2}} \geq 1}$$

$$\boxed{\gamma_{2_{opt}} = (2\gamma_a + \gamma_1)\left(\sqrt{x_m} - 1\right)} \qquad \boxed{\delta_{opt} = \sqrt{x_m} - 1}$$

$$\boxed{P_{out} = A_g I_s \left(\gamma_a + \frac{\gamma_1}{2}\right)\left[\sqrt{x_m} - 1\right]^2} \qquad \boxed{7.5.6}$$

$$P_{th} = \frac{1}{\eta_p}\,\frac{h\nu_{sp}}{\tau}\,\frac{A}{\sigma}\,\gamma \qquad \gamma = \frac{\gamma_1+\gamma_2}{2} + \gamma_i$$

$$\gamma_i = -\ln k_i$$

$$P_{nth} = P_{th}(\gamma_2 = 0) \qquad \gamma_i = -\ln(1-L_i)$$

$$P_{nth} = \frac{P_{th}}{\gamma}\left(\frac{\gamma_1}{2} + \gamma_i\right)$$

$$\boxed{P_{th} = P_{nth}\,\frac{\gamma}{\frac{\gamma_1}{2}+\gamma_i} \qquad (7.5.1)}$$

$$P_{out} = A_b I_s \frac{\gamma_2}{2}\left(\frac{P_p}{P_{nth}}\,\frac{\frac{\gamma_1}{2}+\gamma_i}{\gamma} - 1\right) \qquad x_m \equiv \frac{P_p}{P_{nth}}$$

$$= A_b I_s\left(\gamma_i + \frac{\gamma_1}{2}\right)\underbrace{\frac{\gamma_2/2}{\gamma_i + \frac{\gamma_1}{2}}}_{\equiv S}\left(x_m\,\underbrace{\frac{\gamma_i + \frac{\gamma_1}{2}}{\gamma}}_{=\frac{1}{\frac{\gamma_i+\frac{\gamma_1}{2}+\frac{\gamma_2}{2}}{\gamma_i+\frac{\gamma_1}{2}}} = \frac{1}{S+1}} - 1\right)$$

$$\boxed{P_{out} = \left[A_b I_s\left(\gamma_i + \frac{\gamma_1}{2}\right)\right] S\left(\frac{x_m}{S+1} - 1\right) \qquad (7.5.2)}$$

$$\frac{dP_{out}}{d\gamma_2} = 0 \quad \text{when} \quad \frac{dP_{out}}{dS} = 0$$

$$\frac{dP_{out}}{dS} = [\sim]\left[\left(\frac{x_m}{S+1} - 1\right) + S\left(\frac{-x_m}{(S+1)^2}\right)\right] = 0$$

$$x_m(S+1) - (S+1)^2 - S x_m = 0 \Rightarrow (S+1)^2 = x_m \Rightarrow \boxed{S_{opt} = \sqrt{x_m} - 1 \quad 7.5.5}$$

$$P_{out} = [\sim](\sqrt{x_m} - 1)\left(\frac{x_m}{\sqrt{x_m}} - 1\right)$$

$$\boxed{P_{out_{max}} = \left[A_b I_s\left(\gamma_i + \frac{\gamma_1}{2}\right)\right]\left(\sqrt{x_m} - 1\right)^2 \qquad (7.5.6)}$$

☆ For $P_p = P_{th} \Rightarrow$

$$\gamma_{2opt} = (2\gamma_c + \gamma_1)\left(\sqrt{\frac{\gamma_c + \frac{\gamma_1}{2} + \frac{\gamma_{2opt}}{2}}{\gamma_c + \frac{\gamma_1}{2}}} - 1\right)$$

$$= 2\left[\sqrt{\gamma_c + \frac{\gamma_1}{2} + \frac{\gamma_{2opt}}{2}}\sqrt{\gamma_c + \frac{\gamma_1}{2}} - \gamma_c + \frac{\gamma_1}{2}\right]$$

$$\boxed{\gamma_{2opt} = 0}$$

$$P_{opt} = [w]\left(\sqrt{\frac{\overbrace{\gamma_c + \frac{\gamma_1}{2} + \frac{\gamma_{2opt}}{2}}^{=0}}{\gamma_c + \gamma_1/2}} - 1\right)^2$$

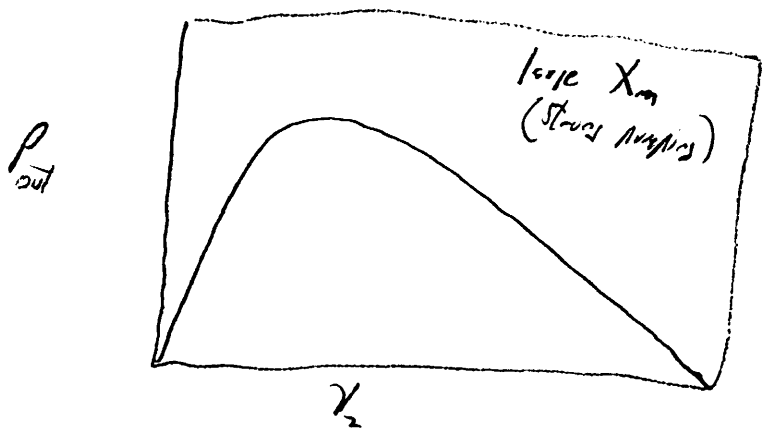
$$\boxed{P_{opt} = 0}$$

☆ The form of $\gamma_{2,opt}$ and P_{opt} is somewhat deceptive. It's written the way it is for calculational convenience.

☆ But we can learn from it.

Near threshold, $\gamma_{2,opt}$ and P_{opt} depend sensitively on the pump. P_{out} depends sensitively on γ_2.

Far from threshold, things are insensitive.



☆ Close to threshold: $X_n \approx 1$

$$X_n = \frac{P_p}{P_{n\,th}} = \frac{P_p}{P_{th}}\left(1 + \overbrace{\frac{\gamma_2/2}{\gamma_e + \frac{\gamma_1}{2}}}^{S}\right)$$

$$\boxed{\text{Let } P_p = P_{th}(1+\Delta)}$$

Note: S must be small for X_n to be close to 1.

$$X_n = (1+\Delta)(1+S) \approx 1 + \Delta + S$$

At optimum coupling:

$$\gamma_{2,opt} = (2\gamma_e + \gamma_1)\left(\sqrt{X_n} - 1\right)$$

$$= (2\gamma_e + \gamma_1)\left[\tfrac{1}{2}(\Delta + S_{opt})\right]$$

$$S_{opt} = \tfrac{1}{2}(\Delta + S_{opt})$$

$$S_{opt} = \Delta$$

$$\boxed{\gamma_{2,opt} = (2\gamma_e + \gamma_1)\Delta}$$

$$\boxed{\begin{aligned} P_{opt} &= [\sim]\tfrac{1}{4}(\Delta + S_{opt})^2 \\ &= \left[A_b I_s\left(\gamma_e + \tfrac{\gamma_1}{2}\right)\right]\Delta^2 \end{aligned}}$$

$\gamma_{2,opt}$ and P_{opt} depend sensitively on pump power.

and

P_{out} depends sensitively on T_2

7.11 Nd:YAG rod, side pumped @ 807 nm by fiber-coupled diode laser.

Rod: D = dia = 4 mm
ℓ = length = 56 mm
z = concentration = 0.9 at. %

TEM 00 mode w/ $w_0 \simeq 1.4$ mm

T_2 = 15%

γ_i = 3.8%

P_p = 370W with P_{pa} = 340 W = optical pump power absorbed

Meant to be a "spatial dependence" problem, but we'll ignore that.

Find: output power and slope efficiency: Get $R_p \to P_{th} \to P_{out}$

$$R_{cp} = \frac{N_c}{\tau} = \frac{\gamma}{\sigma \ell \tau}$$

$$= 3.31 \cdot 10^{20} \ /\mathrm{cm}^3$$

$\boxed{\text{Ex. 7.2}}$ for λ = 1.064 μm: $\sigma = 2.8 \cdot 10^{-19}$ cm^2, τ = 230 μs

$$\gamma = \tfrac{1}{2}\gamma_2 + \gamma_i$$
$$= -\tfrac{1}{2}\ln(1-T_2) + \gamma_i$$
$$= 0.081 + 0.038 = 0.12$$

wall power to light; electrical

$$R_p = \eta_r \eta_t \eta_a \left(\frac{P_p}{h\nu_p}\right)\frac{1}{A\ell}$$

for uniform transverse pumping $\boxed{p223}$

pump injection to medium; optical

$$R_p = \eta_t \eta_a \left(\frac{P_p}{h\nu_p}\right)\frac{1}{A\ell}$$

0.80 (Table 6.3)

0.98 (Table 6.3)

$\eta_t \eta_a = 0.78$

compare

Since we know $\frac{340}{370} = 0.92$

$\eta_t \eta_a = 0.92$

Assumes 1 pump photon yield 1 inversion

$$P_{th} = \frac{R_{pc}}{\eta_t \eta_a} h\nu_p (A\ell) = 62 \ \mathrm{J/s} = 62 \ \mathrm{W}$$

$h\nu_p$: $\frac{hc}{\lambda}$

$= 2.46 \cdot 10^{-19}$ J

$= 1.54$ eV

$A\ell$: $\pi\left(\frac{D}{2}\right)^2 \ell$

$\to 0.704$ cm^3

$$P_{out} = A_b \, I_s \, \frac{\gamma_2}{2}\left(\frac{P_{in}}{P_{th}} - 1\right)$$

$$I_s = \frac{h\nu}{\sigma \tau}$$

$I_s \Rightarrow h\nu = 1.17\,eV$

$= 1.87 \cdot 10^{-19}\,J$

$I_s = 2902\ W/cm^2$

$A_b \Rightarrow \left\{ FWHM = \sqrt{2\ln 2}\, w_c = 1.64\ mm \Rightarrow A = \pi\left(\frac{FWHM}{2}\right)^2 = 0.021\,cm^2 \right\}$

Recall we found

$$P = \frac{\pi w^2}{2} I_p$$

$$A_b = \frac{\pi w_c^2}{2} = 0.031\,cm^2$$

$I_s A_b = 89\,W$

$$P_{out} = (89W)(0.081)\left(\frac{340W}{62W} - 1\right) = 32\,W \checkmark$$

$$\eta_s = A_b I_s \frac{\gamma_2}{2} \frac{1}{P_{th}} = 0.11 = 11\% \checkmark$$

From Ref 36 $\Rightarrow$ $P_{out} = 62\,W$

$\eta_s \approx 25\%$ (I think)

Claims to be the highest reported TEM_{00} mode output for a single laser rod.

[Golla, et. al; Opt Lett 21 210 (1996)]